# EFFECTS OF MEGNETIC FIELD ON PERISTALTIC FLOW OF SECOND GRADE DUSTY FLUID IN AN INCLINED ASYMMETRIC CHANNEL

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# Effects of Magnetic Field on Peristaltic Flow of Second Grade Dusty Fluid in an Inclined Asymmetric Channel

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#### FACULTY OF ENGINEERING AND COMPUTING



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FACULTY OF ENGINEERIING & COMPUTER SCIENCE

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11<sup>th</sup>June, 2024 Date

# ABSTRACT

# Title: Effects of Magnetic Field on Peristaltic Flow of Second Grade Dusty Fluid in an Inclined Asymmetric Channel.

The primary goal of this thesis is to investigate the effects of magnetic field on the peristaltic flow of second-grade dusty fluid in an inclined asymmetric channel. The problem formulation has been developed for peristalsis of MHD second-grade dusty fluid. In addition the inclined asymmetric channels are taken. The modelled problem is solved by applying the perturbation technique. The stream functions are used to simplify the problem by reducing the number of depending variables. The graphs for fluid and solid particles velocity and pressure gradient are achieved using Mathematica software.

# **TABLE OF CONTENTS**

CHAPTER	TITLE	PAGE
	AUTHOR'S declaration	iv
	ABSTRACT	V
	TABLE OF CONTENTS	vi
	LIST OF FIGURES	X
	LIST OF SYMBOLS	xii
	ACKNOWLEDGEMENT	xiv
	DEDICATION	XV

<u>CHAPTER:1</u>	1
INTRODUCTION	1
1.1 <u>Fluid</u>	1
<b><u>1.2 Slip Condition</u></b>	1
<b><u>1.3 Magnetic FIELD</u></b>	2
<u>1.4 Peristalsis</u>	3
1.5 Non-Newtonian Fluid	4

<b><u>1.5.1 Second Grade Fluid</u></b>	4
<u>1.6 Dusty Fluid</u>	
1.7Asymmetric Channel	5
1.8 Thesis Contributions	6
1.9 Thesis Organization	
CHAPTER: 2	
LITERATURE REVIEW	
2.1 Overview of Related Literature	8
<u>CHAPTER 3</u>	13
BASIC DEFINITION	
3.1 Fluid Mechanics	
<u>3.2 Fluid</u>	
3.3 Types of Fluid	
3.3.1 Newtonian Fluid	14
3.3.2 Non-Newtonian Fluid	14
<u>3.3.3 Ideal Fluid</u>	14
3.3.4 Real Fluid	14
3.3.5 Compressible Fluid	15
3.3.6 Incompressible Fluid	15
3.4 Newton's Law of Constant Viscosity	
3.5 Types of Flow	
3.5.1 Steady and Unsteady Flow	
3.5.2 Uniform and Non-Uniform Flow	16
<b><u>3.5.3</u></b> One, Two and Three-Dimensional Flow	16
3.5.4 Rotational or Irrotational Fluid	
3.5.5 Laminar or Turbulent Flow	
<b><u>3.5.6 Creeping Flow</u></b>	17

3.6 Viscosity	17
3.7 Shear Thinning	17
3.8 Shear Thickening	18
3.9 Shear Stress	18
<u>3.10 Shear Strain</u>	18
3.11 Density	18
3.12 No Slip Condition	18
<u>3.13 MHD</u>	19
3.14 Continuity Equation	19
3.15 Newton's Second Law of Motion	19
<u>3.16 Mass</u>	20
3.17 Body Force	20
3.18 Surface Force	20
3.19 Pressure	20
3.20 Dimension	21
3.21 Reynolds Number	21
3.22 Streamline	21
3.23 Stream Function	21
3.24 Perturbation	22
CHAPTER 4	23
PERISTALTIC FLOW OF SECOND-GRADE DUSTY FLUID THROUGH A POROUS	<u>5</u>
MEDIUM IN AN ASYMMETRIC CHANNEL	23
4.1 Introduction	23
<b>4.2 Mathematical Formulation</b>	23
<b>4.3 Method of Solution</b>	28
4.3.1 Zeroth order Syestem	29
4.3.2 First order Syestem	29

<b><u>4.4 Results and Disscusion</u></b>	30
<u>CHAPTER 5</u>	45
Effects of Magnetic Field on Peristaltic Flow of Second Grade Dusty Fluid in an	
Asymmetric Channel	45
5.1Introduction	45
5.2 Mathematical Formulation	45
5.3 Method of Solution	49
5.3.1 Zeroth order Syestem.	49
5.3.2 First Order System	50
5.4 Results and Disscusion	50
CHAPTER 6	60
CONCLUSION.	60
6.1 Significant Results	60
<u>6.2Future Work</u>	61
REFERENCES.	62

# LIST OF FIGURES

FIGURE NO.	TITLE	PAGE
4.1	Portrays the contour patterns for fluid particles for diverse values of $\alpha_1$	32
4.2	Portrays the contour patterns for fluid particles for diverse values of $\delta$	33
4.3	Portrays the contour patterns for fluid particles for diverse values of $\beta$	34
4.4	Portrays the contour patterns for fluid particles for diverse values of k	35
4.5	Portrays the contour patterns for fluid particles for diverse values of Reynolds number (Re)	36
4.6	Portrays the contour patterns for dust particles for diverse values of $\alpha_1$	37
4.7	Portrays the contour patterns for dust particles for diverse values of $\delta$	38

