ANALYSIS OF PERISTALTIC FLOW OF SUTTERBY FLUID WITH VARIABLE LIQUID PROPERTIES IN AN INCLINED CHANNEL

BY

FAIZA ZUBAIR

NATIONAL UNIVERSITY OF MODERN LANGUAGES ISLAMABAD

FEBRUARY 2025

Analysis of Peristaltic Flow of Sutterby Fluid with Variable Liquid Properties in an Inclined Channel

BY
FAIZA ZUBAIR

MS MATH, NATIONAL UNIVERSTY OF MODERN LANGUAGES ISALMABAD 2024

A THESIS SUBMITTED IN PARTIAL FULFILMENT OF
THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF SCIENCE

In **MATHEMATICS**

TO

FACULTY OF ENGINEERING AND COMPUTING

NATIONAL UNIVERSITY OF MODERN LANGUAGES ISLAMABD

NATIONAL UNIUVERSITY OF MODERN LANGUAGES FACULTY OF ENGINEERIING & COMPUTER SCIENCE

THESIS AND DEFENSE APPROVAL FORM

The undersigned certify that they have read the following thesis, examined the defense, are satisfied with overall exam performance, and recommend the thesis to the Faculty of Engineering and Computing for acceptance.

Thesis Title: Analysis of Peristaltic Flow of Sutterby Fluid with Variable Liquid Properties in an Inclined Channel.

Submitted By: Faiza Zubair **Registration #:** 74 MS/MATH/F22

Master of Science in Mathematics (MS Math)
Title of the Degree

Mathematics
Name of Discipline

Dr. Hadia Tariq
Name of Research Supervisor

Signature of Research Supervisor

Dr. Sadia Riaz
Name of HOD (Math)

Signature of HOD (Math)

Dr. Noman Malik
Name of Dean (FEC)

Signature of Dean (FEC)

Date: 3rd February, 2025

AUTHOR'S DECLARATION

I Faiza Zubair

Daughter of M. Zubair Khan

Registration # 74 MS/Math/F22

Discipline Mathematics

Candidate of **Master of Science in Mathematics (MS MATH)** at the National University of Modern Languages do hereby declare that the thesis **Analysis of Peristaltic Flow of Sutterby fluid with Variable Liquid Properties in an Inclined Channel** submitted by me in partial fulfillment of MS Math degree, is my original work, and has not been submitted or published earlier. I also solemnly declare that it shall not, in future, be submitted by me for obtaining any other degree from this or any other university or institution. I also understand that if evidence of plagiarism is found in my thesis/dissertation at any stage, even after the award of a degree, the work may be cancelled and the degree revoked.

Signature of Candidate

Faiza Zubair

Name of Candidate

3rd February, 2025

Date

ABSTRACT

Title: Analysis of Peristaltic Flow of Sutterby Fluid with Variable Liquid Properties in an Inclined Channel

The main focus of this thesis is to investigate the peristaltic transport of Sutterby fluid with variable liquid properties in an inclined channel. The study also takes into consideration porosity. The governing equations for the conservation of mass and momentum for Sutterby fluid in a symmetric channel are introduced. Stream functions are used to reduce the number of dependent variables of governing PDEs. Perturbation method is used to solve these equations in order to obtain velocity and temperature profiles. The effects of diverse parameter on streamlines, velocity, and temperature are investigated. The software Mathematica is used to create the graphs.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF SYMBOLS

ρ	Density
p	Pressure
μ	Viscosity
a	Width
b	Amplitude
c	Wave Speed
λ	Wavelength
δ	Dimensionless wave number
β	Sutterby Fluid parameter
S	Schmidt number
ψ	Stream Function
E_1,E_2, E_3	Wall Properties
u	Velocity of Fluid in x Direction
v	Velocity of fluid in y Direction
Re	Reynold Number
α	Inclination of the Channel
Ec	Eckert Number
σ	Porosity parameter
κ	Thermal conductivity

ACKNOWLEDGMENT

I want to thank and honor Allah Almighty for making this study possible and fruitful. Without the sincere support provided by numerous sources for which I would want to sincerely thank you this project could not have been completed. However, a lot of people helped me succeed, and I will always be grateful for their support. I owe a debt of gratitude to Dr. Hadia Tariq, whose counsel, insight, and steadfast support have been invaluable to me during this study process. I consider myself extremely fortunate to have had you as my mentor because your knowledge and guidance have been helpful.

DEDICATION

This thesis is dedicated to my parents, husband and my teachers who always supported and taught me to work hard for the things that I aspire to achieve. All of them have been a source of motivation and strength during moments of despair and discouragement

CHAPTER 1

INTRODUCTION AND LITERATURE REVIEW

1.1 Introduction

Mechanics looks at the effects of forces on objects, either still or moving. There are two main branches, statics and dynamics. Statics deals with stationary objects, while dynamics analyzes those in motion. The flowing substances such as liquids and gases are called fluids. According to the definition by J. Spurk and N. Aksel [1], the materials which deform continuously under convenient shear stress, i.e., volume does not change but its shape changes. This is crucial for various applications ranging from car design, flow monitoring systems, weather prediction to large structures aerodynamic properties. Fluid mechanics is the study of how forces impact these fluids, whether they are at rest or in motion. It promotes understanding of bodily fluids such as blood and air. Mechanics is much broader than the Fluid Mechanics approach, but there are two specific ways for it.

1. **Macroscopic Approach**: This looks at fluids on a larger scale and is more commonly used in the field.
2. **Microscopic Approach**: This examines fluids at the molecular level.

Fluids are categorized based on traits like viscosity, pressure, surface tension, and density. Knowing these attributes allows scientists and engineers to predict fluid behavior, enhancing technology and expanding our understanding of natural phenomena.

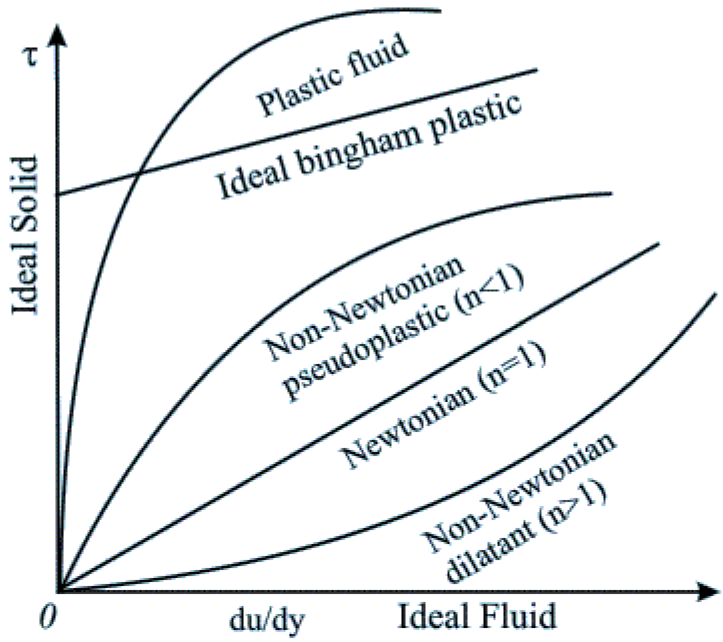


Figure 1.1: Behavior of fluids.

In Figure 1.1, it can be seen that how ideal fluid behavior is represented by the horizontal axis, or abscissa. Ideal fluids exhibit zero resistance to shearing deformation under all flow conditions, resulting in zero shear stress. Essentially, perfect fluids do not experience stress or change shape under force. The vertical axis, or ordinate, represents an ideal solid. Ideal solids do not deform under any load, maintaining their shape regardless of the force applied.

Newtonian fluids have a linear relationship between shear stress and the rate of shear strain or velocity gradient. On a graph of shear stress versus velocity gradient, this relationship appears as a straight line through the origin. The slope of this line represents the fluid's viscosity, which measures its resistance to flow.

In contrast, non-Newtonian fluids exhibit more complex behavior. They are categorized into groups such as dilatant fluids (which become more viscous with increased shear rate), Bingham plastics, and pseudo-plastic fluids (which become less viscous with increased shear rate). Non-

Newtonian fluids, such as certain polymers and slurries, do not follow the simple linear relationship seen in Newtonian fluids like air and water.

Non-Newtonian liquids, for example, Sutterby liquid, which addresses high-polymer arrangements, travel by means of a calculated cylinder. Peristaltic stream, which contains wave-like withdrawals and relaxations of the channel walls, is undifferentiated from a peculiarity tracked down in natural frameworks like the gastrointestinal system. Sutterby liquid has more complicated stream conduct because of its shear-diminishing qualities. This addresses that the liquid's thickness brings down as the shear rate increments. Gravity, the liquid's thickness, changes in the channel walls, and the point of tendency all affect how the liquid moves, including speed, pressure conveyance, and intensity move. This sort of study is helpful in areas like biomedical designing, where understanding peristaltic stream is imperative for drug conveyance, blood stream examinations, and clinical gadget plan. Here is an outline of current investigations did inside this field.

1.2 Peristalsis

In fluid mechanics, peristalsis is the self-propelled flow of fluids caused by a wave-like pattern of contraction and relaxation. This phenomenon is important in a variety of areas of life, especially the human body and a few innovative applications. Peristalsis has applications in stomach related frameworks, blood course, fake muscles, bug development, natural checking, and drug fabricate. Latham [2] introduced the concept of peristalsis by investigating the theoretical and experimental features of urine passage in the ureter using Newtonian compressible fluids. Newtonian fluids have a linear stress-strain relationship, which helps us understand many fluid characteristics. However, it has difficulties in describing the properties of certain biological fluids.

The early studies on peristaltic flows concentrated on Newtonian fluids. However, it is commonly acknowledged that many fluids used in physiology and industry are non- Newtonian, which has a considerable impact on the mathematical and physical elements of the problem. In response, Raju and Devanathan [3] did breakthrough research, becoming the first to study peristaltic flows involving non-Newtonian fluids.

They specifically investigated the peristaltic flow of blood using a power-law fluid model. This trend toward non-Newtonian fluids underscores the significance of understanding more realistic fluid characteristics in a variety of practical applications. Shapiro *et al.* [4] explained the fundamental idea and noted the critical role of the peristaltic flow mechanism in relation to numerous physical parameters affecting fluid flow. A summary of interesting experimental and theoretical studies on peristaltic flow mechanisms involving various fluids is accessible in the extant literature. These investigations often assume low Reynolds numbers and long wavelengths. In our research, we are studying the mechanisms and mathematical models behind peristalsis to better understand how it works. Vaidya *et al.* [5] researched the peristaltic stream of a Carreau-Yasuda liquid in a level microchannel under the impact of an attractive field and halfway slip. To work on the review, the administering conditions were altered considering principal suspicions, for example, long frequency and low Reynolds number. These presumptions improved on the numerical model of liquid stream, giving a clearer comprehension of how the liquid acts in the miniature channel under the gave conditions.

Imran *et al.* [6] examined the intensity move and responses happening in peristaltic transport utilizing the Ellis liquid model. The examination zeroed in on a symmetric channel with adaptable walls. To improve on the numerical examination, suspicions of low Reynolds number and long frequency were consolidated. Utilizing a bother method, the arrangement of the demonstrated issue was gotten. Diagrams were plotted to comprehend how intensity and responses were moved in the peristaltic stream inside a consistent walled channel. Ahmed *et al.* [7] studied the peristaltic movement of a nanofluid with shifting thickness. The review utilized temperature-subordinate thickness and Maxwell's warm conductivity models. Moreover, warm and speed slip limitations were thought of. To improve on the overseeing conditions, the review utilized an oil approach. This study assisted with bettering comprehending how the nanofluid acts during peristalsis in a medium with changing qualities and slip conditions. Salih *et al.* [8] researched the impacts of pivot, evolving consistency, and temperature on peristaltic occasions in a asymmetric channel. Cartesian directions were utilized to foster the conditions controlling movement and intensity, which were then made dimensionless by considering different dimensionless numbers like Reynolds, Hartmann, Grashof, and Prandtl. This permitted us to more readily comprehend how these boundaries influence peristaltic movement and intensity transmission in an unbalanced channel.

Hayat *et al.* [9] studied entropy with regards to liquid exchange utilizing peristalsis. The request took a gander at the blended convective stream in a consistent wall channel with a 3rd-grade liquid. Gravity impacts were thought of, and stream demonstrating integrates magnetohydrodynamics and Joule warming. The energy condition was explicitly tended to, including issues like thick dissemination, and changing warm conductivity. The objective was to comprehend the way that these assorted boundaries impact entropy inside the liquid vehicle peculiarity under study. Balachandra *et al.* [10] investigated a creative method for peristaltic flow of Eyring-Powell liquid across a uniform conduit. It was directed by thinking about the effects of shifting fluid and wall boundaries in a uniform slanted channel, and the stream issue was numerically depicted. Long-wave length and Low Reynolds numbers were utilized to reproduce no-slip conditions on the channel walls. The arrangements were acquired utilizing the conventional twofold irritation procedure, and the nonlinear administering conditions were standardized with appropriate non-layered parts. Graphical portrayals of the impacts of key actual qualities like speed, temperature, focus, and smoothed out were shown and made sense of. The review found that Eyring-Powell liquid boundaries and shifted fluid qualities affected peristaltic stream.

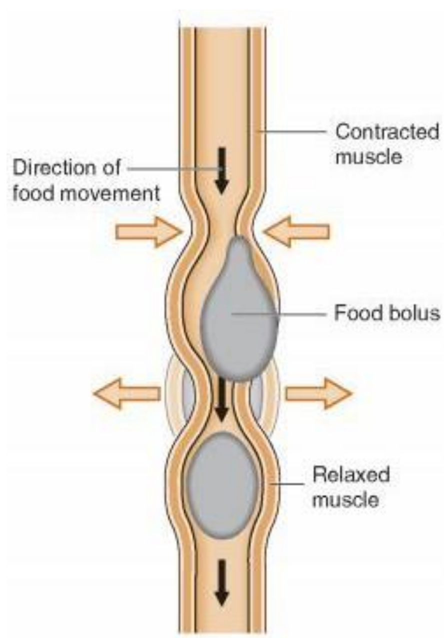


Figure 1.2: Peristaltic movement of bolus in oesophagus.

1.3 Heat Transfer

Heat transfer moves thermal energy between physical systems, happening by conduction, convection and radiation. In a nutshell, heat transfer is the transfer of heat from a hot surface to a cold surface to achieve temperature equilibrium. This self-organized mechanism is important in a large scale of natural and artificial systems where it affects the heat distribution and transport both in solids and fluids. In peristaltic transport with heat transfer, we study the way fluid moves in a manner like how food is digested and how this motion can be affected or controlled by factors like exchange of heat through the flow process, conduction of bio-heat within tissues etc. Some practical problems in this research are kidney transplantation, heat exchangers, solar energy systems, etc. By investigating these constituents, researchers can gain insight into how parasite temperature fluctuations in the gut affects digesting.

In other words, heat transfer is the process of heat coming from a hotter to a cooler area and thus temperatures are even. This mechanism is a vital thing in both nature and humans' life as it influences the distribution of heat and the transport of heat within both solids and fluids. Peristaltic transportation with heat transfer means studying how fluid motion, which is like digestion, is affected by certain factors like thermal exchange, bio-heat conduction in tissues, and other processes. This research deals with not only kidneys and heat exchangers but also solar energy systems and more. The in-depth measurements of these aspects allow researchers to understand how changes in the gastrointestinal tract's temperature affect digestion and overall health of humans.

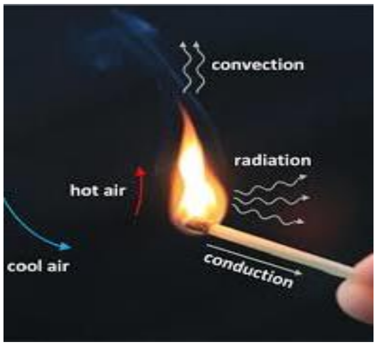


Figure 1.3: Heat transfer in a vacuum environment.

Hasona *et al.* [11] examined how temperature-subordinate thermal conductivity and intensity radiation impact the peristaltic stream of pseudoplastic nanofluids in a slanted, non-uniform, asymmetric channel. The concentrate additionally thought to be the impacts of a calculated attractive field. Hayat *et al.* [12] took a gander at the peristaltic transport of Sutterby fluid in bended molded math while representing temperature-subordinate thermal conductivity. An angled magnetic field was taken into consideration. The lubricating method was used in the formulation. The irregularities in the heat transmission processes were examined using entropy concerns. This analysis used the perturbation method to determine velocity and stream function.

Eldabe and Ramadan [13] looked how heat absorption, chemical reactions, and wall qualities influence the micropolar nanofluid peristaltic flow through a permeable media. The flow's controlling conditions were first stated and then studied using the assumptions of long wavelength and low Reynolds number. The velocity and microrotation velocity were estimated using detailed solutions, whereas the governing conditions for energy and nanoparticles were solved analytically using the homotropy perturbation method.

Elmhedy *et al.* [14] researched heat moves and the impact of an attractive field on peristaltic stream, explicitly utilizing the Rabinowitsch liquid model in a slanted channel. The exploration examined the central issue of peristalsis connected with heat transport within the sight of an attractive field. The review utilized an incompressible Rabinowitsch liquid inside a slanted channel as its reference circumstance. To improve on the review, arrangements were resolved utilizing suppositions about lengthy frequencies and low Reynolds numbers.

Sanil *et al.* [15] investigated the peristaltic mechanism of Sisko fluid under the influence of heat transfer, taking into consideration varying viscosity, slip effects, variable thermal conductivity, and wall features. The nonlinear governing equations were obtained with long wavelength and low Reynolds number approximations. These equations were solved using the regular perturbation method, and a closed-form solution for the stream function was produced for various fluid behavior parameters. The effect of different situations on physiological aspects was visually displayed with MATLAB. The results showed that the velocity and thermal slip parameters had significant effects on heat transfer. It was also found that the bolus volume increased as the velocity slip parameter expanded. Furthermore, the coefficient of pseudo-plasticity played an important part in the study for various fluid behavior index values.

Salahuddin *et al.* [16] examined hybrid nanofluid flow in peristaltic transport within a sinusoidal wavy channel, concentrating on the impacts of heat generation and absorption. The perturbation method was used to solve the governing equations, with low Reynolds numbers and long wavelengths as assumptions. Mathematica was used to develop numerical solutions for temperature and flow, which were then graphed in MATLAB. The data showed that heat generation increased pressure rise, temperature profile, and pressure gradient, whereas greater phase difference and channel width reduced pressure rise and gradient. Furthermore, the bolus size rose with amplitude but reduced with higher heat output.

1.4 Wall properties

According to the structural designs of flexible tubes, peristalsis by wave-like contractions can be influenced by the wall characteristics such as roughness and the type of materials that are used. In turn, these factors will affect the friction, the wall deformation extent, and the fluid-wall interaction in the system. Wear resistance to water leakage is provided by walls that are waterproof, and friction-reducing surface coatings can make them the perfect for heightened interaction with living organisms. In addition, wall properties are also responsible for the thermal transfer efficiency in peristaltic systems. I. M. Eldesoky, *et al.* [17] examined the characteristics of elastic walls that contribute heavily to the fluid flow behavior, such as the mean axial velocity, pressure, and the net flow rate. These wall features are frequently modeled with a flexible wall model, which includes an elastic spring to absorb vertical wall displacement and a wall damper to minimize normal wall velocity. Simply said, the flexibility of the walls influences how the fluid travels, and these models use springs and dampers to account for this impact.

Boujelbene *et al.* [18] portrayed a clever investigation of the peristaltic flow of Eyring-Powell liquid over a uniform course. The investigation thought about the effect of wall characteristics, variable fluid properties, and numerical advancement for the stream issue. The channel walls go through speed and warm slip conditions, and the investigation utilized long-frequency and low Reynolds number approximations. The overseeing nonlinear halfway differential conditions were standardized with significant non layered elements, and arrangements were determined by applying a normal irritation methodology. Manjunatha *et al.* [19] study the effects of changing liquid and wall properties were included into the peristaltic process of Rabinowitsch fluid. The study focused on a two-dimensional non-uniform porous channel. Heat transfer characteristics were investigated under convective environments, whereas mass transfer was considered with slip conditions on the channel walls. The model was built on the assumptions of a long-frequency and low Reynolds number. Exact solutions were identified for concentration, velocity, and streamlines, whereas the perturbation method was used to find the temperature solution. The effect of different parameters on temperature, velocity, and streamlines was investigated for dilatant, Newtonian, and pseudoplastic fluid models. Researchers discovered that changing liquid characteristics raised fluid temperature for shear-thinning, shear-thickening, and Newtonian fluids.

Manjunatha *et al.* [20] investigated the effect of magnetohydrodynamics on the peristaltic development of the Eyring-Powell fluid. The assessment relied upon the Navier-Works up conditions, which are broadly problematic. To simplify the survey, the model was smoothed out by including explicit assumptions, for instance, a long recurrence and a low Reynolds number. This concentrate similarly investigated the effect of wall features on peristalsis inside seeing an appealing field, addressing changing fluid variables like adjusting thickness and warm conductivity. The speed, temperature, concentration, and smooth out profiles were procured by settling the nonlinear circumstances that direct the system under various slip conditions. Besides, the effects of different waveforms on speed profiles were investigated. A parametric evaluation gave more information, and the results were obviously shown using MATLAB R2023a programming. The results showed that the alluring limit and variable thickness essentially impacted the fluid's approach to acting.

1.5 Sutterby Fluid

Akram *et al.* [21] considered the Sutterby fluid model defines blood's shear-thinning properties. Sutterby fluid model, developed by Sutterby in 1966, is used to represent viscosity data for various polymer solutions and melts. Imran *et al.* [22] kept a strong emphasis on biologically inspired propulsion systems. Theoretical analysis was used to better understand how reactions, especially heterogeneous-homogeneous responses, influence intensity and warm transmission in incompressible Sutterby liquids. To work on the intricate conditions controlling liquid vehicle issues, suspicions, for example, low Reynolds number and long frequency were made. These simplified calculations were then solved with a perturbation approach. This technique aids in understanding how biologically inspired propulsion systems respond under the impact of these reactions in the Sutterby fluid setting. Aasma and Hummady [23] examined the impact of the revolution variable on the peristaltic stream of Sutterby liquid in an unbalanced channel with heat move. The demonstrating of science was made within the sight of the impact of pivot, utilizing constitutive conditions following the Sutterby liquid model. In stream examination, suspicions, for example, long frequency estimation and low Reynolds number were used. The subsequent nonlinear condition was mathematically settled utilizing the irritation strategy.

Faseeha et al. [24] contemplated the peristaltic system of Sutterby liquid in a symmetric channel with mass and intensity move centers around how the liquid acts under conditions. The examination is done with the understanding of a little Reynolds number and a long frequency. For little upsides of the Sutterby liquid boundary, conditions for speed, temperature, and fixation are gotten. The Sutterby liquid model is especially valuable for portraying polymer-rich watery arrangements. Attractive powers are becoming known to be a more successful therapy for specific ailments than conventional prescriptions. Peristaltic stream with mass and intensity move is utilized for various clinical treatments, including tissue obliteration, hemodialysis, and oxygenation. This study adds to a more noteworthy comprehension of how such liquid streams are capable in different settings, bringing about upgraded clinical applications and treatments.

Hayat *et al.* [25] meant to look at the impact of consistent walls on the peristaltic stream of Sutterby liquid in an upward channel. The stream was impacted by a cross over attractive field. Heat, not set in stone by convective limit conditions, were likewise thought of. The energy condition thought about the impacts of intensity dissemination and radiative intensity transmission. The issue was planned under the reason of insignificant inertial impacts and a long frequency estimate. The shooting technique was utilized with Mathematica's NDSolve to create mathematical outcomes for the stream capability, temperature, and intensity move coefficient. Neeran and Hayat [26] kept an eye on the mathematical showing of peristaltic flow of Sutterby fluid across a pit between coaxial chambers. The internal chamber remained stable, but the outer chamber showed sinusoidal discontinuous changes along the channel walls. The managing conditions for development, temperature, and obsession were written in round and empty bearings under the assumptions of little Reynolds number and a long frequency. The smart solution for the fluid's temperature and center was achieved with Mathematica 11.3, and the shut construction speed profile was induced using the aggravation strategy. The movements in center speed, stream capacity, temperature, obsession, and power drop were graphically bankrupt down, considering the effects of huge limits.

1.6 Inclined Channel

In peristalsis, it means that the channels on the upward slope which can be varied are used to manipulate the motion of fluid by the variance of the channel's slope. This incidence allows more intricate key issues between liquid and the channel walls. The mechanisms of these slanted tubes of the digestive system are the conveyor belt of food particles, which naturally smooth the digestive process and assist the absorption phase. The distinct flow types and the variances in the current that are the attraction of these channels are the features of variety. Scientists such as Balachandra *et al.* [27] examined how slip conditions, eroding thermal conductivity, and fluctuating atmospheric conditions affect the flow of Ree-Eyring Powell fluid in an inclined tube. Axisymmetric flow refers to the flow that is induced by periodic contraction and expansion of the circular symmetry around the central axis. In this peristaltic flow, even though the fluctuating velocity and pressure exist, they remain in equilibrium. Rafiq *et al.* [28] inspected Jeffrey liquid in an asymmetric channel, with tightening impacts. This approach was provoked by the way that numerous human physiological frameworks and modern hardware have complex calculations, requiring tightening channel contemplations. Non- Newtonian liquids, such as those described in the Jeffrey fluid model, have high molecular weights, rendering the usual no-slip condition ineffective. As a result, slip effects were integrated into the analysis. The work investigated the peristaltic flow of a Jeffrey nanofluid via a tapered asymmetric channel in the presence of a magnetic field.

Aasma and Hummady [29] explored the impact of rotation and other factors on the peristaltic flow of Sutterby fluid within a slanted asymmetric tube, which was filled with a heat conducting porous medium. A Sutterby fluid model was used to derive constitutive equations for mathematical modelling in the presence of rotation. A long wavelength approximation ($k \ll 1$) and low Reynolds number ($Re \ll 1$) were used to analyze the flow. A perturbation method was used to analytically solve the resulting nonlinear ordinary differential equation. Graphical investigations of the effects of various parameters on the stream function and pressure gradient were discussed including Grashof number, Hartmann number, Reynolds number, Froude number, Hall parameter, Darcy number, magnetic field, Sutterby fluid parameter, and heat transfer. The numerical results were obtained with the MATHEMATICA software tool. It was found that the size of boluses reduced as some parameters increased, although the pressure gradient was directly proportional to most of these parameters.

Hamed *et al.* [30] addressed the peristaltic transport of a nano-hyperbolic tangent fluid in an inclined asymmetric channel with convective boundary conditions. Copper and aluminum oxide nanoparticles were considered. The mathematical modeling was performed under long wavelength and low Reynolds number assumptions. The governing equations were analytically solved using a perturbation technique with a low Weissenberg number. The effect of the nanoparticle volume fraction, slip parameter, Biot number, and other important parameters on the stream function, axial velocity, pressure rise per wavelength, temperature, and Nusselt numbers were investigated using graphical representations. Copper and aluminum oxide nanoparticles had a minor effect on the axial velocity field and heat transfer coefficients. Retrograde pumping rate dropped as n increased but reversed in peristaltic and co-pumping flows. Furthermore, the pressure rise per wavelength did not alter much as the Weissenberg number increased in the presence of copper and aluminum oxide nanoparticles.

1.7 Slip Effects

Slip effects in peristaltic flow occur when the fluid partially deviates from the walls of the channel or tube that moves the fluid. This means that there is some shift or relative motion between the fluid and the solid surface. These slip effects can now act as control parameters of the peristaltic flow by impacting factors like fluid speed, pressure, and the whole fluid dynamics process. Slip at the fluid-solid interface can thus transform the issue. Understanding slip effects is essential for the most exact peristaltic transport even when the solving liquid is in contact with hydrophobic surfaces, mainly, when it comes to model fluids. Researchers often interrogate slip effects as a means of improving the validity of models that elucidate peristaltic flow both in biological systems and engineered devices. Ibrahim *et al.* [31] examined the mixed convection magnetized nanoflow of a Prandtl fluid in a non-uniform channel undergoing peristalsis. External considerations like activation energy and non-constant velocity slip were carefully considered. The fluid's governing equations were developed, and these equations were then translated into a set of ordinary differential equations with non-dimensional parameters. This technique streamlines the study and allows for a more thorough understanding of the fluid's behavior under the stated parameters.

Chinnasamy *et al.* [32] concentrated on heat move in a two-layered deformable cylinder loaded up with attractive viscoelastic nanofluids. The review thought about a few elements simultaneously, including peristaltic waves, corridor current, particle slip, and others. The Sutterby liquid model was utilized to portray the rheology of nanofluids, while the Buongiorno model consolidated nanoscale impacts. A novel strategy to mathematical calculation was introduced, joining a Multi-facet Perceptron feed-forward back-engendering fake brain organization (ANN) with the Levenberg-Marquardt calculation. The objective of this system was to propose an imaginative and proficient answer for the difficult intensity transport issue in the depicted situation. Vaidya *et al.* [33] focused in on the new effect of slip on the peristaltic stream of a non-Newtonian Jeffrey fluid through a skewed chamber. The consequences of changing fluid limits, like thickness and warm conductivity, as well as wall qualities, were investigated. The directing circumstances were dimensionless using fitting comparability changes, and a series plan was found for speed, temperature, and concentration. MATLAB 2019b was utilized to make graphical depictions of the impacts of key components on physiological stream estimations. The results showed that rising the thickness limit achieved better speed and temperature scatterings. Additionally, the effect of moving consistency possibly extended the size of the got bolus. This concentrates also perceived potential purposes for getting a handle on the movement of cellulose through the gastrointestinal plot.

Rajashekhar *et al.* [34] investigated the impact of wall slip on the progression of a 3rd grade liquid through a slanted peristaltic channel, considering varieties in thickness, warm conductivity, and wall properties. The overseeing conditions were improved with long-frequency and low Reynolds number approximations. These adjusted conditions were tackled utilizing the annoyance approach. Key physiological qualities like speed, smoothed out, temperature, and focus were determined for a scope of elements. The outcomes showed that rising the upsides of the variable thickness and slip term upgraded the speed profile. Moreover, flexibility factors advanced liquid stream, yet damping eased back liquid particles. Furthermore, expanding the slip boundary, variable consistency, and tendency qualities expanded the size of the caught bolus, which caused the formation of more boluses. The joining of variable attributes worked on how we might interpret the complex rheological way of behaving of blood moving through tight or miniature corridors.

Divya *et al.* [35] zeroed in on the peristaltic system of Jeffrey liquid dropping down a versatile cylinder, considering the impacts of speed slip, convective limit conditions, and shifted liquid qualities. Shut structure arrangements were found for the speed, motion, and temperature fields. To linearize the temperature condition, the irritation approach was utilized. Moreover, the transition was determined hypothetically utilizing both the Rubinow-Keller and Mazumdar methods, and the outcomes were looked at graphically. The impact of different key factors on liquid stream should be visible and explored with graphical portrayals. The outcomes showed the significance of versatile variables in improving the transition of non-Newtonian liquids. It was additionally found that rising the variable consistency brought about higher speed and temperature, however diminished stream. Besides, the catching peculiarities exhibited that as the variable thickness and speed slip boundaries expanded so did the volume of the bolus.

Vaidya *et al.* [36] researched the impact of slip and intensity move on the peristaltic movement of Bingham liquid in a slanted cylinder. Various waveforms were explored, including sinusoidal, multi-sinusoidal, triangle, square, and trapezoidal. The investigation was done under the suppositions of long frequency and low Reynolds number. Shut structure arrangements were created for speed, plug stream speed, pressure slope, smoothed out, and temperature. Pressure rise and frictional power were dissected utilizing mathematical combination. The effect of significant boundaries on physiological qualities was explored and introduced utilizing charts. The discoveries showed that both speed and warm slip diminished the liquid's speed and temperature. Also, the investigation discovered that when the speed slip boundary expanded so did the caught bolus volume.

Ajithkumar *et al.* [37] researched how halfway slip and gyrotactic microorganisms influence the peristaltic transport of a magnetohydrodynamic Ree-Eyring nanofluid across an adjusted permeable course. The examination thought about warm radiation, energy age, cross-dissemination, and twofold dispersion impacts. By embedding fitting nondimensional boundaries, the first overseeing conditions were transformed into a bunch of dimensionless fractional differential conditions. To find scientific answers for this arrangement of halfway differential conditions, the review utilized the homotopy annoyance approach.

1.8 Variable liquid properties

The variable liquid qualities in peristalsis are the fluid properties that may be variable. These properties are viscosity, thermal conductivity, density, and specific heat capacity along with chemical composition. The influence of these qualities on the fluid can be caused by external factors, resulting in the fluid behavior during the peristaltic movements to be different. Divya *et al.* [38] set to exist a mathematical model which exactly represents the blood flow. In this study, peristaltic transport channel investigation was conducted using oblique walls that are both porous and compliant. The peculiarity of this study is the use of two variable properties, viscosity and thermal conductivity, within the electrically conducting blood confined to a radial magnetic field. This is one of the first studies that have tried to address the complex features of blood rheology. Also, the study includes convective boundary conditions for heat transmission and slip conditions for mass transfer.

The qualities of fluids are urgent in characterizing how they act in different regular and modern cycles. One significant property is consistency, which estimates a fluid's protection from stream and decides how thick or dainty it is. Consistency diminishes with expanding temperature, permitting fluids, for example, honey to stream more promptly when warmed, and it can likewise vary with pressure. One more fundamental trademark is thickness, which is characterized as the mass per unit volume of a fluid. Thickness ordinarily diminishes as temperature climbs, yet it can increase at higher tensions. The presence of disintegrated mixtures can likewise impact a fluid's thickness. Surface pressure is how much energy important to grow the surface region of a fluid, or the power that makes the surface agreement. This property commonly reduces with expanding temperature and can be changed by foreign substances or surfactants in the fluid. Warm conductivity is a fluid's ability to lead heat. This trait can change considering temperature and strain, with different fluids having fluctuating measures of warm conductivity. The intensity limit of a fluid is how much intensity is expected to expand the temperature of a unit mass by one degree Celsius. This characteristic fluctuates with temperature and tension and may contrast relying upon the fluid's creation.

Rajashekhar *et al.* [39] reproduced the peristaltic component with a non-Newtonian 3rd grade liquid, considering variable liquid qualities, electroosmosis, slip, and synthetic responses. The administering conditions considered low Reynolds numbers and long-frequency approximations. The review explored the effect of various liquid boundaries on speed, temperature, fixation, and catching. Graphical portrayals of speed and temperature profiles were utilized to represent the effect of these different liquid qualities, which uncovered a critical drop in these boundaries because of varieties in fluid properties. Rajashekha *et al.* [40] Investigated how variable viscosity and thermal conductivity affect the peristaltic movement of Casson fluid in a convectively heated inclined porous tube. The analysis considered temperature-dependent thermal conductivity as well as variations in viscosity along the radial axis. The governing nonlinear equations were solved using the perturbation approach, with long wavelengths and low Reynolds numbers as assumptions. Analytical solutions were developed for velocity, streamlines, pressure rise, frictional force, and temperature, considering factors such as slip and convective boundary conditions. The results were evaluated and discussed using graphics. It was discovered that varying viscosity considerably improved velocity profiles. Furthermore, the study found that increasing the velocity slip parameter reduced the amount of the trapped bolus.

Balachandra *et al.* [41] investigated an innovative way to peristaltic transport of Eyring-Powell fluid across a uniform conduit. The study examined the effects of varying liquid and wall parameters in a uniform inclined channel, and the flow problem was mathematically stated. Low Reynolds numbers and long-wavelength approximations were used to approximate no-slip conditions along the channel walls. The solutions were derived using the traditional double perturbation technique, and the nonlinear governing equations were normalized with suitable non-dimensional components. Graphs were utilized to evaluate and discuss the effects of major physical characteristics such velocity, temperature, concentration, and streamlines.

1.9 Porous medium

Porous materials, such as rocks and sponges, contain microscopic voids. These compartments can hold water, air, and other materials. They are necessary for filters and a variety of other activities, as well as groundwater flow. The study of fluid mechanics employing porous media is important in a variety of technological domains, including biology. Porous mediums are used in fluid mechanics because they can alter the behavior of fluid flows in a variety of ways. It improves heat and mass transfer, reduces drag in fluid flow, and is utilized in filtering and separation operations. Funmilay and Moses [42] used a porous channel to study heat exchange via naturally occurring convection in an unstable magneto hydrodynamic stream of non-Newtonian fluids. Darcy [43] defined the flow rate in porous materials. He investigated the resistance factor generated by the porous medium's permeability experimentally. Nazeer *et al.* [44] investigated the thermal transport of the Jeffrey fluid in a porous media with flexible walls and discovered that radiation alters temperature profiles and reduces fluid velocity by adding thermal energy.

Vijayaragawan et al. [46] studied the peristaltic motion of a Jeffrey fluid in a permeable medium, accounting for the fluid's graphical representations, velocity slip parameters, and an external magnetic field. Ahmed et al. [45] examined heat and mass transfer parameters in an asymmetric porous channel with peristaltically flowing nanofluids influenced by magnetic fields and temperature-dependent viscosity.

1.10 Thesis Contribution

This thesis includes a full analysis of Faseeha's work. It studies how inclined channel, wall features, and effects of heat and mass transfer affect the peristaltic flow of the Sutterby fluid. Important influences of viscous dissipation are investigated in the mathematical formulation. When the long wavelength and low Reynolds number requirements are met, a perturbation solution emerges. The computational investigation was carried out using Mathematica software, and the results will be shown graphically.

1.10 Thesis Organization

The rest of thesis is coordinated in the accompanying way:

Chapter 1: Chapter 1 provides the detailed overview of introduction and literature review.

Chapter 2: This portion of thesis manages the fundamental definition and regulations that are fundamental to comprehend the examination work.

Chapter 3: This chapter addresses the research work done by Faseeha Atlas *et al.* [24]. The results are achieved by solving partial differential equations and then applying perturbation techniques.

Chapter 4: Chapter 4 presents the extension work of chapter 3. We have added slip conditions along with wall properties and using variable liquid properties. Numerical conditions are tackled by comparable strategies utilized in survey work. The outcomes are obtained by representing charts.

Chapter 5: Thesis portion of thesis includes the works introduced in chapter 4 and contains the future work

In the end the reference list shows every one of the sources which research have been used in entirety.

CHAPTER 2

FUNDAMENTAL LAWS AND BASIC DEFINITIONS

2.1 Fluid

A fluid is a substance that flows easily and has no fixed shape. It includes both liquids (like water and oil) and gases (like air and steam). Fluids adhere to the shape of their container and respond to factors such as pressure and gravity. For example, water and air are fluids because they flow and adhere to the shape of any container in which they are placed. The term "fluid" can also refer to something that changes quickly or moves easily, indicating flexibility or adaptability. In physics, fluids are studied for their flow behavior, viscosity, pressure, and density [47].

2.2 Types of fluids

Fluids can be classified according to their flow properties and physical qualities. Here are the primary types:

1) Ideal Fluid: Ideal fluids are theoretical concepts. It is assumed to be incompressible and have no viscosity (flow resistance). It flows without friction and, while it does not exist, it helps to simplify fluid dynamics equations [47].

2) Real Fluid: Real fluids are those we meet in daily life. They have viscosity and encounter resistance as they flow. Examples include water, oil, and air [47].

3) Newtonian Fluids: Newtonian fluids have a constant viscosity, which means their flow properties do not alter with stress or force. Water and air are Newtonian fluids [47].

4) Non-Newtonian Fluids: Non-Newtonian fluids vary in viscosity based on applied force or shear rate. Their flow characteristics change under stress. Examples include ketchup, toothpaste, and blood [47].

5) Compressible fluid: Compressible fluids have a density that changes significantly with pressure. Gases, for example, are compressible because their volume can be reduced rapidly under pressure [47].

6) Incompressible fluid: An incompressible fluid maintains constant density despite pressure changes. In real life, most liquids are considered incompressible [47].

7) Viscous fluid: Viscous fluids are thick and have significant internal resistance, which causes slow flow. Examples include honey and molasses [47].

8) Non-Viscous fluid: A non-viscous fluid has little internal resistance to flow. Ideal fluids are non-viscous and are used in theoretical circumstances to simplify fluid studies [47]. Each of these fluid types reacts differently to forces and strains, making it critical to understand their behavior, particularly in fields such as fluid mechanics and engineering.

2.3 Fluid Mechanics

The study of fluid mechanics focuses on understanding, predicting, and managing fluid behavior. An essential handle of liquid mechanics becomes vital for day to day living since we live in a gas-filled environment on a planet fundamentally canvassed in water. Engineers consider it a critical area of applied science with several practical and intriguing applications. Municipal water, sewage, and electrical systems all rely largely on fluid devices, hence fluid mechanics are important for their proper operation [47].

2.3.1 Applications of Fluid Mechanics

Fluid mechanics has broad applications in many fields

1) Science and Engineering: Fluid mechanics principles play an important role in the design and maintenance of water supply systems, providing effective water distribution. It is also used in sewage systems to help manage the flow of trash through pipelines and treatment units. Pumps and turbines, such as those used in hydroelectric power plants, rely on fluid mechanics to move fluids.

2) Automotive and Flight experiences: Liquid mechanics updates vehicle streamlined features by chopping down air block, which increases eco-amiability and adequacy. It is additionally immense in plane course of action, as understanding the breeze stream around the plane guarantees ideal lift and control. Also, liquid mechanics is utilized in gas powered motors to refresh fuel combination, air affirmation, and fumes stream.

3) Medical applications: Liquid mechanics unites zeroing in on flow framework in the human body to look at cardiovascular issues. It besides assists with building clinical contraptions like cardiovascular siphons, stents, and dialysis machines, which depend after understanding liquid course to fittingly work.

4) Environmental science: liquid parts are utilized to expect barometrical circumstances by meteorologists showing air stream. It is also used to examine sea streams and tides, which assists specialists with better figuring out environment designs and ocean life normal structures.

5) Chemical and Cycle attempts: The substance and direct affiliations depend on liquid mechanics in various ways, including building pipelines for huge oil and gas transmission and guaranteeing authentic liquid blending in conveyed reactors. Liquid mechanics is other than enormous in energy age, especially hydropower, where dams and turbines are made utilizing liquid stream standards to make power from water. Likewise, liquid mechanics helps wind energy structures by redesigning wind turbine means to substantially more immediately get wind energy.

6) Marine Preparation: Liquid mechanics adds to convey game-plan by extra making steady quality, decreasing evasion, and extending eco-cheerfulness. It additionally adds to submarine progress by zeroing in on how submarines drop cut down. Considering everything, liquid mechanics is a substitute science with applications in various endeavors, influencing both standard utilities and clear level mechanical forward skips.

2.4 Newtonian Fluids

Newtonian liquids are liquids that act typically while going through force. The consistency, or how thick or tacky the liquid is, stays consistent regardless of how hard you push or pull on it. This demonstrates that the liquid moves in an orderly fashion with the power applied [47]. In straightforward terms, the more energy you apply, the quicker the liquid streams, and this is a persistent association. Newton's law of thickness depicts this way of behaving as follows:

$$\tau = \mu \frac{du}{dy}.$$

Here, τ is the shear stress, which is the force applied per unit area, μ is the dynamic viscosity, which is a measure of the fluid's thickness and stays constant for Newtonian fluid and $\frac{du}{dy}$ is the speed slope, showing how rapidly the liquid's speed changes.

Normal instances of Newtonian liquids are water, air, and most regular oils. These liquids stream in a clear way, regardless of how much power you use.

2.4.1 Applications of Newtonian fluids

Newtonian fluids can't avoid being liquids with a consistent consistency, and that infers their thickness or security from stream doesn't change regardless of what how much power or shear given to them. Their stream lead is clear and consistent with Newton's law of consistency, making them completely important in various applications.

1) Lubrication: Oil is one of Newtonian fluids' most major use. Engine oils and water driven fluids, for example, rely upon their anticipated thickness to work outstandingly at different temperatures and pressures. These fluids guarantee that mechanical parts, similar to engines and turbines, work effortlessly and without amazing changes in execution.

2) Household Things: Newtonian fluids like water can moreover be found in family things. Other individual thought things, like shampoos and liquid cleaning agents, show Newtonian qualities, achieving a smooth surface and obvious stream when used.

3) Food Industry: In the food area, Newtonian liquids like water, milk, and squeeze are easy to deal with, siphon, and cycle. Their uniform way of behaving empowers makers to effectively blend, bundle, and transport fluid food items. This trademark reaches out to flavorings and synthetics utilized in food handling, where controlled stream is basic for exact dosing.

4) Piping and Transport Frameworks: Newtonian liquids additionally assume a significant part in channeling frameworks, especially in water appropriation in civil pipelines, guaranteeing smooth and consistent stream. Unrefined petroleum, while going about as a Newtonian liquid, might be effectively shipped through pipelines, making it a significant piece of the energy business.

5) Cooling Designs: Cooling structures, for example, those found in power plants, vehicles, and climate control systems, depend upon Newtonian liquids like water to give consistent cooling and power trade. Finally, Newtonian liquids are basic in firefighting considering the way that water's normal stream areas of strength for ponders and control during fire mask.

6) Pharmaceuticals: In the drug business, different fluid cures, like syrups, are Newtonian liquids, considering cautious dosing and fundamental affiliation. Newtonian liquids are particularly basic in fabricated dealing with, where solvents with clear way to deal with acting are utilized in responses and extractions.

Generally, Newtonian liquids' steady thickness and reliable direct make them huge in various undertakings, empowering smooth activity under propelling conditions.

2.5 Non-Newtonian Fluids

Non-Newtonian fluids are fluids whose thickness, or security from stream, changes with the power applied to them. This is one of a kind corresponding to Newtonian fluids, for instance, water or air, which have a predictable thickness despite how much power is applied. Non-Newtonian fluids can act particularly dependent upon the conditions, making them seriously baffling. They could turn out to be more slim, thicker, or go probably as both a solid and a liquid under different powers or over an extended time [47].

2.5.1 Types of Non-Newtonian liquids

There are different types of non-Newtonian liquids, each with its own arrangement of properties.

1) **Shear-thinning fluids:** Shear-thinning fluids, such as ketchup and paint, become thinner and easier to flow as greater force is applied, such as squeezing or stirring. This occurs when the fluid's internal structure adapts to the force, reducing friction [47].

2) **Shear-thickening Fluids:** Shear-thickening fluids, like cornstarch mixed with water (also known as Oobleck), do the opposite: they thicken and become more difficult to move when pressured. This happens when the particles in the fluid stick together under pressure, making it harder to flow [47].

3) **Bingham Plastic Fluids:** Bingham plastics are fluids that behave like solids until a specific amount of force is applied. When this barrier is crossed, they start to flow like liquids. Toothpaste is a good example; it stays in the tube until you squeeze it, after which it flows out. Another interesting group is thixotropic fluids, which thin over time when disturbed or stressed but recover to their previous thickness when left alone. Some paints and clays exhibit this characteristic [47].

4) **Rheopectic Fluids:** Rheopectic fluids, on the other hand, thicken with time when continuously strained. Some lubricants behave similarly, becoming more resistant to flow the longer the force is applied [47].

5) **Viscoelastic Fluids:** Viscous fluids show both liquid and solid properties. When a force is applied slowly, they flow like a liquid; when stretched or pulled quickly, they snap back like an elastic solid. Viscoelastic fluids include Silly Putty and blood plasma [47].

2.5.2 Factors Influencing Non-Newtonian Fluid Behavior

Several key components determine how non-Newtonian fluids react to stress. These factors determine whether the fluid thickens, thins, or alters its flow properties over time. The following are the primary factors that influence the behavior of non-Newtonian fluids:

1) **Shear Rate:** The shear rate indicates how fast or hard a force is exerted to a fluid. Applying more pressure (such as squeezing or quickly swirling) causes some fluids, such as ketchup, to flow more easily. In contrast, other fluids, such as oatmeal, resist and thicken as you apply force faster. The variation in viscosity with shear rate is an important characteristic of several non-Newtonian fluids [47].

2) **Duration of Stress:** non-Newtonian fluids' behavior can also be influenced by the duration of the applied force. Some fluids, such as thixotropic fluids (e.g., certain paints), lose viscosity as they are swirled or under pressure. When you stop applying force, they gradually return to their previous state. Other fluids, such as rheopectic fluids, thicken the longer they are stressed [47].
3) **Fluid Composition:** A non-Newtonian fluid's internal composition is also an essential factor. These fluids frequently contain suspended particles, polymers, or molecules that react to external stimuli in distinct ways. When stress is applied to these particles within the fluid, their arrangement and interaction can change. This causes shifts in viscosity. For example, the particles in a shear-thickening fluid pack together under pressure, making it difficult for the fluid to move [47].

To put it plainly, the way of behaving of non-Newtonian liquids is impacted by the rate at which a power is applied, the span of the power, and the liquid's inside structure. Understanding these viewpoints assists with making sense of why these liquids act in such remarkable and frequently astounding ways.

2.5.3 Everyday Applications

Non-Newtonian liquids are tracked down in different normal items and organizations.

1) **Cosmetics and Individual Consideration Items:** non-Newtonian liquids in beauty care products and individual consideration items, like moisturizers, creams, toothpaste, and shampoos, are intended to stream promptly when applied while staying sufficiently thick to remain on the skin or hair. This trademark guarantees that the program moves along as planned, and the client has an agreeable encounter. Likewise, non-Newtonian liquids are utilized in paints and coatings on the grounds that their shear-diminishing properties consider basic application with brushes or rollers while staying sufficiently thick to forestall trickles.

2) **Printing and Ink Assembling**: Non-Newtonian liquids likewise help in the printing and ink fabricating processes. Many inks have shear-diminishing characteristics, which permit them to stream flawlessly during printing however stay adequately thick to adhere to paper without

spreading. At last, non-Newtonian oils and lubes utilized in hardware safeguard parts by becoming slenderer when moving and thicker while inactive, bringing about smooth activity and less wear.

3) **Body Covering and Defensive Stuff:** Shear-thickening liquids are utilized in body shield and defensive stuff, for example, fluid body reinforcement, which stays adaptable during typical development however solidifies rapidly upon effect on give assurance. This property makes the protective layer more agreeable and lightweight than commonplace covering materials.

4) **Oil and Gas Industry:** non-Newtonian penetrating liquids, frequently known as muds, are significant in the oil and gas area since they grease up and cool bores, convey cuttings to the surface, and oversee well tension all through penetrating tasks. These liquids can acclimate to changing circumstances inside the well, making them incredibly productive.

5) **Biomedical Field:** Non-Newtonian liquids, for example, manufactured blood substitutions and wound care gels, are utilized in biomedical applications considering their capacity to copy natural liquids and really convey prescription to determined areas. Blood plasma, for instance, is viscoelastic, meaning it can stream under ordinary circumstances yet answers flexibly to pressure.

Overall, non-Newtonian fluids are important in many sectors due to their adaptability, efficiency, and variety in diverse applications.

2.6 Viscosity

Viscosity denotes the extent to which a fluid opposes flow. It determines how easily liquids or gases move when a force, such as pressure or gravity, is applied. Fluids with high viscosity, such as honey, flow slowly, whereas fluids with low viscosity, such as water, flow rapidly and easily [47]. This resistance is caused by friction between the fluid's molecules. Viscosity is often measured in

Pascal-seconds. There are two main types of viscosity:

1) Dynamic Viscosity: Dynamic (or absolute) viscosity indicates how much a fluid opposes movement.

2) Kinematic Viscosity: Kinematic viscosity is the dynamic viscosity divided by the fluid density. Simply put, viscosity indicates whether a fluid is thick or thin.

2.7 Density

Density is a property that describes how much mass is contained in each amount of space within a material or object. It indicates how tightly packed the particles in a substance are. To determine density, divide an object's mass by its volume [47].

$$Density = \frac{Volume}{Mass}.$$

The common unit for density is kilograms per cubic meter (kg/m^3), but it can also be measured in grams per cubic centimeter (g/cm^3).

Simply put, a high density indicates that something is heavy for its size (such as a chunk of metal). A low density means that something is lightweight in comparison to its size.

2.8 Pressure

Pressure is a measurement of how much force is delivered to a certain area. It depicts how much force is spread out or concentrated on a surface, which is generally caused by particles in a gas or liquid pushing on it [47].

$$Pressure = \frac{Force}{Area}.$$

The SI unit fir pressure is the pascal (Pa), where 1 pascal equals 1 newton of force applied over 1 square meter $\left(\frac{\mathrm{N}}{\mathrm{m}^2}\right)$. Other units include atmosphere (atm), $bars$ and psi.

Simply put, pressure indicates how powerfully a force is acting on a specific location. For example, the air around us constantly presses against everything, and in water, pressure increases as you get deeper because of the weight of the water above.

2.9 Stress Tensor

A stress tensor is a mathematical tool used in solid mechanics and fluid dynamics to depict the internal forces that occur within a material when external forces are applied. It takes the basic concept of stress—force per unit area—and expands it into many dimensions, allowing researchers to investigate how a material resists or deforms under different types of stresses. The stress tensor describes the amount and direction of stresses operating on various planes within the material, providing a comprehensive picture of how forces are distributed within the item [47].

2.9.1 Cauchy Stress Tensor

The Cauchy stress tensor mirrors the all-out pressure (ordinary and shear) working on a spot inside a material. It gives a top to bottom portrayal of the inward powers following up on a material in response to outer powers. This pressure tensor is in many cases utilized in strong mechanics and liquid elements to describe a material's pressure in its current (twisted) setup [47].

The Cauchy stress tensor contains:

a) Normal strains act perpendicular to a surface.
b) Shear strains (which occur parallel to a surface).

The Cauchy stress tensor, abbreviated as σ, is a second-order tensor with 9 components in 3D space (3 normal stresses and 6 shear stresses).

$$\sigma = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix}.$$

The Cauchy stress tensor is utilized to portray the all-out pressure state in the two liquids and solids, and it is shaped from the powers following up on little pieces of a material. It is basically utilized in solids, although it additionally has applications in liquids.

2.9.2 Extra Stress Tensor

The extra stress tensor is a fundamental concept in fluid mechanics, particularly when dealing with non-Newtonian fluids. Non-Newtonian fluids have a more complex relationship between stress and deformation than Newtonian fluids, which are directly proportional to the rate of deformation. The extra stress tensor expresses stress in a fluid that is distinct from isotropic pressure, which works uniformly in all directions. In layman's terms, it focuses on the stress associated with fluid deformation, specifically those connected to viscosity [47].

The extra stress tensor typically accounts for the following components:
a) Shear stresses are created by the flow of fluids.
b) Viscous stresses refer to the fluid's viscosity and how it reacts to deformation.
Mathematically:

$$\sigma = -pI + \tau.$$

Where p represents the isotropic pressure, I is the identity matrix, τ is the extra stress tensor, depicting the stress influences that are not related to pressure. The extra stress tensor is largely utilized in fluid mechanics, particularly when analyzing non-Newtonian fluids with complex deformation responses. It is important in many models that describe the behavior of viscoelastic materials, including the Oldroyd-B and Maxwell models.

2.10 Wavelength

The wavelength is the distance between two successive points in a wave that are moving in the same direction, such as two neighboring crests or troughs. It is typically denoted by the Greek letter λ $(lambda)$ and measured in $meters$ (m). Wavelength is an important feature of waves that changes with the type of wave [47]. Wavelength is connected to a wave's speed (v) and frequency (f) through the equation:

$$\lambda = \frac{v}{f},$$

where the wave's wavelength is denoted by λ, its speed by v (like the speed of sound in air or light in a vacuum), and its frequency by f (measured in Hz). This equation demonstrates that when frequency increases, wavelength decreases, and vice versa.

2.11 Reynolds Number

The Reynolds number is a dimensionless quantity developed in liquid mechanics to figure how a liquid will stream. It shows the harmony between the liquid's latency (propensity to move) and consistency (protection from stream). The Reynolds number permits us to survey whether the stream is laminar (smooth and consistent) or tempestuous (unpredictable and turbulent) [47].
The recipe for computing the Reynolds number (Re) is as per the following:

$$Re = \frac{iniertial\ forces}{viscous\ forces}.$$

2.11.1 Flow Types Based on Reynolds Number

a) Fierce stream ($Re > 4000$): is tumultuous, with twirling movements and swirls, regularly at more prominent speeds or with less thick liquids.
b) Laminar stream ($Re < 2000$): happens when a liquid goes in smooth, requested layers, for the most part at lower speeds or thickness
c) Momentary stream ($2000 - 4000$): The stream becomes shaky, displaying both laminar and tempestuous highlights.

2.11.2 Applications

a) Avionics: It helps conjecture how air moves over wings, which further develops airplane execution.
b) Lines: Specialists utilize the Reynolds number to assess whether liquid in a line will stream without a hitch (laminar) or violently.
c) Marine designing: Marine designing is the investigation of how water streams around transport frames and submerged developments.

All in all, the Reynolds number is basic for understanding and gauging how liquids act in various designing and logical applications.

2.12 Equation of Continuity

The condition of congruity is a principal thought in liquid elements that depicts mass protection inside a liquid stream. It implies that how much liquid entering a framework should match the sum leaving, assuming that no liquid is added or taken out. Basically said, it guarantees that liquid isn't framed or obliterated as it travels through a framework [47]. For compressible liquids (where thickness can change), the condition of congruity is communicated as follows:

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho v) = 0,$$

where t is time, ρ is the density of fluid, v is velocity and $\nabla.(\rho v)$ shows change in mass over space.

- For compressible fluids, the equation accounts for variations in density as the fluid travels.
- For incompressible fluids, the result of region and speed stays consistent all through the stream.

2.12.1 Applications

The congruity condition is helpful in liquid mechanics for understanding how liquids act in different settings, like pipelines, air conduits, and open channels. It is likewise helpful in areas like avionics (grasping wind stream over wings) and hydrology (concentrating on water stream in waterways). This hypothesis helps engineers in planning liquid vehicle frameworks that are effective while keeping up with mass preservation all through the activity.

2.13 Force

Force is a quantity of how objects interact, causing them to move, slow down or change direction. Simply described, force is a push or pull applied to an item. It is measured in newtons (N) and is classified as a vector quantity because it has both magnitude and direction [47].

Newton's Second Law of Motion states that the force acting on an object is proportional to its mass and acceleration. This relationship is stated by the following formula:

$$F = m\,a.$$

Where F symbolizes force, m is the object's mass, and a is the acceleration generated by the force. This law describes how an object's mass and the force applied to it affect its motion. For example, a heavier object requires more force to achieve the same acceleration as a lighter one.

2.13.1 Body Forces

Body forces act across the entire volume of an object. They influence the entire thing, not just its surface. These forces are typically induced by external fields such as gravity or electromagnetic forces, and they are proportional to the object's mass or volume [47].

Examples of body forces are:

- Gravitational force is a force that pulls objects toward the Earth's core, acting on all components of the item based on its mass.
- Electromagnetic Force: Forces exerted on charged particles within a substance by electric or magnetic fields.
- Centrifugal force is the force that pushes an object outward as it moves in a round path.

2.13.2 Surface Force

Surface forces affect only an object's outside surface or border. These forces are delivered to a specific area and are typically caused by direct contact between objects or fluid pressures. Surface forces are commonly represented as force per unit area, such as pressure or stress [47].

Examples of Surface Forces:

- Frictional force is the resistance to movement between two surfaces that are in touch.

- Fluid pressure is the force exerted by a liquid or gas on a surface.
- Normal force is the power applied by a surface to help the heaviness of an article laying on it.

2.14 Thermal Conductivity

Thermal conductivity measures how well a material carries heat. It describes how quickly heat may go through a substance when there is a temperature differential. Metals have high thermal conductivity, allowing heat to pass easily through them, whereas insulators have poor thermal conductivity, slowing heat transfer [47]. Fourier's Law of Heat Conduction describes how heat moves through a substance, as follows:

$$q = -k\frac{\Delta T}{\Delta x},$$

Where The heat flow (q) is measured in W/m^2, thermal conductivity (k) in $\left({}^{W}/_{m.\,k}\right)$, temperature differential (ΔT), and material thickness (Δx). Thermal conductivity is essential in a variety of applications, including building insulation, electronics cooling, and the design of systems such as heat exchangers that require precise heat flow management.

2.15 Stream Functions

A stream function is a mathematical tool used in fluid dynamics to model the flow of incompressible fluids. It facilitates the investigation and observation of fluid motion, particularly in two dimensions. The stream function allows us to readily determine the flow patterns and trajectories that fluid particles take, often known as streamlines, without having to explicitly compute the fluid's velocity components [47]

CHAPTER 3

EFFECTS OF HEAT AND MASS TRANSFER ON THE PERISTALTIC MOTION OF SUTTERBY FLUID IN AN INCLINED CHANNEL [24]

3.1 Introduction

The study focused on intensity and mass transmission, thick scattering, and a slanted channel. The emphasis was on a non-Newtonian liquid inside the channel with consistent wall properties. The conditions were created using mass, momentum, and energy conservation concepts, as well as proper boundary conditions. The modeled equations were solved using a perturbation approach, specifically using a small parameter β for the Sutterby fluid. The resulting formulas were then applied to study the effects of different physical quantities.

3.2 Mathematical Formulation

Consider a non-Newtonian steady, incompressible sutterby fluid via complaint walls flowing in a symmetric channel which control the peristaltic construction of the sinusoidal wave

The geometry of channel is provided by:

$$y = \pm\eta(x,t) = \pm\left[l + a\sin\frac{2\pi}{\lambda}(x - ct)\right], \tag{3.1}$$

in the above equation, λ represents the wavelength, c expresses wave speed, $\pm\eta$ signifies the movements of the upper and lower walls, and t shows time.

The appropriate velocity field provided is as follows:

$$V = (u(x,y,t), v(x,y,t), 0), \tag{3.2}$$

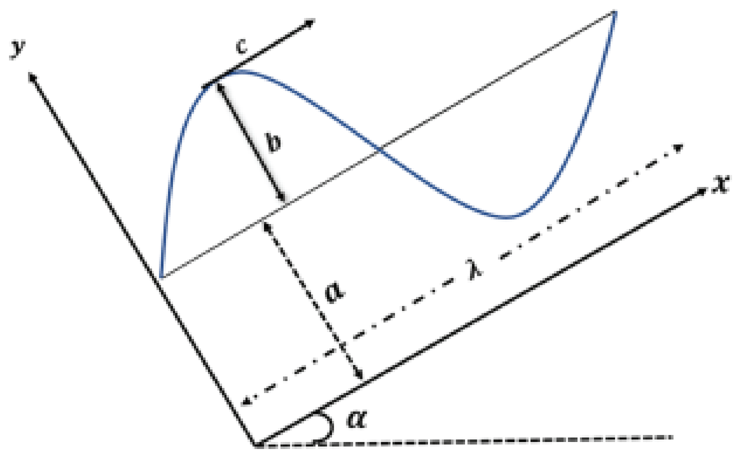


Figure 3.1: Geometry of problem

The fundamental equations that describe how incompressible fluids flow are stated as follows:

The continuity equation

$$div\, V = 0. \tag{3.3}$$

The momentum equation:

$$\rho\frac{dV}{dt} = div\tau + \rho g, \tag{3.4}$$

$$\tau = -pI + S, \tag{3.5}$$

where u and v are x and y components of fluid velocity, ρ represent fluid density, P is the pressure, t shows the time, S is the additional stress tensor, κ denotes the permeability of the porous medium, μ is the dynamic viscosity, I is identity matrix, τ shows the tensor and α shows the angle of inclination.

The temperature equation is given as:

$$\rho C_p \frac{dT}{dt} = \kappa \nabla^2 T + \tau . L, \tag{3.6}$$

where C_p represent specific heat, T represents temperature and κ represents thermal conductivity. $\tau . L$ represent viscous dissipation.

The concentration equation is:

$$\frac{dC}{dt} = D \nabla^2 C, \tag{3.7}$$

where C represents concentration, D represents coefficient of mass diffusivity.

If the velocity components are (u, v) according to the coordinates (x, y) in the wave frame, then

$$x = X - ct, \qquad y = Y, \qquad p(x, y) = P(X, Y, t),$$

$$T(x, y) = T(x, y, t), \qquad u(x, y) = U(X, Y, t) - c,$$

$$v(x, y) = V(X, Y, t). \tag{3.8}$$

Now, the two-dimensional form of Eqs (3.1) - (3.7) can be composed as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{3.9}$$

$$\rho \frac{du}{dt} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x} S_{xx} + \frac{\partial}{\partial y} S_{xy} + \rho g \sin \alpha, \tag{3.10}$$

$$\rho \frac{dv}{dt} = -\frac{\partial p}{\partial y} + \frac{\partial}{\partial x} S_{yx} + \frac{\partial}{\partial y} S_{yy} - \rho g \cos\alpha, \tag{3.11}$$

$$\rho C_p \frac{dT}{dt} = \kappa \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + S_{xx} \frac{\partial u}{\partial x} + S_{yy} \frac{\partial v}{\partial y} + S_{xy} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right), \tag{3.12}$$

$$\frac{dC}{dt} = D \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) + D \left(\frac{\hbar_T}{T_m} \nabla^2 T \right), \tag{3.13}$$

where $\hbar_T$ represents thermal diffusion ratio and T_m represents mean temperature.

The extra stress tensor for sutterby fluid is given by

$$S = \mu \left[\frac{\sinh^{-1} \beta \dot{\gamma}}{\beta \dot{\gamma}} \right]^m A_1, \tag{3.14}$$

$$\dot{\gamma} = \sqrt{\frac{1}{2} tras(A_1)^2}, \tag{3.15}$$

$$A_1 = \nabla V + (\nabla V)^T, \tag{3.16}$$

where A_1 represents the first Rivilin-Ericksen tensor, S is additional stress tensor, β and m represent the material constants of sutterby fluid and μ is the dynamic viscosity $sinh^{-1}$ is roughly equals to

$$\sinh^{-1} \beta \dot{\gamma} = \beta \dot{\gamma} - \frac{\beta^3 \dot{\gamma}^3}{6}, \left| \frac{\beta^5 \dot{\gamma}^5}{6} \right| \ll 1. \tag{3.17}$$

The stress tensors components are defined as follows:

$$S_{xx} = \mu \left[1 - \frac{mB^2}{6} \left\{ 2 \left(\frac{\partial v}{\partial y} \right)^2 + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)^2 + 2 \left(\frac{\partial u}{\partial x} \right)^2 \right\} \right] 2 \left(\frac{\partial u}{\partial x} \right), \tag{3.18}$$

$$S_{xy} = \mu\left[1 - \frac{mB^2}{6}\left\{2\left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2 + 2\left(\frac{\partial u}{\partial x}\right)^2\right\}\right]\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right), \tag{3.19}$$

$$S_{yy} = \mu\left[1 - \frac{mB^2}{6}\left\{2\left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2 + 2\left(\frac{\partial u}{\partial x}\right)^2\right\}\right]2\left(\frac{\partial v}{\partial y}\right), \tag{3.20}$$

The boundary conditions are

$$u = 0 \qquad at \qquad y = \pm\eta, \tag{3.21}$$

$$T = \begin{Bmatrix} T_0 \\ T_1 \end{Bmatrix} \qquad at \qquad y = \pm\eta, \tag{3.22}$$

$$C = \begin{Bmatrix} C_0 \\ C_1 \end{Bmatrix} \qquad at \qquad y = \pm\eta, \tag{3.23}$$

$$\frac{\partial L(\eta)}{\partial x} = \frac{\partial p}{\partial x}$$

$$= \frac{\partial}{\partial x}S_{xx} + \frac{\partial}{\partial y}S_{xy} + \rho g \sin\alpha - \rho\left(\frac{\partial}{\partial t}u + u\frac{\partial}{\partial x}u + v\frac{\partial}{\partial y}v\right) \qquad at \qquad y = \pm\eta \tag{3.24}$$

The following equation describes the relationship between the stream function (Ψ) and the velocity components. By defining Ψ as a stream function given below:

$$u = \frac{\partial\psi}{\partial y}, v = -\frac{\partial\psi}{\partial x}, \tag{3.25}$$

The dimensionless quantities are given below,

$$x^* = \frac{x}{\lambda},\ \ \psi^* = \frac{\psi}{cl},\ \ t^* = \frac{ct}{\lambda},\ \ \delta = \frac{l}{\lambda},\ \ y^* = \frac{y}{l},\ \ F = \frac{\mu c}{\rho g l^2},$$

$$E_1 = \frac{\tau l^3}{\lambda^3 \mu c},\ \ E_2 = \frac{mcl^3}{\lambda^3 \mu},\ \ E_3 = \frac{dl^3}{\lambda^2 \mu},\ \ p^* = \frac{l^2 p}{c\lambda\mu},\ \ Sc = \frac{\mu}{\rho D},$$

$$Sr = \frac{\rho D_{\mathcal{k}_T}(T_1 - T_0)}{\mu T_m(C_1 - C_0)}, \;\; \theta = \frac{T - T_0}{T_1 - T_0}, \;\; \varphi = \frac{C - C_0}{C_1 - C_0}, \;\; Ec = \frac{c^2}{C_p(T_1 - T_0)},$$

$$S^*{}_{ij} = S_{ij}\frac{l}{\mu c}, \;\; \beta = \frac{mc^2b^2}{6l^2}, \;\; Re = \frac{\rho cl}{\mu},$$

$$Pr = \frac{\mu C_p}{\kappa}, \;\; Br = EcPr, \;\; \eta^* = \frac{\eta}{l}.$$

where Er represent the Eckert number, δ is the dimensionless wave number, β denotes Sutterby fluid parameter, Re represents Reynolds number, Sc shows Schmidt number, Pr shows Prandtl number, Br is the Brinkman number and ψ is the stream function.

The dimensionless value of η is

$$\eta = 1 + \varepsilon \sin 2\pi(x - t), \tag{3.26}$$

After removing the asterisk and for longer wavelength, Eqs (3.10-3.13) become

$$\frac{\partial p}{\partial x} = \psi_{yyy}\left\{1 - \beta\left(\psi_{yy}\right)^2\right\} - \frac{\sin\alpha}{F}, \tag{3.27}$$

$$\frac{\partial p}{\partial y} = 0, \tag{3.28}$$

$$\frac{\partial^2\theta}{\partial y^2} + Br\left(\psi_{yy}\right)^2\left\{1 - \beta\left(\psi_{yy}\right)^2\right\} = 0, \tag{3.29}$$

$$\frac{1}{Sc}\frac{\partial^2\psi}{\partial y^2} + Sr\left(\frac{\partial^2\theta}{\partial y^2}\right) = 0, \tag{3.30}$$

with

$$u = 0 \qquad at \qquad y = \pm\eta, \tag{3.31}$$

$$\frac{\partial p}{\partial x} = \Omega(x,t) = \psi_{yyy}\left\{1-\beta\left(\psi_{yy}\right)^2\right\} - \frac{\sin\alpha}{F}, \qquad at \qquad y = \pm\eta, \tag{3.33}$$

$$\Omega(x,t) = 4\varepsilon\pi^2\{\sin 2\pi(x-t)\,(E_3) - 2\pi(E_2 - E_1)\cos 2\pi(x-t), \tag{3.34}$$

$$\theta = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \qquad at \qquad y = \pm\eta, \tag{3.35}$$

$$\phi = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \qquad at \qquad y = \pm\eta, \tag{3.36}$$

3.3 Problem solution

As temperature velocity, and concentration equations are non-linear and it is difficult to gain the desired solution so we will use perturbation techniques to acquire solution. We will utilize β to expand the series of θ, ψ and ϕ as perturbation coefficients.

$$\psi = \sum_{n=0}^{\infty} \beta^n \psi_n = \psi_0 + \beta\psi_1 + O(\beta^2), \tag{3.37}$$

$$\theta = \sum_{n=0}^{\infty} \beta^n \theta_n = \theta_0 + \beta\theta_1 + O(\beta^2), \tag{3.38}$$

$$\phi = \sum_{n=0}^{\infty} \beta^n \phi_n = \phi_0 + \beta\phi_1 + O(\beta^2), \tag{3.39}$$

3.3.1 Zeroth Order System

$$\frac{\partial^4 \psi_0}{\partial y^4} = 0, \tag{3.40}$$

$$\frac{\partial^2 \theta_0}{\partial y^2} + Br\left(\frac{\partial^2 \psi_0}{\partial y^2}\right)^2 = 0, \tag{3.41}$$

$$\frac{1}{Sc}\frac{\partial^2 \phi_0}{\partial y^2} + Sr\frac{\partial^2 \theta_0}{\partial y^2} = 0, \tag{3.42}$$

along with the boundary conditions:

$$\frac{\partial \psi_0}{\partial y} = 0, \qquad \frac{\partial p}{\partial x} = \frac{\partial^3 \psi_0}{\partial y^3} - \frac{\sin\alpha}{F} \qquad at \qquad y = \pm\eta, \tag{3.43}$$

$$\theta_0 = 0 \qquad \phi_0 = 0 \qquad at \qquad y = +\eta, \tag{3.44}$$

$$\theta_0 = 1 \qquad \phi_0 = 1 \qquad at \qquad y = -\eta, \tag{3.45}$$

3.3.2 First Order System

$$\frac{\partial^4 \psi_1}{\partial y^4} - 2\left(\frac{\partial^3 \psi_0}{\partial y^3}\right)^2\left(\frac{\partial^2 \psi_0}{\partial y^2}\right) - \left(\frac{\partial^2 \psi_0}{\partial y^2}\right)^2\frac{\partial^4 \psi_0}{\partial y^4} = 0, \tag{3.46}$$

$$\frac{\partial^2 \theta_1}{\partial y^2} + 2Br\left(\frac{\partial^2 \psi_0}{\partial y^2}\right)\left(\frac{\partial^2 \psi_1}{\partial y^2}\right) - Br\left(\frac{\partial^2 \psi_0}{\partial y^2}\right)^4 = 0, \tag{3.47}$$

$$\frac{1}{Sc}\frac{\partial^2 \phi_1}{\partial y^2} + Sr\frac{\partial^2 \theta_1}{\partial y^2} = 0, \tag{3.48}$$

along with the boundary conditions:

$$\frac{\partial \psi_1}{\partial y} = 0, \quad \frac{\partial^3 \psi_1}{\partial y^3} - \left(\frac{\partial^3 \psi_0}{\partial y^3}\right)\left(\frac{\partial^4 \psi_0}{\partial y^4}\right) \quad at \quad y = \pm\eta, \tag{3.4}$$

$$\theta_1 = 0 \quad \phi_1 = 0 \quad at \quad y = +\eta, \tag{3.50}$$

$$\theta_1 = 0 \quad \phi_1 = 0 \quad at \quad y = -\eta, \tag{3.51}$$

3.4 Results and Discussion

The way of behaving of the boundaries utilized in the temperature (θ), velocity (u), pressure and streamlines expressions is explained in this part. The behavior of the parameters used in the axial velocity (u), temperature (θ) , pressure and streamlines expressions is explained in this section. This section explores the impact of several limitations on the velocity supply, particularly wall parameters E_1, E_2, and E_3, the Sutterby fluid parameter, β slip parameter, and angle of inclination α. To examine the effects of these settings, graphs were made using the MATHEMATICA programming language.

Figure 3.2 illustrates as the Sutterby fluid parameter β grows, so does the velocity. The flow accelerates in the axial direction as β increases. Figure 3.3 depicts the effect of the inclination angle on the velocity profile, which shows that as the angle steepens, the velocity increases. Figure 3.4 shows that a higher amplitude causes an increase in velocity. Additionally, the velocity profile has a parabolic shape when the parameters are kept constant.

Figure 3.5-3.8 show the response of a temperature profile to different evolving influences. As the Sutterby fluid parameter (β) rises, the temperature profile falls (Figure 3.5). It shows that viscous fluid has higher temperature compared to Sutterby fluid. In Figure 3.6, temperature increase with different angles of inclination (α). The effect of the Brinkman number (Br) on the temperature profile is presented in Fig. 3.7, as it is raised. The Brinkman number (Br) measures the intensity of dissipation effects. The effect of the Brinkman number: Since the heat transfer effects return to considering the heat dissipation effects only, we expect an increase in temperature with an increase in the Brinkman number (fig 3.7). Figure 3.8 shows the influence of the epsilon parameter (ϵ) on the temperature profile demonstrates that as its value increases, so does the temperature profile.

Figure 3.9 shows that increasing the angle of inclination α increases the concentration profile. Figure 3.10 shows that more noteworthy upsides of the Sutterby liquid boundary β bring about an expansion in the focus profile. Figures 3.11 and 3.12 show that rising the Brinkman number, Schmidt number, and wall qualities further develops the focus profile, exhibiting a positive connection between these boundaries and fixation levels.

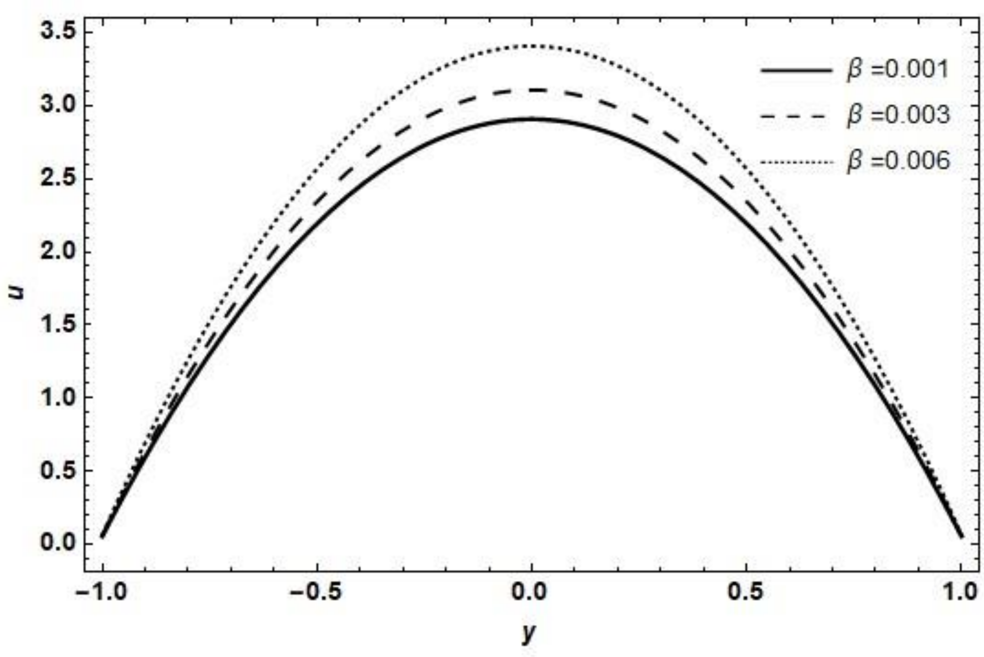


Figure 3.2: Variation of sutterby fluid parameter β on velocity field.

$F = 0.1, \quad \alpha = \frac{\pi}{6}, \quad x = 0.2, \quad t = 0.001, \quad \epsilon = 0.01, \quad E_2 = 0.2, \quad E_3 = 0.1,$

$E_1 = 0.5.$

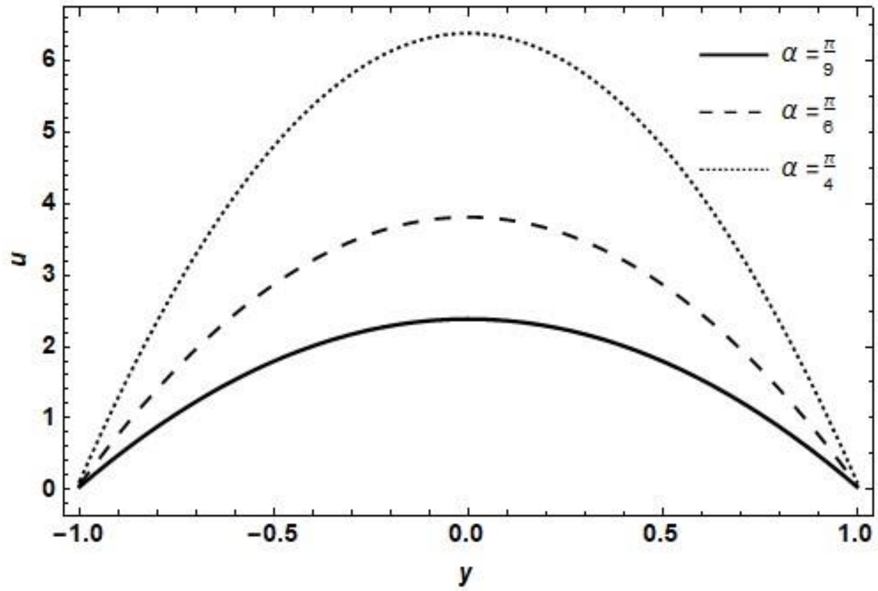


Figure 3.3: Variation of inclination parameter α on fluid velocity.

$\beta = 0.01$, $F = 0.1$, $x = 0.2$, $t = 0.001$, $\epsilon = 0.01$, $E_2 = 0.2$, $E_3 = 0.1$, $E_1 = 0.5$.

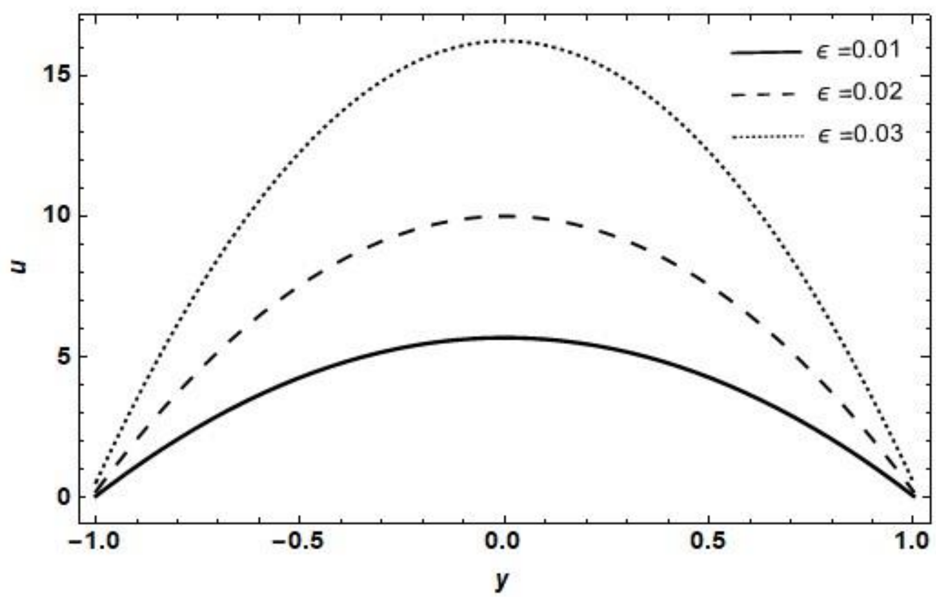


Figure 3.4: Variation of amplitude parameter ϵ on fluid velocity.

$\beta = 0.01$, $F = 0.1$, $x = 0.2$, $t = 0.001$, $\alpha = \frac{\pi}{6}$, $E_2 = 0.2$, $E_3 = 0.1$, $E_1 = 0.5$.

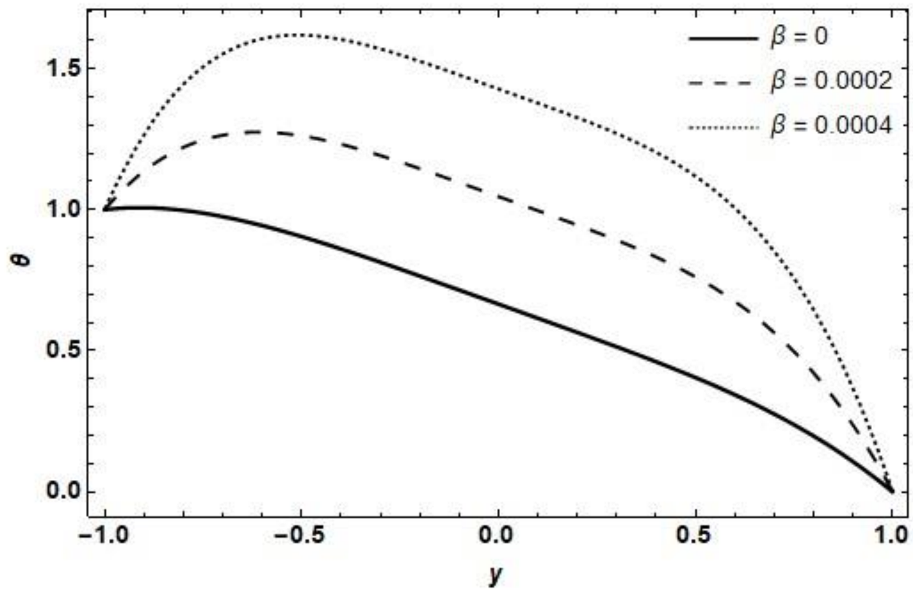


Figure 3.5: Variation of sutterby fluid parameter β on temperature field.

$Br = 0.02,\quad F = 0.01,\quad x = 0.4,\quad t = 0.01,\quad \alpha = 0.1,\quad E_2 = 0.1,\quad E_3 = 0.1,\quad E_1 = 0.1,\quad \epsilon = 0.001.$

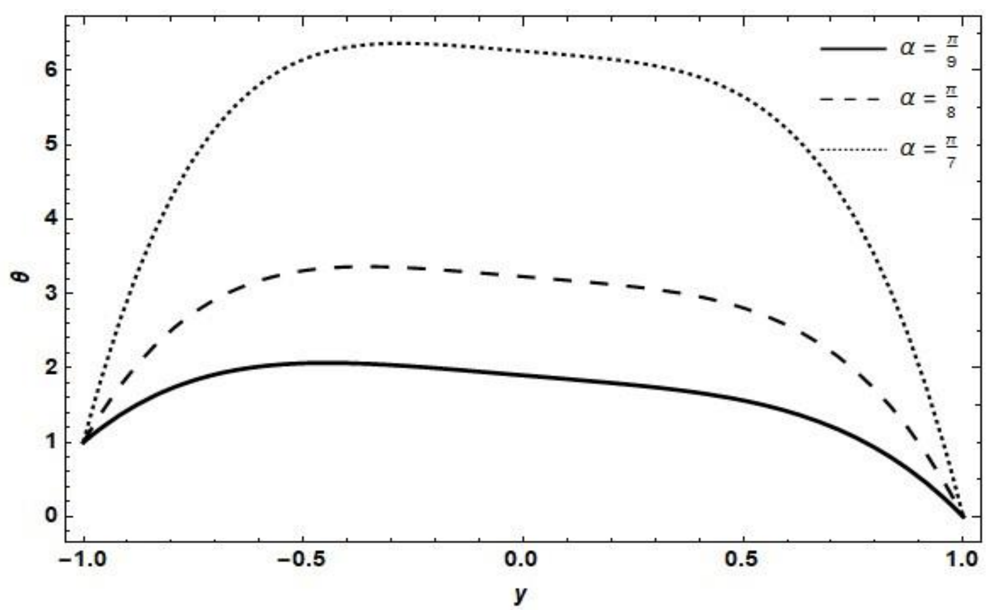


Figure 3.6: Variation of sutterby inclination parameter α on temperature field.

$\beta = 0.001,\quad F = 0.01,\quad x = 0.5,\quad t = 0.01,\quad Br = 0.001,\quad E_2 = 0.1,\quad E_3 = 0.1,\quad E_1 = 0.1,\quad \epsilon = 0.001.$

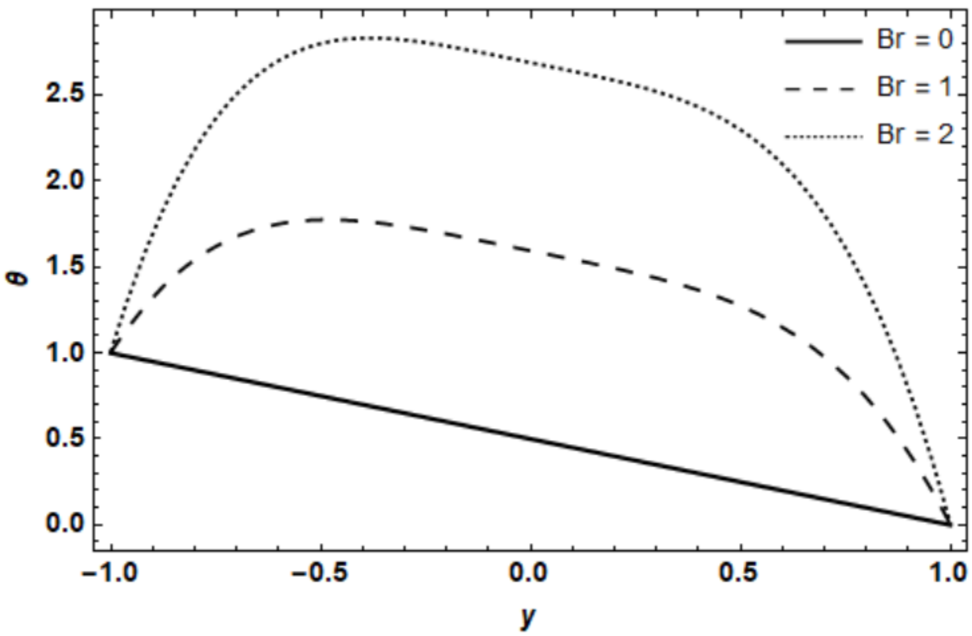


Figure 3.7: Variation of sutterby Brinkman number Br on temperature field.

$\alpha = 0.03, \quad F = 0.01, \quad x = 0.5, \quad t = 0.01, \quad \beta = 0.004, \quad \epsilon = 0.001, \quad E_2 = 0.1, \quad E_3 = 0.1, \quad E_1 = 0.1.$

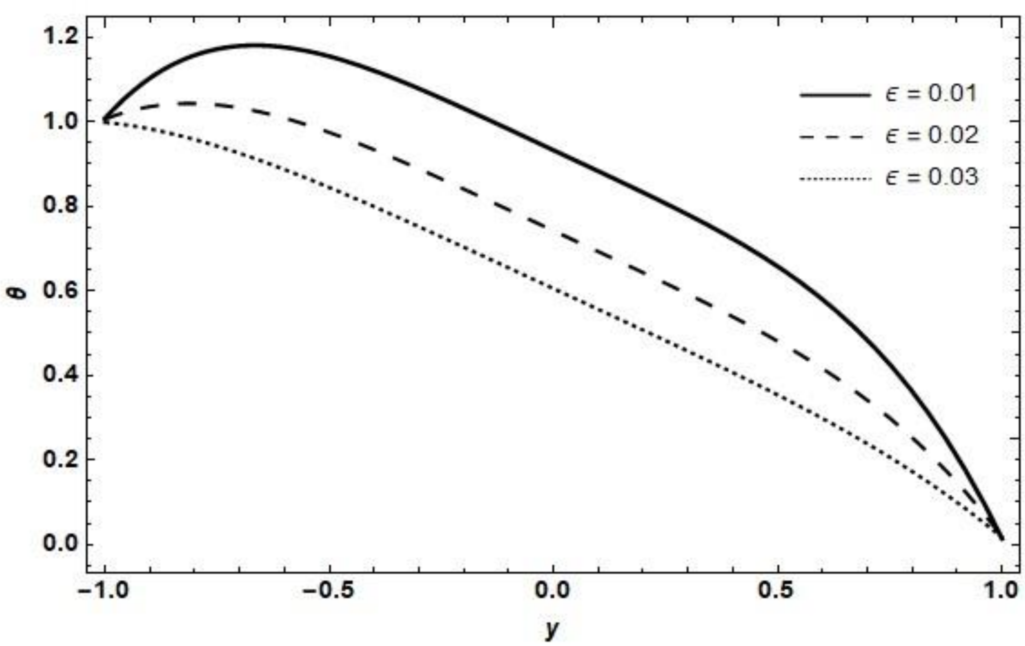


Figure 3.8: Variation of amplitude parameter ϵ on temperature field.

$\alpha = 0.2, \quad F = 0.01, \quad x = 0.5, \quad t = 0.01, \quad \beta = 0.001, \quad E_2 = 0.1, \quad E_3 = 0.1, \quad E_1 = 0.1, \quad Br = 2.$

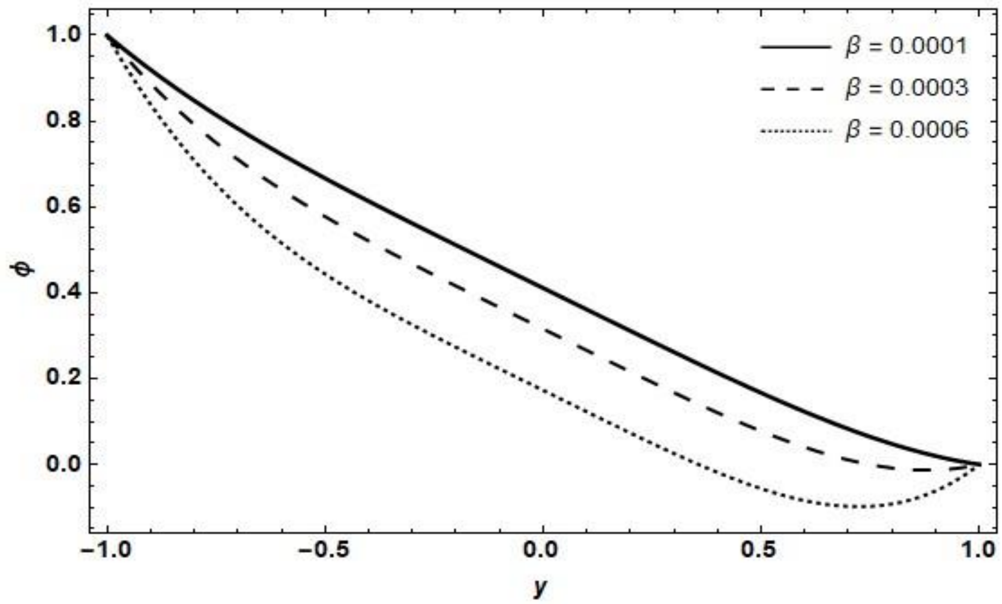


Figure 3.9: Variation of sutterby fluid parameter β on concentration field.

$\alpha = 0.1,\ F = 0.01,\ S = 2.5, s = 0.001\ \ t = 0.01,\ E_2 = 0.1, E_3 = 0.1, E_1 = 0.1,\ B = 2,$

$\epsilon = 0.01, \qquad x = 0.4.$

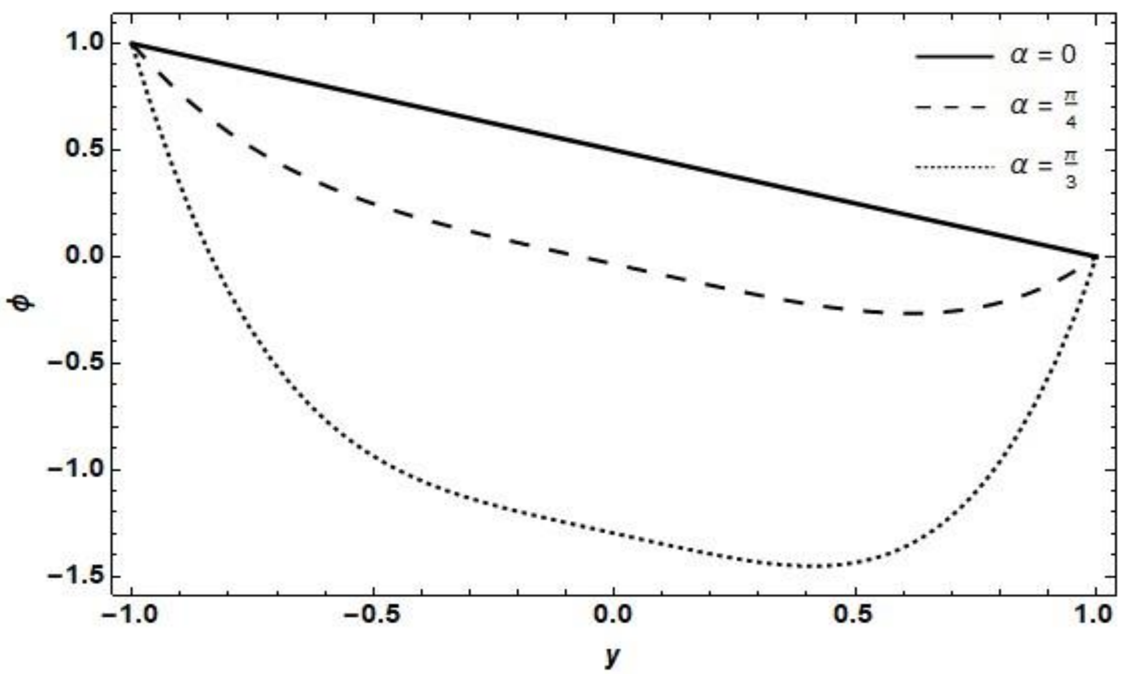


Figure 3.10: Variation of sutterby inclination parameter α on concentration field.

$\beta = 0.0001, \qquad F = 0.01, \qquad S = 0.001, \qquad s = 0.001, \qquad t = 0.01,$

$E_2 = 0.1, \quad E_3 = 0.1,\ E_1 = 0.1, \quad Br = 2, \quad \epsilon = 0.01,\ x = 0.4.$

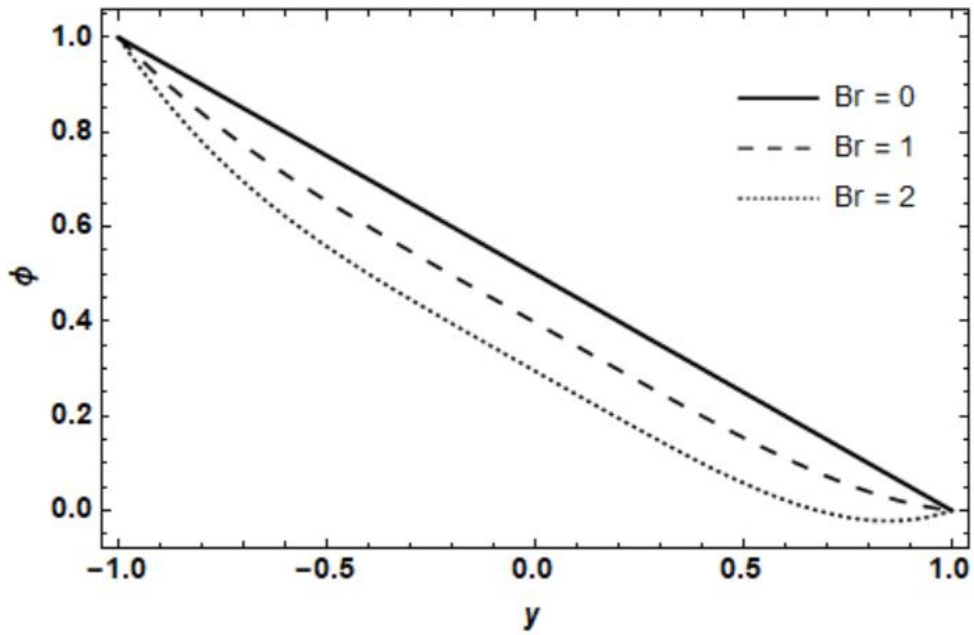


Figure 3.11: Variation of sutterby Brinkman number Br on concentration field.

$\beta = 0.001$, $F = 0.01$, $S = 0.001$, $s = 1$, $t = 0.01$, $E_2 = 0.1$, $E_3 = 0.1$, $E_1 = 0.1$, $\alpha = 0.1, \epsilon = 0.01$, $x = 0.4$.

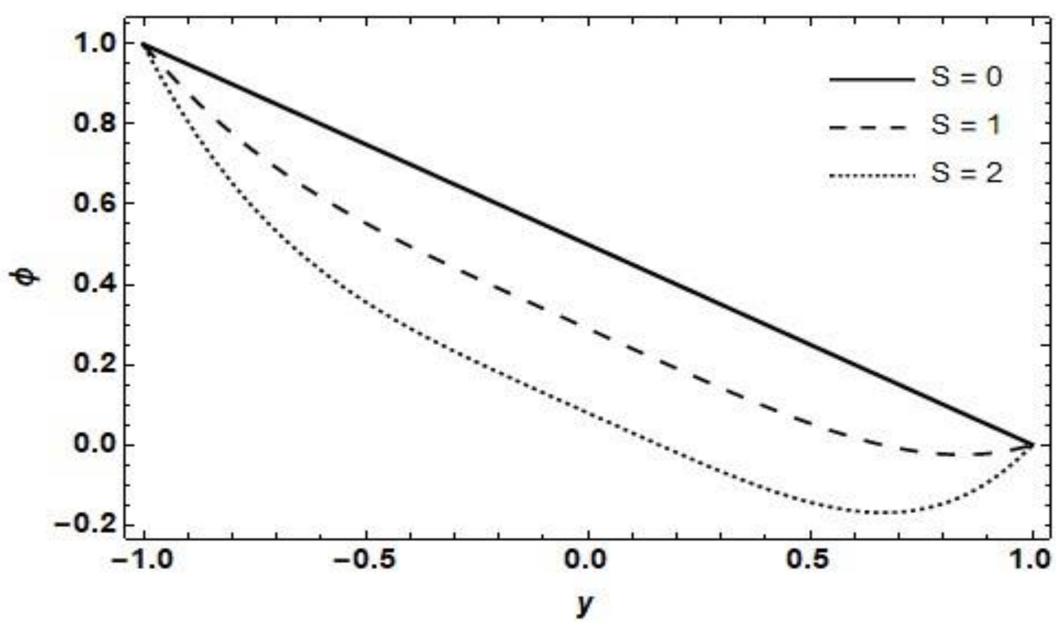


Figure 3.12: Variation of Schmidth number S on concentration field.

$Br = 2$, $F = 0.01$, $\beta = 0.001$, $s = 0.001$, $t = 0.2$, $E_2 = 0.1$, $E_3 = 0.1$, $E_1 = 0.1$, $\alpha = 0.1$, $x = 0.5$, $\epsilon = 0.01$.

3.5 Final Remarks

In this study, we investigated the behavior of the peristaltic mechanism, with an emphasis on mass and heat transmission within an inclined channel. The plots show how different factors affect the velocity, concentration, and temperature profiles across the system.

3.6 Main Findings

As the Sutterby fluid parameter β grows, so does the axial velocity. A larger angle of inclination α improves the velocity profile. Higher wall parameters E_1, E_2, and E_3 result in increased velocity. However, the impact of the wall damping boundary E_3 on the flow behaves differently than E_1 and E_2. As the Brinkman number. Br increases, the temperature rises accordingly. Increasing the angle of inclination α reduces the concentration profile. As the Sutterby fluid parameter β grows, the temperature profile lessens.

CHAPTER 4

ANALYSIS Of PERISTALTIC FLOW OF SUTTERBY FLUID WITH VARIABLE LIQUID PROPERTIES IN AN INCLINED CHANNEL.

4.1 Introduction

This chapter focuses on the behavior of the Sutterby fluid model, taking into consideration variable fluid characteristics and slip factors. Similarity transformations simplify the system by reducing the number of dependent variables. Inspired by the review work [24], this study examines the impacts of altering liquid qualities and slip boundaries on the Sutterby liquid. The perturbation technique is utilized to tackle outcomes. The effect of various circumstances is introduced through charts and inspected.

4.2 Mathematical Formulation

Consider a viscous incompressible liquid that courses through adaptable walls in a porous slanted channel. The non-Newtonian Sutterby fluid model governs the flow, which is characterized by peristaltic motion caused by sinusoidal wave trains. Slip conditions, fluctuating heat conductivity, and changeable viscosity are all assumed in this arrangement. Peristaltic motion causes distortion of the tube walls.

The geometry of channel is provided by:

$$y = \pm\eta(x,t) = \pm\left[l + a\sin\frac{2\pi}{\lambda}(x - ct)\right], \tag{4.1}$$

in the above equation, c represents wave speed, λ represents the wavelength, $\pm\eta$ signifies the changes of the upper and lower walls and t represents time.

The appropriate velocity field given is as follows:

$$V = (\ u(X,Y,t), v(X,Y,t),\ 0), \tag{4.2}$$

The equation of continuity, momentum and energy are given by

The continuity equation:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \tag{4.3}$$

The momentum equation:

$$\rho\frac{dV}{dT} = div\tau - \frac{\mu}{\kappa}V + \rho g, \tag{4.4}$$

$$\tau = -pI + S, \tag{4.5}$$

$$\rho\left(\frac{\partial U}{\partial t} + V\frac{\partial U}{\partial Y} + U\frac{\partial U}{\partial X}\right) = \frac{\partial}{\partial X}S_{XX} + \frac{\partial}{\partial Y}S_{XY} - \frac{\partial P}{\partial X} - \frac{\mu}{\kappa}U + \rho g\sin\alpha, \tag{4.6}$$

$$\rho\left(\frac{\partial V}{\partial t}+V\frac{\partial V}{\partial Y}+U\frac{\partial V}{\partial X}\right)=\frac{\partial}{\partial X}S_{YX}+\frac{\partial}{\partial Y}S_{YY}-\frac{\partial P}{\partial Y}-\frac{\mu}{\kappa}V-\rho g\cos\alpha, \tag{4.7}$$

where U and V are X and Y components of fluid velocity, ρ represent fluid density, P is the pressure, t shows the time, S is the additional stress tensor, κ denotes the permeability of the porous medium, μ is the dynamic viscosity, I is identity matrix, τ shows the tensor and α shows the angle of inclination.

The Energy equation:

$$\rho C_p\left(\frac{\partial T}{\partial t}+V\frac{\partial T}{\partial Y}+U\frac{\partial T}{\partial X}\right)=\kappa\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)$$
$$+S_{YY}\frac{\partial V}{\partial Y}+S_{XX}\frac{\partial U}{\partial X}+S_{XY}\left(\frac{\partial V}{\partial X}+\frac{\partial U}{\partial Y}\right), \tag{4.8}$$

where C_p represent specific heat, T represents temperature and κ represents thermal conductivity. The extra stress tensor for sutterby fluid is given by

$$S=\mu\left[\frac{\sinh^{-1}\beta\dot{\gamma}}{\beta\dot{\gamma}}\right]^m A_1, \tag{4.9}$$

$$\dot{\gamma}=\sqrt{\frac{1}{2}tras(A_1)^2}, \tag{4.10}$$

$$A_1=\nabla V+(\nabla V)^T, \tag{4.11}$$

where m and β shows the material coefficients of sutterby liquid, μ is the unique consistency, S is additional pressure tensor and A_1 represents the first Rivilin-Ericksen tensor. $sinh^{-1}$ are roughly equivalents to

$$\sinh^{-1}\beta\dot{\gamma}=\beta\dot{\gamma}-\frac{\beta^3\dot{\gamma}^3}{6},\left|\frac{\beta^5\dot{\gamma}^5}{6}\right|\ll 1, \tag{4.12}$$

the stress tensors are defined as follows

$$S_{XX} = \mu(y)\left[1 - \frac{mB^2}{6}\left\{2\left(\frac{\partial U}{\partial X}\right)^2 + \left(\frac{\partial V}{\partial X} + \frac{\partial U}{\partial Y}\right)^2 + 2\left(\frac{\partial V}{\partial Y}\right)^2\right\}\right] 2\left(\frac{\partial U}{\partial X}\right), \quad (4.13)$$

$$S_{XY} = \mu(y)\left[1 - \frac{mB^2}{6}\left\{2\left(\frac{\partial U}{\partial X}\right)^2 + \left(\frac{\partial V}{\partial X} + \frac{\partial U}{\partial Y}\right)^2 + 2\left(\frac{\partial V}{\partial Y}\right)^2\right\}\right] \left(\frac{\partial V}{\partial X} + \frac{\partial U}{\partial Y}\right), \quad (4.14)$$

$$S_{YY} = \mu(y)\left[1 - \frac{mB^2}{6}\left\{2\left(\frac{\partial U}{\partial X}\right)^2 + \left(\frac{\partial V}{\partial X} + \frac{\partial U}{\partial Y}\right)^2 + 2\left(\frac{\partial V}{\partial Y}\right)^2\right\}\right] 2\left(\frac{\partial V}{\partial Y}\right), \quad (4.15)$$

If the velocity components are (u, v) according to the coordinates (x, y) in the wave frame, then

$$x = X - ct, \qquad y = Y, \qquad p(x, y) = P(X, Y, t),$$

$$T(x, y) = T(x, y, t), \qquad u(x, y) = U(X, Y, t) - c,$$

$$v(x, y) = V(X, Y, t). \quad (4.16)$$

The following equation describes the relationship between the stream function (Ψ) and the velocity components. By defining Ψ as a stream function,

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}. \quad (4.17)$$

The dimensionless equations are given below,

$$x^* = \frac{x}{\lambda}, \ \psi^* = \frac{\psi}{ac}, \qquad t^* = \frac{ct}{\lambda}, \ \delta = \frac{a}{\lambda}, \ y^* = \frac{y}{a}, \ Fr = \frac{c^2}{ga}, \ Re = \frac{\rho ca}{\mu_0},$$

$$p^* = \frac{l^2 p}{c\lambda\mu}, \ Sc = \frac{\mu_0}{\rho D}, \quad Sr = \frac{\rho D K_T (T_0 - T_1)}{\mu_0 T_m (C_0 - C_1)}, \quad \theta = \frac{T - T_0}{T_0},$$

$$Ec = \frac{c^2}{C_p T_0}, \quad Pr = \frac{\mu_0 C_p}{\kappa}, \quad Br = Ec \times Pr, \quad , \quad \eta^* = \frac{\eta}{a}, \quad \sigma^2 = \frac{a^2}{\kappa}. \tag{4.18}$$

The dimensionless value of η is

$$\eta = 1 + \varepsilon \sin 2\pi(x - t), \tag{4.19}$$

Where Er represent the Eckert number, δ is the dimensionless wave number, β denotes Sutterby fluid parameter, Re represents Reynolds number, Sc shows Schmidt number, Pr shows Prandtl number, Br is the Brinkman number, σ^2 is the porosity parameter and ψ is the stream function.

After removing the asterisk and for longer wavelength, Eqs (4.6-4.8) and Eqs (4.13-4.15) becomes

$$S_{xy} = \mu(y)\left\{1 - \beta\left(\psi_{yy}\right)^2\right\}\psi_{yy}, \tag{4.20}$$

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial y} S_{xy} - \mu(y)\sigma^2\left(\psi_y + 1\right) + \frac{Re}{Fr}\sin\alpha, \tag{4.21}$$

$$\frac{\partial p}{\partial y} = 0, \tag{4.22}$$

$$\frac{\partial^2 \theta}{\partial y^2} + \mu(y)(Br)(\psi_{yy})S_{xy} = 0, \tag{4.23}$$

The expression for variable viscosity is:

$$\mu(y) = 1 - \gamma(y)\ for\ \gamma \ll 1, \tag{4.24}$$

where γ is the coefficient of viscosity.

The non-dimensional boundary conditions are

$$\psi = 0; \qquad \frac{\partial^2 \psi}{\partial y^2} = 0 \qquad at \qquad y = 0, \tag{4.25}$$

$$\psi = F; \qquad \frac{\partial \psi}{\partial y} + \Gamma S_{xy} = -1 \qquad at \qquad y = h(x), \tag{4.26}$$

$$\theta = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \qquad at \qquad y = \pm\eta, \tag{4.27}$$

4.3 Solution Methodology

The velocity and temperature equations are nonlinear, which makes them difficult to solve directly. To address this, the perturbation method is used to determine a solution. Perturbation coefficients are used to create formulas for temperature and velocity, as seen below.

$$\left.\begin{aligned} \psi &= \psi_0 + \beta\psi_1 + \ldots.. \\ F &= F_0 + \beta F_1 + \ldots.. \\ p &= p_0 + \beta p_1 + \ldots.. \\ \theta &= \theta_0 + \beta\theta_1 + \ldots.. \end{aligned}\right\}, \tag{4.28}$$

where

$$\left.\begin{aligned} \psi_0 &= \psi_{00} + \gamma\psi_{10} + \ldots.. \\ \psi_1 &= \psi_{10} + \gamma\psi_{11} + \ldots.. \\ F_0 &= F_{00} + \gamma F_{10} + \ldots.. \\ F_1 &= F_{10} + \gamma F_{11} + \ldots.. \\ p_0 &= p_{00} + \gamma p_{10} + \ldots.. \\ p_1 &= p_{10} + \gamma p_{11} + \ldots.. \\ \theta_0 &= \theta_{00} + \gamma\theta_{10} + \ldots.. \\ \theta_1 &= \theta_{10} + \gamma\theta_{11} + \ldots.. \end{aligned}\right\}, \tag{4.29}$$

Now we use Eq. (28) into Eq. (21)-(27) and separate the terms of differential order in β and γ, we get the following systems of partial differential equation for pressure gradients, temperature and stream function with boundary conditions:

4.3.1 Zeroth Order System:

Case I

$$\frac{\partial^4 \psi_{00}}{\partial y^4} - \sigma^2 \frac{\partial^2 \psi_{00}}{\partial y^2} = 0, \tag{4.30}$$

$$\frac{dp_{00}}{dx} = \frac{\partial^3 \psi_{00}}{\partial y^3} - \sigma^2 \left(\frac{\partial \psi_{00}}{\partial y} + 1\right) + \frac{Re}{Fr} \sin \alpha, \tag{4.31}$$

$$\frac{\partial^2 \theta_{00}}{\partial y^2} + Br \left(\frac{\partial^2 \psi_{00}}{\partial y^2}\right)^2 = 0, \tag{4.32}$$

along with the boundary conditions

$$\psi_{00} = 0; \qquad \frac{\partial^2 \psi_{00}}{\partial y^2} = 0 \qquad at \qquad y = 0, \tag{4.33}$$

$$\psi_{00} = F_{00}; \qquad \frac{\partial \psi_{00}}{\partial y} + \Gamma \frac{\partial^2 \psi_{00}}{\partial y^2} = -1 \qquad at \qquad y = h(x), \tag{4.34}$$

Case II

$$\frac{\partial^4 \psi_{01}}{\partial y^4} - \sigma^2 \frac{\partial^2 \psi_{01}}{\partial y^2} = y \frac{\partial^4 \psi_{00}}{\partial y^4} - \sigma^2 y \frac{\partial^2 \psi_{00}}{\partial y^2}, \tag{4.35}$$

$$\frac{dp_{01}}{dx}=\frac{\partial^3\psi_{01}}{\partial y^3}-\sigma^2\ \frac{\partial\psi_{01}}{\partial y}-y\left\{\frac{\partial^3\psi_{00}}{\partial y^3}-\sigma^2\left(\frac{\partial\psi_{00}}{\partial y}+1\right)\right\}, \tag{4.36}$$

$$\frac{\partial^2\theta_{01}}{\partial y^2}+Br\left\{2\left(\frac{\partial^2\psi_{00}}{\partial y^2}\right)\left(\frac{\partial^2\psi_{01}}{\partial y^2}\right)-y\left(\frac{\partial^2\psi_{00}}{\partial y^2}\right)\right\}=0, \tag{4.37}$$

along with the boundary conditions

$$\psi_{01}=0; \qquad \frac{\partial^2\psi_{01}}{\partial y^2}=0 \qquad at \qquad y=0, \tag{4.38}$$

$$\psi_{01}=F_{01}; \qquad \frac{\partial\psi_{01}}{\partial y}+\Gamma\ \frac{\partial^2\psi_{01}}{\partial y^2}-y\Gamma\frac{\partial^2\psi_{00}}{\partial y^2}=-1 \qquad at \qquad y=h(x), \tag{4.39}$$

4.3.2 First Order System

Case III

$$\frac{\partial^4\psi_{10}}{\partial y^4}-2\left(\frac{\partial^3\psi_{00}}{\partial y^3}\right)^2\left(\frac{\partial^2\psi_{00}}{\partial y^2}\right)-\left(\frac{\partial^2\psi_{00}}{\partial y^2}\right)^2\left(\frac{\partial^4\psi_{00}}{\partial y^4}\right)-\sigma^2\ \frac{\partial^2\psi_{10}}{\partial y^2}=0, \tag{4.40}$$

$$\frac{dp_{10}}{dx}=\frac{\partial^3\psi_{10}}{\partial y^3}+3\left(\frac{\partial^2\psi_{00}}{\partial y^2}\right)\left(\frac{\partial^3\psi_{00}}{\partial y^3}\right)-\sigma^2\ \frac{\partial\psi_{10}}{\partial y}, \tag{4.41}$$

along with the boundary conditions

$$\psi_{10}=0; \qquad \frac{\partial^2\psi_{10}}{\partial y^2}=0 \qquad at \qquad y=0, \tag{4.42}$$

$$\psi_{10}=F_{10}; \qquad \frac{\partial\psi_{10}}{\partial y}+\Gamma\ \frac{\partial^2\psi_{10}}{\partial y^2}-\Gamma\left(\frac{\partial^2\psi_{00}}{\partial y^2}\right)^3=0 \qquad at \qquad y=h(x), \tag{4.43}$$

and so forth.

4.4 Discussion of graphical results

This section explains the behavior of the parameters used in the formulas of axial velocity (u), temperature (θ), pressure (ΔP), and streamline. The study focusses on the inclination of channel (α), Brinkman number (B), porosity parameter (σ), sutterby fluid parameter (β), slip parameter (Γ), Froude number (Fr), Amplitude ratio (ϕ) , Flow rate (Q) and Reynolds number (Re). To investigate the consequences of these parameters, graphs were created with the MATHEMATICA computer language.

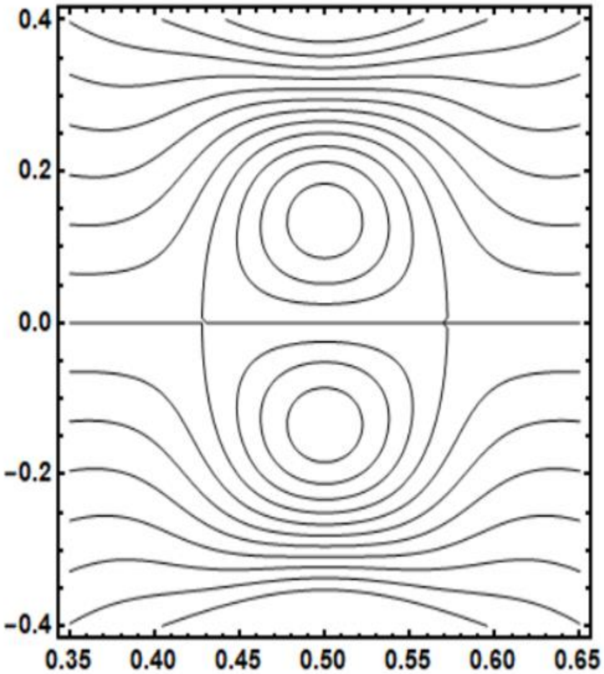
Trapping is one of the most important processes in peristaltic movement. Trapping occurs when streamlines separate and produce a circulating bolus under certain conditions. Because it is entirely surrounded by peristaltic waves, the trapped bolus moves at the same rate as the wave. Figures 4.1(a), (b) and (c) depict the influence of the Sutterby fluid parameter β. The size of the confined bolus and the number of streamlines both decrease as (β) increases. Figures 4.2(a) (b) and (c) show how inclination parameter (α) affects streamline patterns. It can be seen that as (α) increases, the number of circulations and the size of the trapped bolus increases.

Figures 4.3(a), (b) and (c) demonstrate that increasing the value of the magnetic porosity parameter (σ) increases bolus size and the number of circulations. Figures 4.4(a), (b) and (c) illustrate the similar behavior when the slip parameter (Γ) increases.

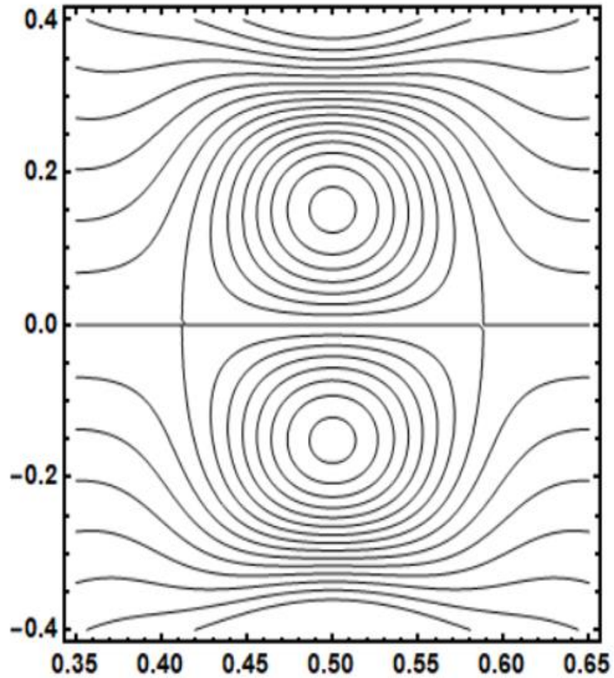
The velocity field is the most noticeable aspect of fluid movement. The analytical solution for velocity is calculated first, followed by the determination of the remaining flow variables. Figure 4.5 shows that when the value of porosity parameter (σ) increases, the velocity profile decreases. Figure 4.6 shows how the fluid's velocity drops when the Sutterby fluid parameter (β) value increases. Figure 4.7 demonstrates that when the value of inclination (α) grows, the velocity decreases because the resistive force increases. Figure 4.8 shows that when the slip parameter (Γ) rises, the fluid velocity decreases within the channel.

Figures 4.9–4.13 demonstrate the impact of various emergent flow features on the fluid's temperature θ. Figure 4.9 illustrates that when the value of Brinkman number (B) increases, temperature drops. Figure 4.10 shows the same trend of the temperature profile, increase in porosity parameter (σ) causes temperature drop. Figure 4.11 shows how the temperature profile decreases as the value of the inclination angle (α) increases. Figure 4.12 shows that increasing slip parameter (Γ) results in a lower temperature profile.

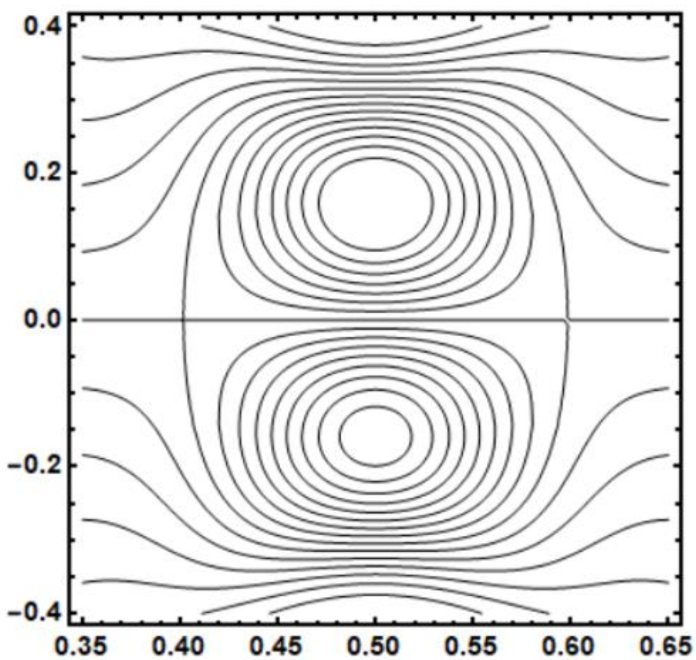
This section includes graphical results that show how the parameters vary. Figure 4.14 shows that decrease in pressure (ΔP) cause decrease in flow rate (Q) in case of Reynold number (Re). The change of pressure rises (ΔP) with (Q) for different values of Sutterby fluid parameter (β), as shown in Figure 4.15. In the pumping region, (Q) drops as (β) grow, while it increases in both free and co-pumping regions. Figure 4.16 show a graph of pressure versus (Q) illustrates that when Fr increases, the pumping rate decreases across all locations. Figures 4.17 show the plot of pressure rise (ΔP) vs flow rate (Q) for amplitude ratio (ϕ). (Q) reduces with increasing (ϕ) in pumping and free pumping zones, but increases with increasing (ϕ) in co-pumping regions. Figure 4.18 shows the graph of pressure rise (ΔP) vs flow rate (Q) for the case of inclination parameter (α). As (α) grows, (Q) drops in the pumping region, but increases in both free and co-pumping regions.



(a) $\beta = 0.01$

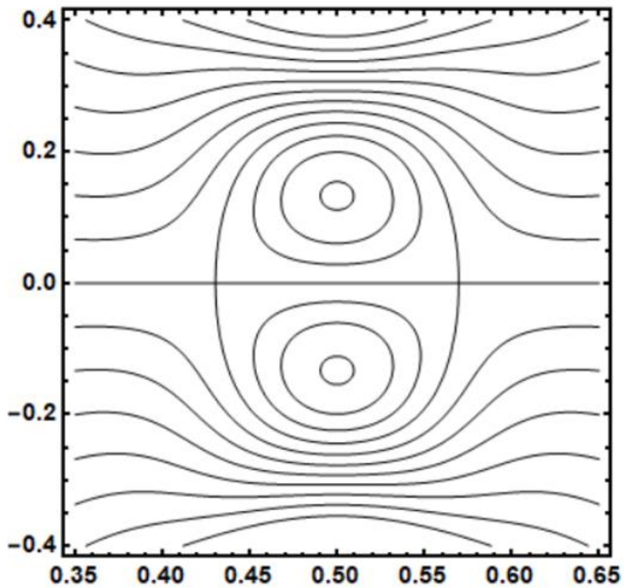


(b) $\beta = 0.015$

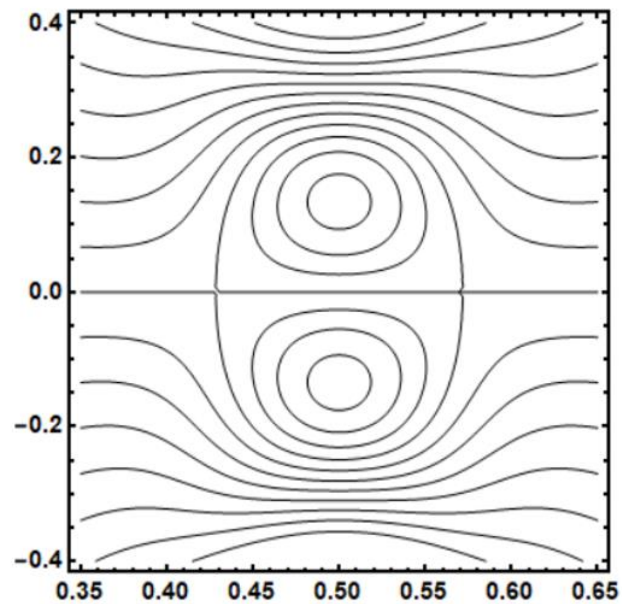


(c) $\beta = 0.02$

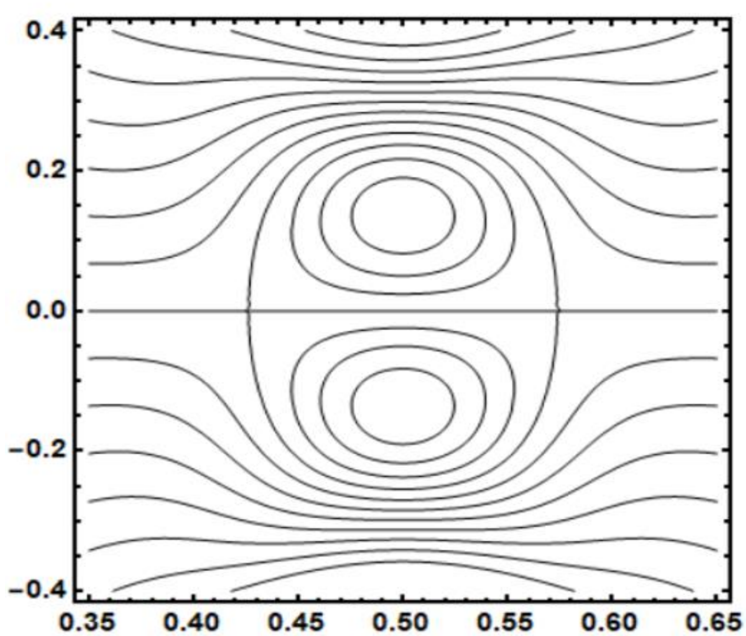
Figure 4.1 Variation of Sutterby fluid parameter β on Streamlines.



(a) $\alpha = 0$

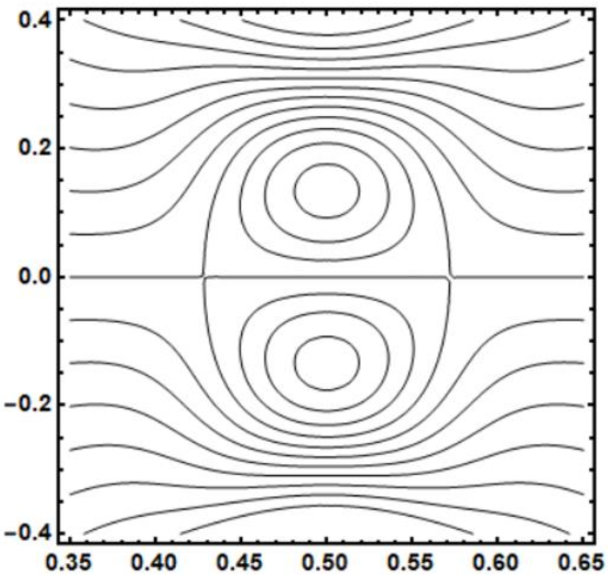


(b) $\alpha = 0.2$

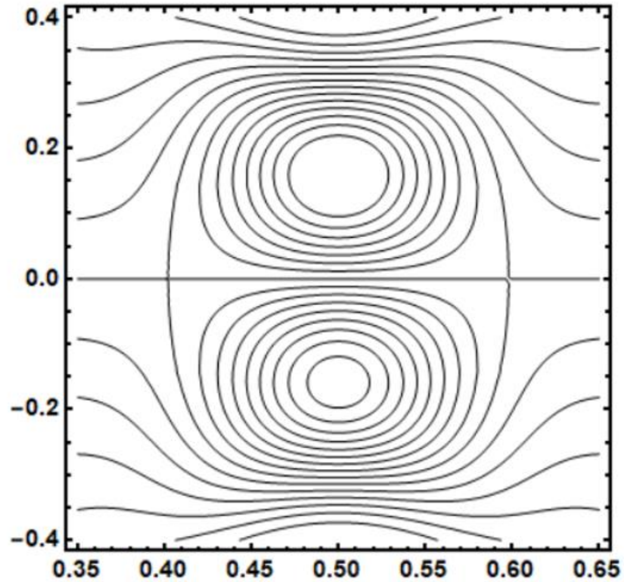


(c) $\alpha = 0.4$

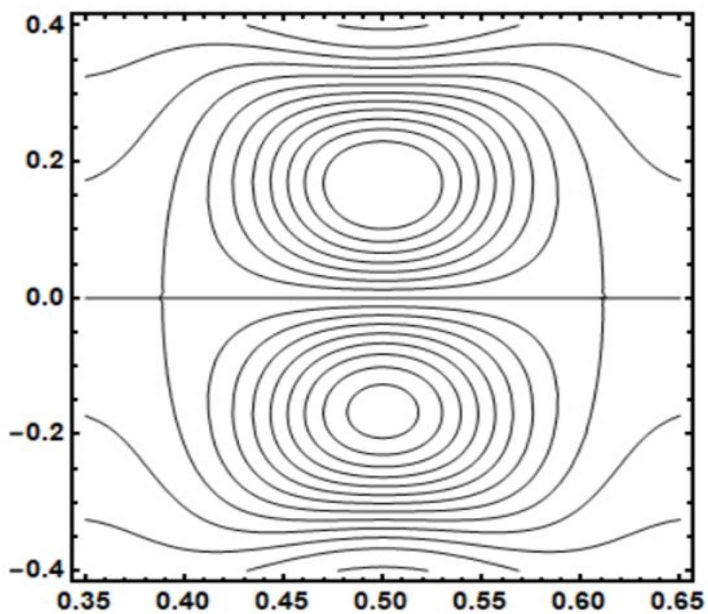
Figure 4.2 Variation of inclination parameter α on Streamlines.



(a) $\sigma = 0.01$

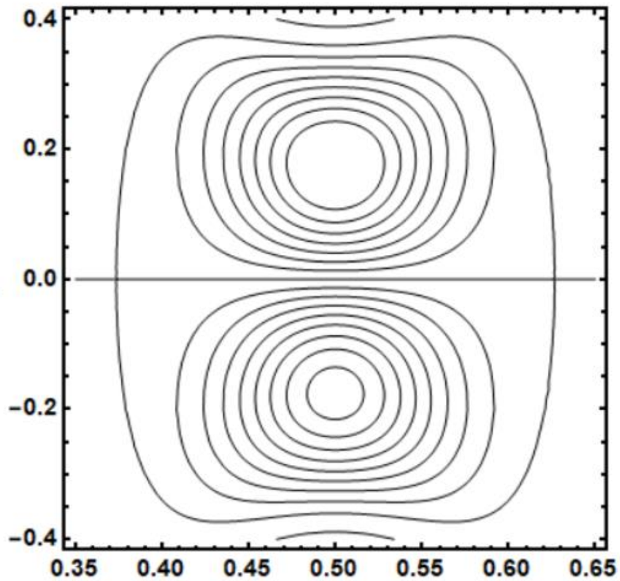


(b) $\sigma = 0.03$

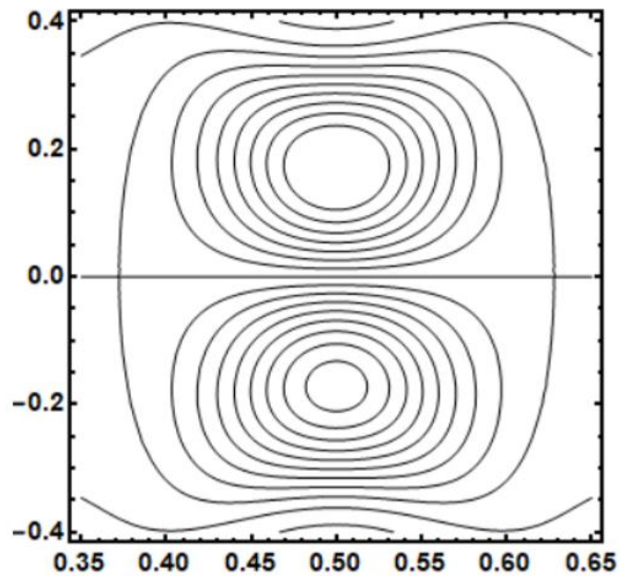


(c)) $\sigma = 0.09$

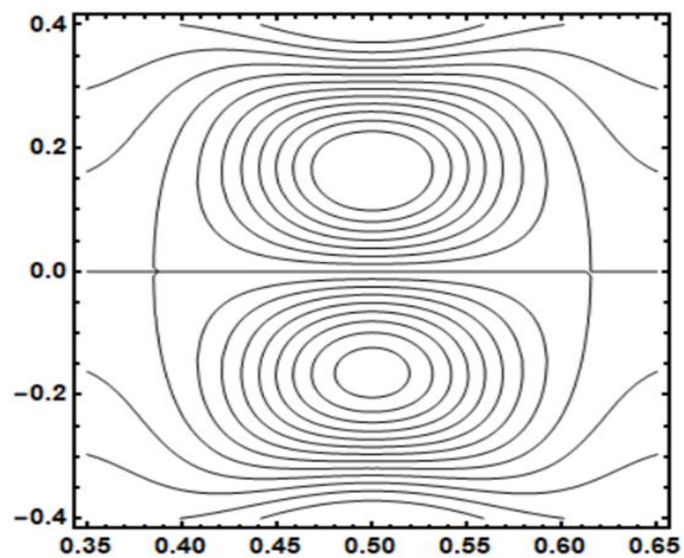
Figure 4.3 Variation of porosity parameter σ on streamlines.



(a) $\Gamma = 0$



(b) $\Gamma = 0.2$



(c) $\Gamma = 0.6$

Figure 4.4 Variation of slip parameter Γ on streamlines.

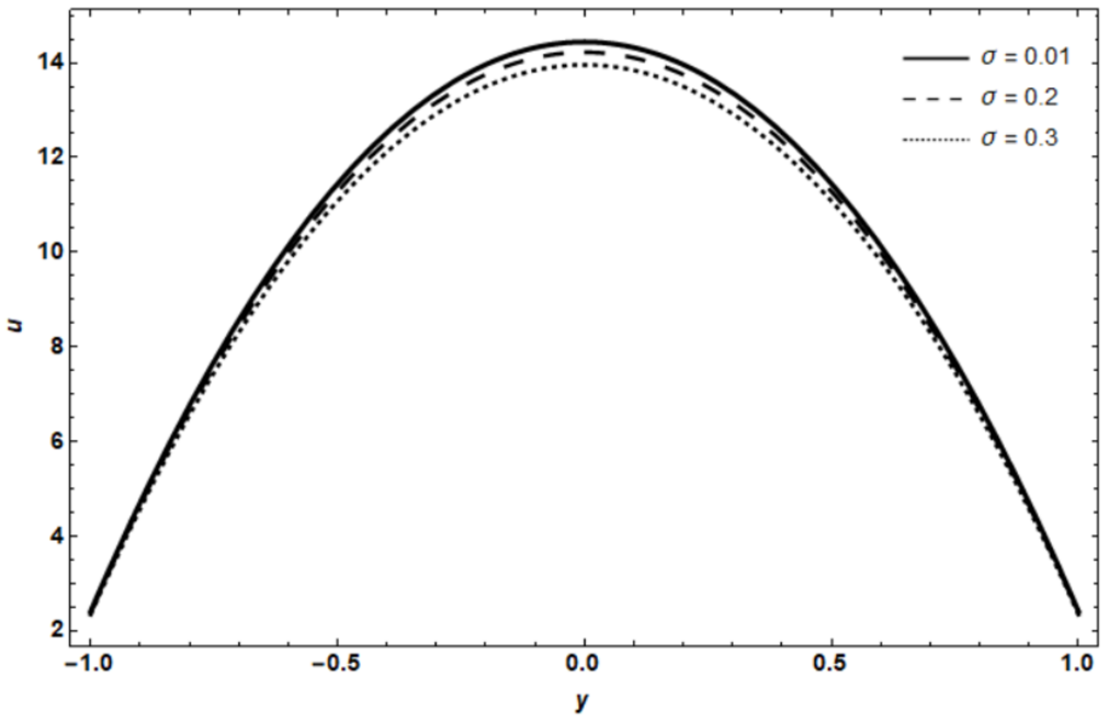


Figure 4.5 Variation of porosity parameter σ on Velocity field.

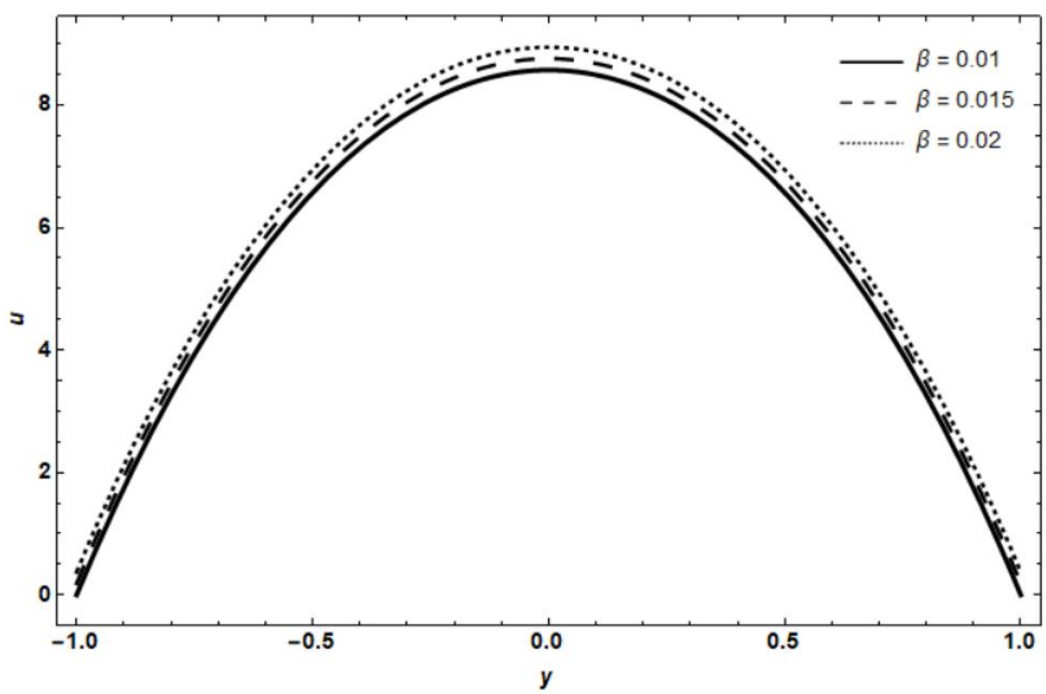


Figure 4.6 Variation of Sutterby fluid parameter β on Velocity field.

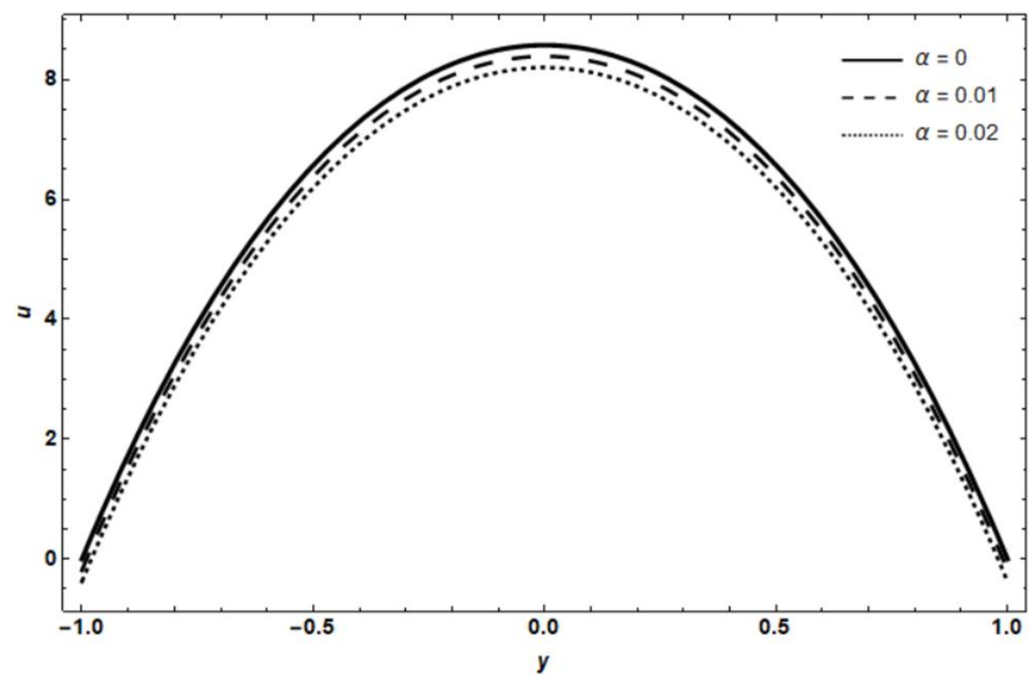


Figure 4.7 Variation of inclination α on Velocity field.

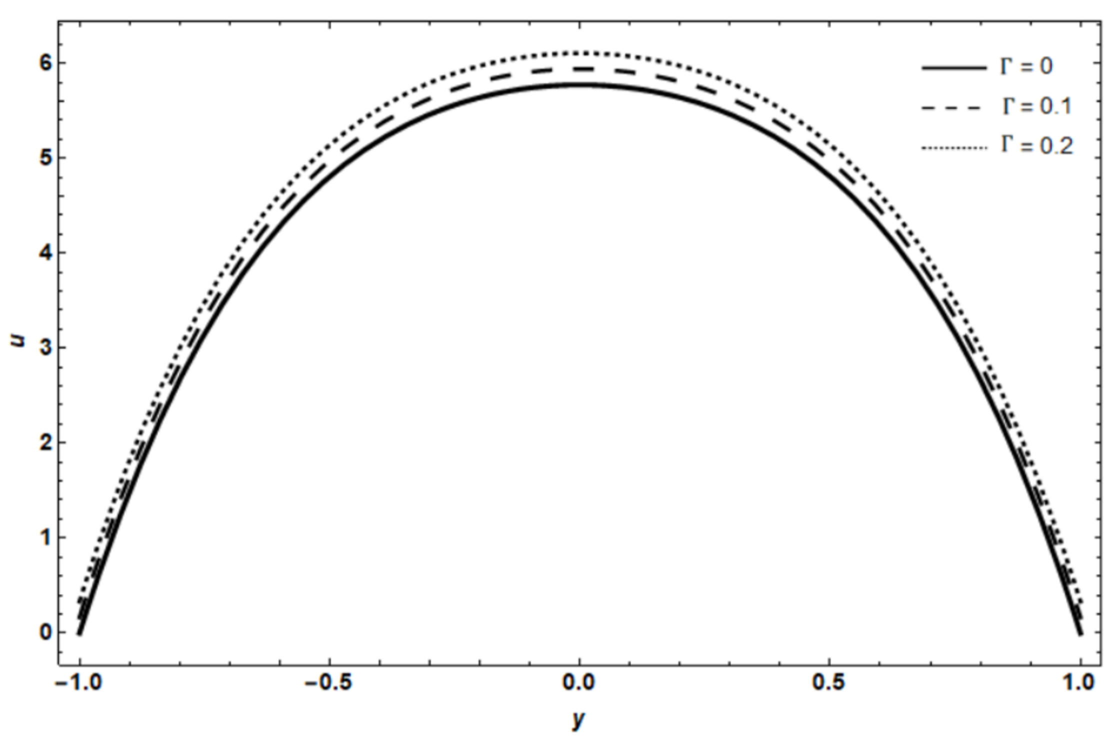


Figure 4.8 Variation of slip parameter Γ on velocity field.

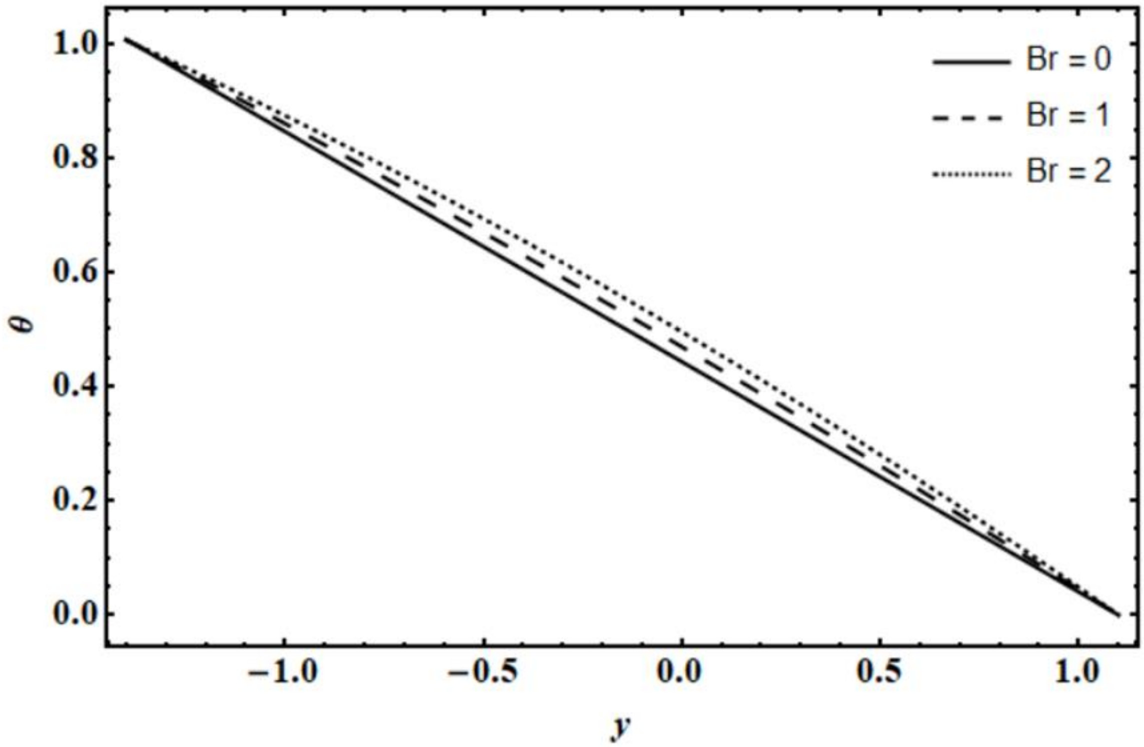


Figure 4.9 Variation of Brinkman number Br on Temperature field.

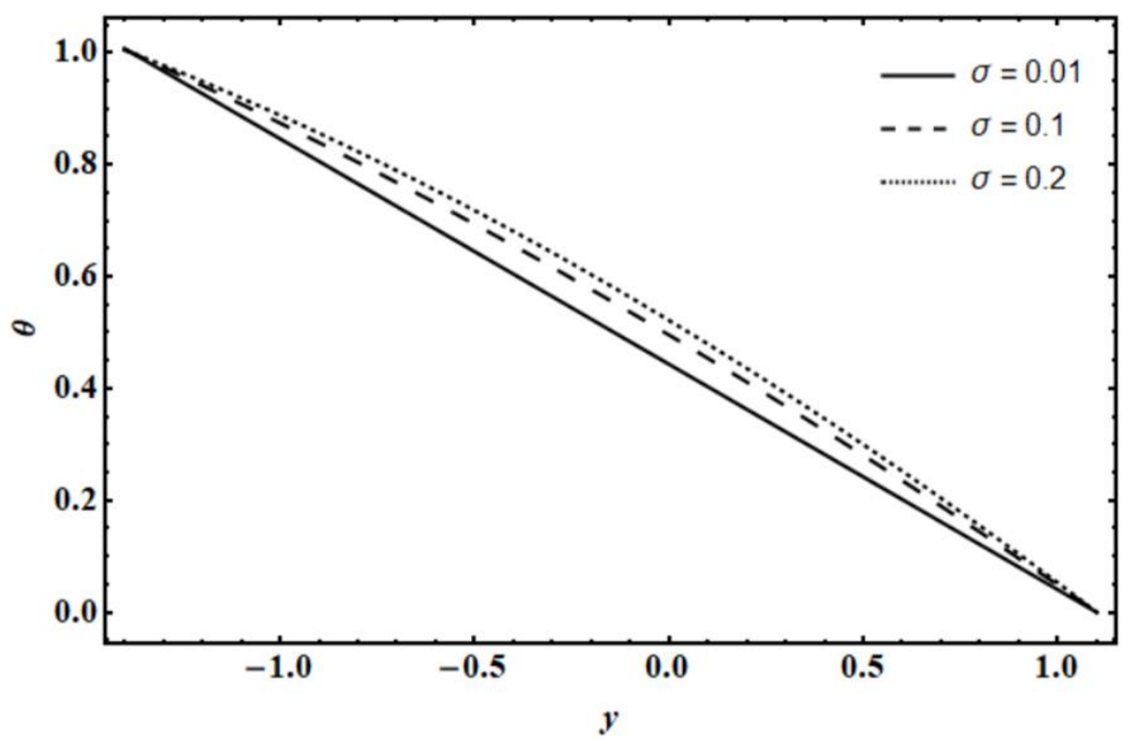


Figure 4.10 Variation of porosity parameter σ on Temperature field.

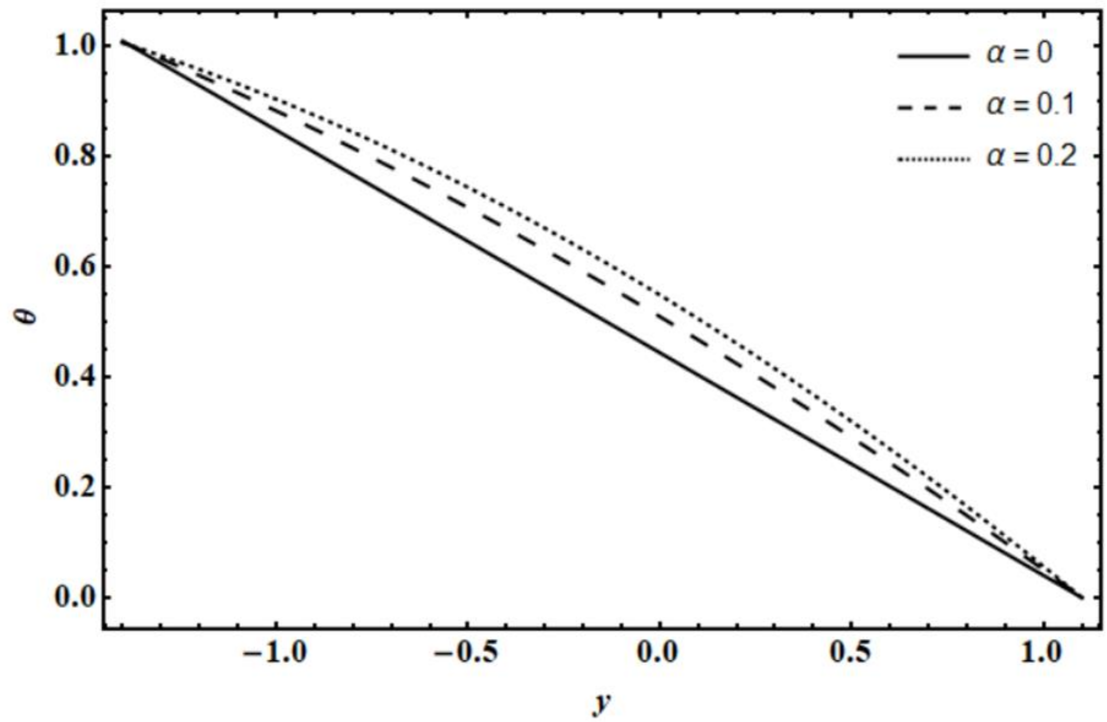


Figure 4.11 Variation of inclination α on Temperature field.

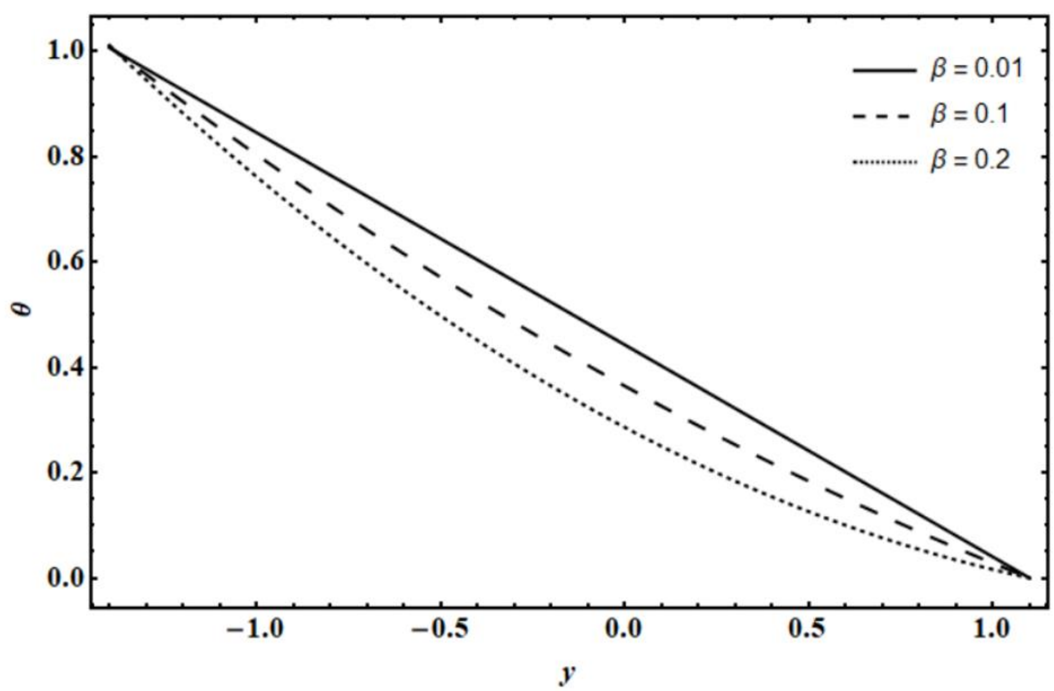


Figure 4.12 Variation of Sutterby fluid parameter β on Temperature field.

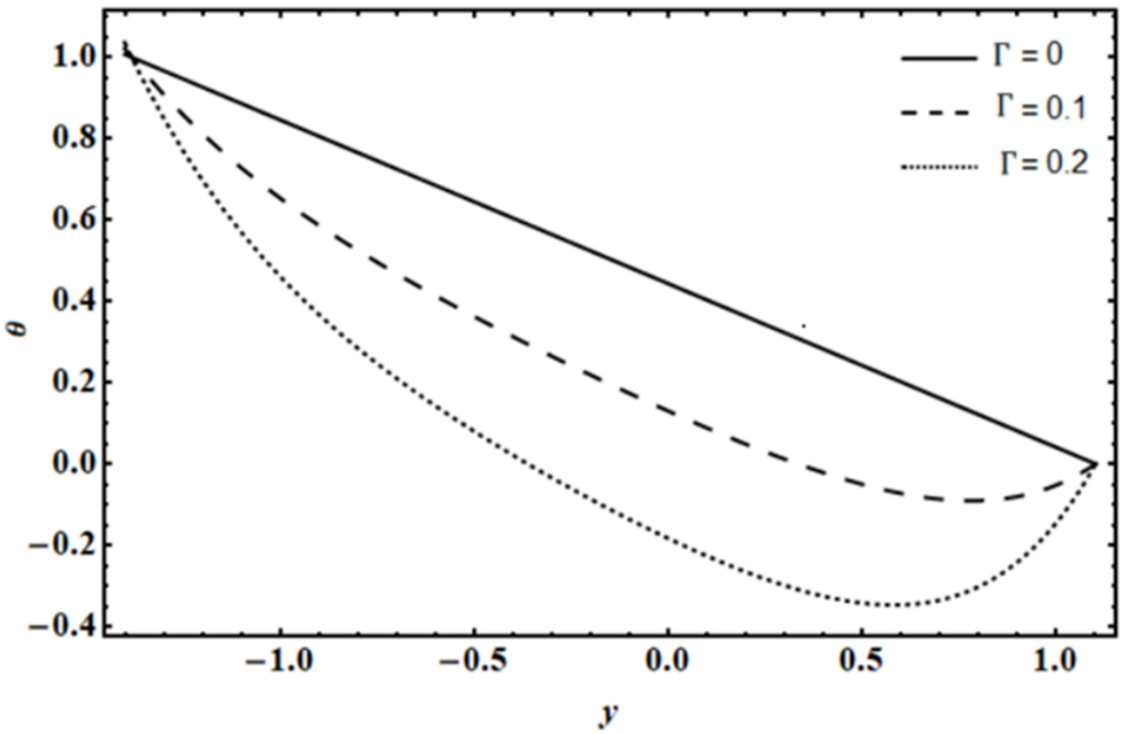


Figure 4.13 Variation of slip parameter Γ on Temperature field.

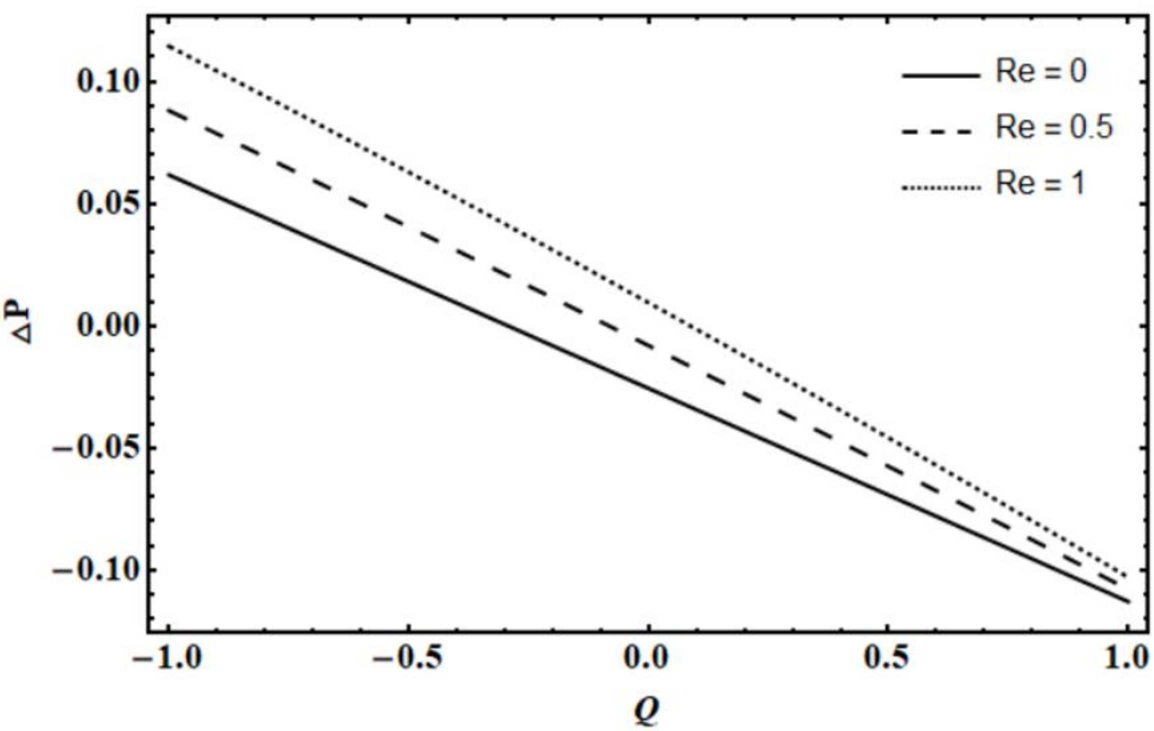


Figure 4.14 Variation of Reynolds number Re on pressure.

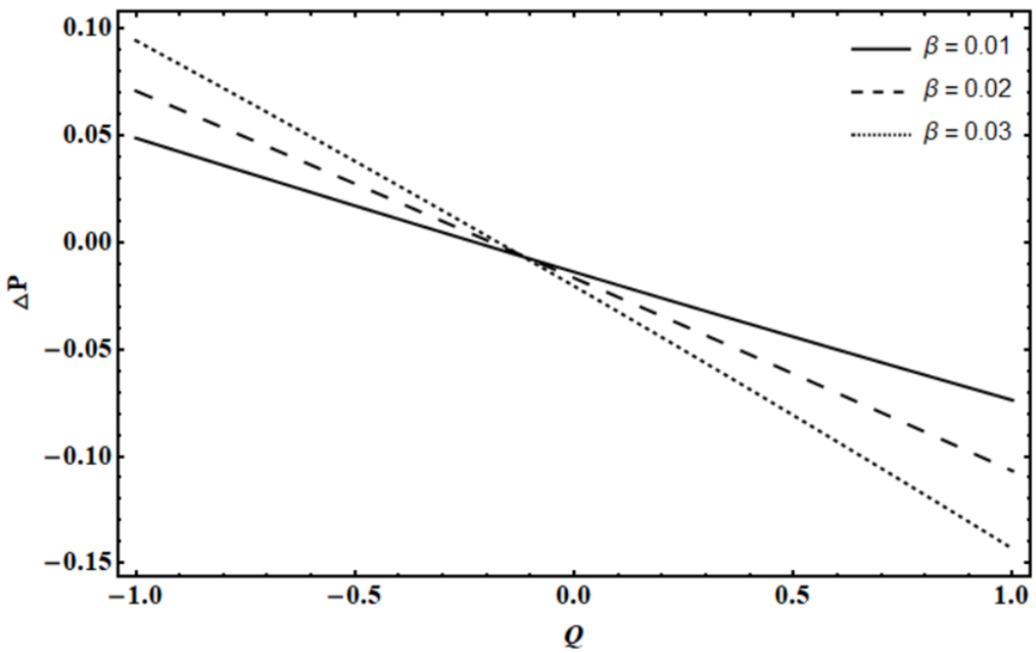


Figure 4.15 Variation of Sutterby fluid parameter β on Pressure.

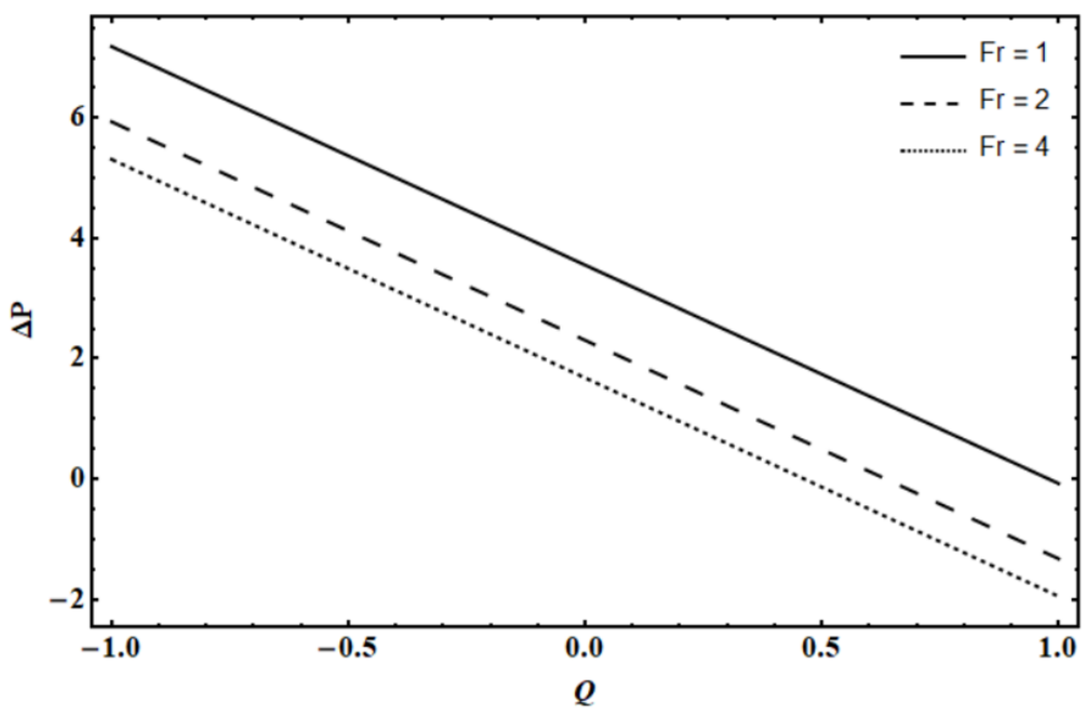


Figure 4.16 Variation of Froude number Fr on Pressure.

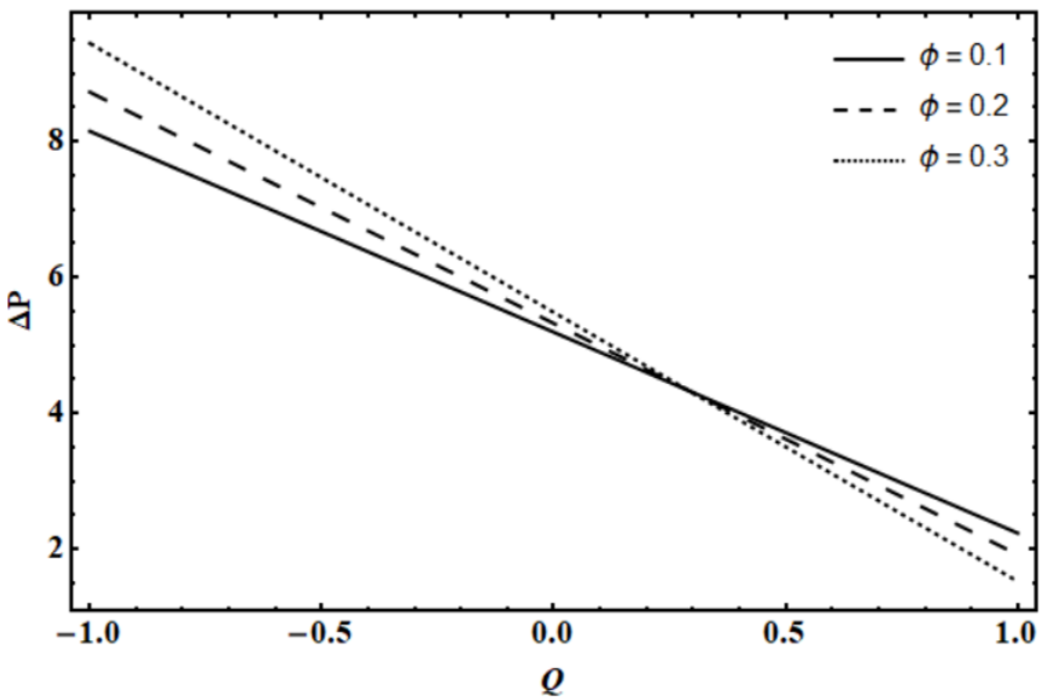


Figure 4.17 Variation of Amplitude ratio ϕ on Pressure.

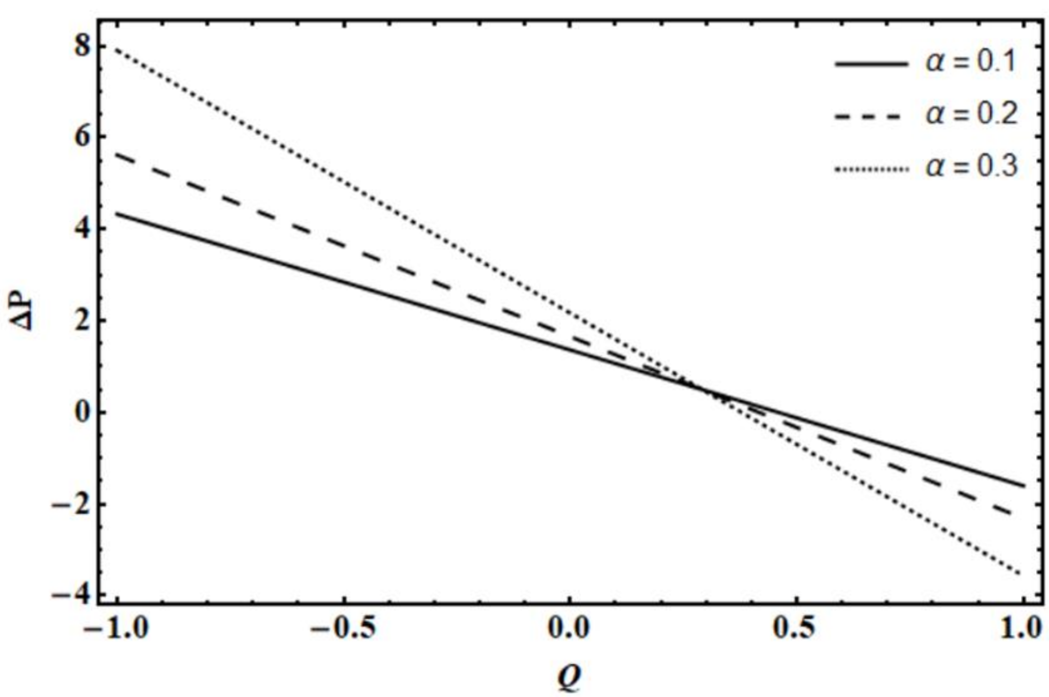


Figure 4.18 Variation of inclination α on pressure.

CHAPTER 5

CONCLUSION AND FUTURE WORK

5.1 Conclusion

This study centers around the peristaltic movement of a Sutterby non-Newtonian liquid in a slanted channel with walls that move in a sinusoidal wave example to duplicate peristalsis. Utilizing the suppositions of long frequencies and a low Reynolds number, the liquid's stream is researched inside a moving reference outline that compares to the wave speed. The stream conduct is characterized by a bunch of fractional differential conditions (PDEs) that typify the channel's liquid elements. Graphical investigation is utilized to show what different actual boundaries mean for essential qualities like temperature, strain, and speed, offering a total comprehension of how these components interface inside the liquid stream framework.

5.2 Significant Results

Several elements are important in determining streamline behavior. Increasing the Sutterby fluid parameter (β) results in a smaller trapped bolus and fewer streamlines. Increasing the channel inclination (α) leads to larger trapped boluses and increased circulation rates. Higher values of the porosity parameter (σ) and slip parameter (Γ) lead to larger bolus sizes and increased circulation. Several parameters affect the fluid's velocity within the channel. As the porosity parameter (σ) increases, velocity falls, indicating slower flow. As the Sutterby fluid parameter (β) increases, velocity decreases. raising the channel inclination (α) decreases velocity due to resistance, but raising the slip parameter (Γ) also reduces velocity within the channel.

Temperature behavior is affected by characteristics like the Brinkman number (B) and the porosity parameter (σ). As these factors rise, the temperature normally falls. Increasing the tilt (α) and slip parameter (Γ) reduces temperature, indicating that these factors contribute to a cool flow environment.

The influence of Sutterby fluid parameter (β) varies by area. In the pumping zone, increasing (β) decreases (Q), but in the free and co-pumping regions, (Q) increases. The Froude number (Fr) has a similar effect as it grows, the pumping rate falls in all locations. The amplitude ratio (ϕ) impacts (Q) differently depending on the region: it decreases (Q) in pumping and free pumping regions while increasing it in co-pumping regions. Increasing channel inclination (α) decreases flow rate in the pumping region, but increases it in the free and co-pumping regions.

5.3 Future Work

This examination can be created in the future to incorporate different liquid models. For instance, higher-request liquids, like 3rd or 4th grade liquids, could be examined to assemble additional data. Elective liquid models, like the Williamson, Ellis, Maxwell, and Jaffery models, could likewise be examined to decide the effect of various liquid qualities under comparative slip circumstances. The review might be extended by incorporate other body powers in the model. Besides, considering different calculations, like a porous medium or a tube-shaped channel, would give a more complete information on these liquid elements in fluctuating circumstances.

REFERENCES

[1] Spurk, J., & Aksel, N. (2007). *Fluid mechanics*. Springer Science & Business Media.

[2] Latham, T. W. (1966). *Fluid motions in a peristaltic pump* (Doctoral dissertation, Massachusetts Institute of Technology).

[3] Raju, K. K., & Devanathan, R. (1972). Peristaltic motion of a non-Newtonian fluid. *Rheologica Acta*, *11*(2), 170-178.

[4] Shapiro, A. H., Jaffrin, M. Y., & Weinberg, S. L. (1969). Peristaltic pumping with long wavelengths at low Reynolds number. *Journal of fluid mechanics*, *37*(4), 799-825.

[5] Vaidya, H., Prasad, K. V., Choudhari, R., Keriyappa, S., Gudekote, M., & Shetty, J. (2023). Partial slip effects on MHD peristaltic flow of Carreau-Yasuda fluid (CY) through a planner micro- channel. *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences*, *104*(2), 65-85.

[6] Imran, N., Tassaddiq, A., Javed, M., Alreshidi, N. A., Sohail, M., & Khan, I. (2020). Influence of chemical reactions and mechanism of peristalsis for the thermal distribution obeying slip constraints: Applications to conductive transportation. *Journal of Materials Research and Technology*, *9*(3), 6533- 6543.

[7] Ahmed, B., Nisar, Z., & EI-Sherbeeny, A. M. (2024). Numerical study for MHD peristaltic flow of nanofluid with variable viscosity in the porous channel. *ZAMM-Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik*, *104*(3), e202300694.

[8] Salih, A. W. (2021). Influence of Rotation, Variable Viscosity and Temperature on Peristaltic Transport in an Asymmetric Channel. *Turkish Journal of Computer and Mathematics Education (TURCOMAT)*, *12*(6), 1047-1059.

[9] Hayat, T., Nawaz, S., Alsaedi, A., & Ahmad, B. (2020). Entropy analysis for the peristaltic flow of third grade fluid with variable thermal conductivity. *The European Physical Journal Plus*, *135*(5), 1-18.

[10] Hadimani, Balachandra, Rajashekhar Choudhari, Prathiksha Sanil, Hanumesh Vaidya, Manjunatha Gudekote, Kerehalli Vinayaka Prasad, and Jyoti Shetty. "The influence of variable fluid properties on peristaltic transport of Eyring Powell fluid flowing through an inclined uniform channel." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 102, no. 2 (2023): 166-185.

[11] Hasona, W. M., Almalki, N. H., ElShekhipy, A. A., & Ibrahim, M. G. (2020). Combined effects of variable thermal conductivity and electrical conductivity on peristaltic flow of pseudoplastic nanofluid in an inclined non-uniform asymmetric channel: applications to solar collectors. *Journal of Thermal Science and Engineering Applications*, *12*(2), 021018.

[12] Hayat, T., Khan, A. A., Bibi, F., & Alsaedi, A. (2021). Entropy minimization for magneto peristaltic transport of Sutterby materials subject to temperature dependent thermal conductivity and non-linear thermal radiation. *International Communications in Heat and Mass Transfer*, *122*, 105009.

[13] Eldabe, N. T., & Ramadan, S. F. (2020). Impacts of peristaltic flow of micropolar fluid with nanoparticles through a porous medium under the effects of heat absorption and wall properties: Homotopy perturbation method. *Heat Transfer*, *49*(2), 889-908.

[14] Elmhedy, Y., Abd-Alla, A. M., Abo-Dahab, S. M., Alharbi, F. M., & Abdelhafez, M. A. (2024). Influence of inclined magnetic field and heat transfer on the peristaltic flow of Rabinowitsch fluid model in an inclined channel. *Scientific Reports*, *14*(1), 4735.

[15] Sanil, Prathiksha, Manjunatha Gudekote, Rajashekhar Choudhari, Hanumesh Vaidya, and Kerehalli Vinayaka Prasad. "Heat transfer analysis on peristaltic transport of Sisko fluid in an inclined uniform channel." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 103, no. 2 (2023): 157-179.

[16] Salahuddin, T., Muhammad Habib Ullah Khan, Mair Khan, Sayer Obaid Alharbi, and Yu-Ming Chu. "Peristaltically driven flow of hybrid nanofluid in a sinusoidal wavy channel with heat generation." *Physica Scripta* 96, no. 2 (2020): 025205.

[17] Eldesoky, I. M., Abumandour, R. M., Kamel, M. H., & Abdelwahab, E. T. (2019). The combined influences of heat transfer, compliant wall properties and slip conditions on the peristaltic flow through tube. *SN Applied Sciences*, *1*, 1-16.

[18] Boujelbene, M., Hadimani, B., Choudhari, R., Sanil, P., Gudekote, M., Vaidya, H., ... & El Din, S. M. (2023). Impact of Variable Slip and Wall Properties on Peristaltic Flow of Eyring–Powell Fluid Through Inclined Channel: artificial intelligence based perturbation technique. *Fractals*, *31*(06), 2340140.

[19] Gudekote, Manjunatha, Hanumesh Vaidya, Divya Baliga, Rajashekhar Choudhari, and Kerehalli Vinayaka Prasad. "The effects of convective and porous conditions on peristaltic transport of non-Newtonian fluid through a non-uniform channel with wall properties." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 63, no. 1 (2019): 52-71.

[20] Gudekote, Manjunatha, Rajashekhar Choudhari, Prathiksha Sanil, Balachandra Hadimani, Hanumesh Vaidya, and Kerehalli Vinayaka Prasad. "MHD Effects on the Peristaltic Transport of Non-Newtonian Eyring–Powell Fluid with Heat and Mass Transfer in an Inclined Uniform Channel." *Arabian Journal for Science and Engineering* (2024): 1-17.

[21] Akram, J., Akbar, N. S., & Tripathi, D. (2020). Blood-based graphene oxide nanofluid flow through capillary in the presence of electromagnetic fields: A Sutterby fluid model. *Microvascular Research*, *132*, 104062.

[22] Imran, N., Javed, M., Sohail, M., Thounthong, P., & Abdelmalek, Z. (2020). Theoretical exploration of thermal transportation with chemical reactions for sutterby fluid model obeying peristaltic mechanism. *Journal of Materials Research and Technology*, *9*(4), 7449-7459.

[23] Mohammed, A. A., & Hummady, L. Z. (2023). Influence of heat transform and rotation of sutterby fluid in an asymmetric channel. *Iraqi Journal of Science*, 5766-5777.

[24] Atlas, Faseeha, Maryiam Javed, and Naveed Imran. "Effects of heat and mass transfer on the peristaltic motion of Sutterby fluid in an inclined channel." *Multidiscipline Modeling in Materials and Structures* 16, no. 6 (2020): 1357-1372.

[25] Hayat, T., Hina Zahir, M. Mustafa, and A. Alsaedi. "Peristaltic flow of Sutterby fluid in a vertical channel with radiative heat transfer and compliant walls: a numerical study." *Results in physics* 6 (2016): 805-810.

[26] Ammar, Neeran, and Hayat A. Ali. "Mathematical modelling for peristaltic flow of sutterby fluid through tube under the effect of endoscope." *Iraqi Journal of Science* (2023): 2368-2381.

[27] Balachandra, H., Rajashekhar, C., Mebarek-Oudina, F., Manjunatha, G., Vaidya, H., & Prasad, K. V. (2021). Slip effects on a ree-eyring liquid peristaltic flow towards an inclined channel and variable liquid properties. *Journal of Nanofluids, 10*(2), 246-258.

[28] Rafiq, M., Sajid, M., Alhazmi, S. E., Khan, M. I., & El-Zahar, E. R. (2022). MHD electroosmotic peristaltic flow of Jeffrey nanofluid with slip conditions and chemical reaction. *Alexandria Engineering Journal, 61*(12), 9977-9992.

[29] Mohammed, Asmaa A., and Liqaa Zeki Hummady. "The effect of heat transformation and rotation on sutterby fluid peristaltic flow in an inclined, asymmetric channel with the porousness." *Baghdad Science Journal* 9 (2023): 13.

[30] Sayed, Hamed M., Emad H. Aly, and K. Vajravelu. "Influence of slip and convective boundary conditions on peristaltic transport of non-Newtonian nanofluids in an inclined asymmetric channel." *Alexandria Engineering Journal* 55, no. 3 (2016): 2209-2220.

[31] Ibrahim, M. G., and M. Y. Abou-Zeid. "Influence of variable velocity slip condition and activation energy on MHD peristaltic flow of Prandtl nanofluid through a non-uniform channel." *Scientific Reports* 12, no. 1 (2022): 18747.

[32] Chinnasamy, P., Sivajothi, R., Sathish, S., Abbas, M., Jeyakrishnan, V., Goel, R., ... & Loganathan, K. (2024). Peristaltic transport of Sutterby nanofluid flow in an inclined tapered channel with an artificial neural network model and biomedical engineering application. *Scientific Reports, 14*(1), 555.

[33] Vaidya, Hanumesh, Oluwole Daniel Makinde, Rajashekhar Choudhari, Kerehalli Vinayaka Prasad, Sami Choudhari, Rajashekhar, Oluwole D. Makinde, Fateh Mebarek-Oudina, Hanumesh Vaidya, Kerehalli V. Prasad, and Palluru Devaki. "Analysis of third-grade liquid

under the influence of wall slip and variable fluid properties in an inclined peristaltic channel." *Heat Transfer* 51, no. 7 (2022): 6528-6547.

[34] Ullah Khan, and Kuppalapalle Vajravelu. "Peristaltic flow of non-Newtonian fluid through an inclined complaint nonlinear tube: application to chyme transport in the gastrointestinal tract." *The European Physical Journal Plus* 135, no. 11 (2020): 1-15.

[35] Divya, B., G. Manjunatha, C. Rajashekhar, Vaidya Hanumesh, and K. Prasad. "Impact of variable liquid properties on Peristaltic mechanism of convectively heated Jeffrey fluid in a slippery elastic tube." *Frontiers in Heat and Mass Transfer* 12 (2018): 15.

[36] Divya, B., G. Manjunatha, C. Rajashekhar, Vaidya Hanumesh, and K. Prasad. "Impact of variable liquid properties on Peristaltic mechanism of convectively heated Jeffrey fluid in a slippery elastic tube." *Frontiers in Heat and Mass Transfer* 12 (2018): 15.

[37] Divya, B. B., Manjunatha, G., Rajashekhar, C., Vaidya, H., & Prasad, K. V. (2020). The hemodynamics of variable liquid properties on the MHD peristaltic mechanism of Jeffrey fluid with heat and mass transfer. *Alexandria Engineering Journal*, *59*(2), 693-706.

[38] Ajithkumar, M., Lakshminarayana, P., & Vajravelu, K. (2023). Peristaltic flow of bioconvective Ree– Eyring nanofluid through an inclined elastic channel with partial slip effects. *Journal of Applied Physics*, *134*(15).

[39] Choudhari, Rajashekhar, Hanumesh Vaidya, Kerehalli Vinayaka Prasad, Manjunatha Gudekote, M. Ijaz Khan, Mehdi Akermi, Rym Hassani, Hala A. Hejazi, and Shahid Ali. "Analysis of peristalsis blood flow mechanism using non-newtonian fluid and variable liquid characteristics." *Results in Engineering* 21 (2024): 101842.

[40] Rajashekhar, C., G. Manjunatha, Hanumesh Vaidya, B. Divya, and K. Prasad. "Peristaltic flow of Casson liquid in an inclined porous tube with convective boundary conditions and variable liquid properties." *Frontiers in Heat and Mass Transfer (FHMT)* 11 (2018).

[41] Choudhari, Rajashekhar, Hanumesh Vaidya, Kerehalli Vinayaka Prasad, Manjunatha Gudekote, M. Ijaz Khan, Mehdi Akermi, Rym Hassani, Hala A. Hejazi, and Shahid Ali. "Analysis of peristalsis blood flow mechanism using non-newtonian fluid and variable liquid characteristics." *Results in Engineering* 21 (2024): 101842.

[42] F. H. Oyelami and M. S. Dada, “Unsteady magnetohydrodynamic flow of some non-Newtonian fluids with slip through porous channel,” *Int. J. Heat Technol.*, vol. 36, no. 2, pp. 709–713, 2018, doi: 10.18280/ijht.360237.

[43] Whitaker, Stephen. "Flow in porous media I: A theoretical derivation of Darcy's law." *Transport in porous media* 1 (1986): 3-25.

[44] M. Nazeer, A. Al-Zubaidi, F. Hussain, F. Z. Duraihem, S. Anila, and S. Saleem, “Thermal transport of two-phase physiological flow of non-Newtonian fluid through an inclined channel with flexible walls,” *Case Stud. Therm. Eng.*, vol. 35, Jul. 2022, doi: 10.1016/j.csite.2022.102146.

[45] Vijayaragavan, R., P. Tamizharasi, and A. Magesh. "Peristaltic motion of Non-Newtonian fluid under the influence of inclined magnetic field, porous medium and chemical reaction." Scientia Iranica (2024).

[46] B. Ahmed, D. Liu, Y. Zhang, and M. A. Hussien, “Peristaltic pumping of convective nanofluid with magnetic field and thermal radiation in a porous channel,” *Case Stud. Therm. Eng.*, vol. 53, no. October 2023, p. 103918, 2024, doi: 10.1016/j.csite.2023.103918.

[47] Shaughnessy, Edward J., Ira M. Katz, and James P. Schaffer. *Introduction to fluid mechanics*. Vol. 8. New York: Oxford University Press, 2005.