4.8	Portrays the contour patterns for dust particles for diverse values of $\beta$	39
4.9	Portrays the contour patterns for dust particles for diverse values of k	40
4.10	Portrays the contour patterns for dust particles for diverse values of Reynolds number (Re)	41
4.11	Deviation of $\Delta p(P)$ with Q for diverse values of Re.	42
4.12	Deviation of $\Delta p(P)$ with Q for diverse values of $\delta$ .	42
4.13	Deviation of $\Delta p(P)$ with Q for diverse values of $\beta$ .	43
4.14	Deviation of $\Delta p(P)$ with Q for diverse values of k.	43
4.15	Deviation of $\Delta p(P)$ with Q for diverse values of $\alpha_1$ .	44
5.1	Geometry of the problem.	46
5.2	Velocity distribution of the fluid for diverse values of M.	53
5.3	Velocity distribution of the fluid for diverse values of $\beta$	53
5.4	Velocity distribution of the fluid for diverse values of $\delta$ .	54
5.5	Velocity distribution of the fluid for diverse values of $\alpha_1$ .	54
5.6	Velocity distribution of the dust particles for diverse values of M.	55
5.7	Velocity distribution of the dust particles for diverse values of $\beta$	55
5.8	Velocity distribution of the dust particles for diverse values of $\delta$	56
5.9	Velocity distribution of the dust particles for diverse values of $\alpha_1$ .	56
5.10	Effects of M on pressure.	57

5.11	Effects of $\beta$ on pressure.	57
5.12	Effects of $\delta$ on pressure.	58
5.13	Effects of $\alpha_1$ on pressure.	58
5.14	Effects of Re on pressure.	59
5.15	Effects of $\gamma$ on pressure.	59

# LIST OF SYMBOLS

S	Extra Stress Tensor
ρ	Density
Р	Pressure
μ	Viscosity
$u_s$ , $v_s$	Velocities of Solid Particles
a	Amplitude
c	Wave Speed
λ	Wavelength
М	Magnetic Field
α <sub>1</sub>	Second Grade Parameter
$\psi, arphi$	Stream Function
β	Slip Parameter
u	Velocity of Fluid in x Direction
v	Velocity of fluid in y Direction
Re	Reynold Number
γ	Inclination of the Channel
d	Half Width of the Channel
Ø	Phase difference
δ	Wave Number

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# **DEDICATION**

# To Ammi Gi

#### **CHAPTER 1**

# INTRODUCTION

#### 1.1 Fluid

Material that constantly changes shape under applied shear stress is known as a fluid. Examples include water, blood, and honey. Two primary branches further classify fluids. One is Newtonian and the other is non-Newtonian based on the stress-strain connection. Such fluids that follow Newton's law of constant viscosity are known as Newtonian fluids. In contrast, fluids that are known as non-Newtonian are such fluids that do not follow Newton's law of constant viscosity. It is commonly recognized that for industrial, medicinal, and technical uses, non-Newtonian fluids are preferable to Newtonian fluids. Numerous consumer items include significant levels of glass or carbon fibers, paints, lubricants that have polymer additives, and biological fluids that are non-Newtonian in origin. According to how they behave to applied shear stresses, non-Newtonian fluids may be classified into several groups. The viscosity may vary when force is applied to either more liquid or more solid in non-Newtonian fluids. A non-Newtonian fluid is, for instance, ketchup, which thins out when disturbed. A non-Newtonian fluid may possess just one or all of the given characteristics, such as creeping, shear thinning, shear thickening, etc.

#### 1.2 Slip Condition

The basic premise of the slip boundary condition, also identified as the velocity-offset boundary condition where the velocity function is discontinuous, i.e., that there is comparative motion among the fluid and the boundary. Navier introduced the concept of a slip first. Effects of fluid slipping on the wall may be noticed in many different kinds of domains, including microand nanochannels. It is frequently employed when circumstances where a moving plate is sprayed using a very thin layer of oil. The thermal emission has a clear detrimental effect on weak radiation strength under laminar flow configurations. At low shear rates, this condition will be sufficient (Navier slip). However, the slip condition of Navier declines as the slip length rapidly expands with the slide rate. Due to this, numerous situations (a non-Navier slip) also take into consideration the second-order slip boundary condition. The existence of slip has previously been documented and required for years in the industrial area since the Fluid Mechanics state of no-slip boundary is not always appropriate for multiple situations that involve complicated fluids. Complex fluids such as mixtures, suspensions, foam, and polymer solutions are examples that generally generate a wall slip with a boundary slip at the wall. For technical uses like cleaning prosthetic heart valves and interior cavities, this phenomenon is important. On the other hand, the original investigation into the boundary circumstances for linear slip was carried out by Navier and Maxwell. There are a variety of Newtonian and non-Newtonian fluids, with particulate fluids such as emulsions, suspensions, and polymer solutions in which there may be a slip among the fluid and the boundary. As a result, slip flow problems are crucial on both the fixed and moving boundaries. As can be shown from a review of the literature, the presence of a slip boundary condition can have a considerable impact on velocity profiles.

As far as we are aware, the study of the slip impact on the fluid flow has not received a great deal of interest. Many researchers did, however, address the importance of the slip condition for minimizing skin friction and provide a notion of what it entails. It has been determined that the slip condition considerably raises flow pressure when the typical size of the flow system is low. This has been done by examining the slip impact on various fluid flows. As a result of the ability, they have to minimize skin-friction drag, superhydrophobic surfaces have recently gained significant relevance and are the focus of extensive study. In stream-wise and span-wise paths, the slip-length values of the superhydrophobic surfaces change depending on the direction in which they travel. Also receiving significant consideration are the impacts on flows over various surfaces.

#### **1.3 Magnetic Field**

A magnetic field communicates the magnetic effect when moving electric charges, electric currents, and magnetic materials are present. A force upright to the magnetic field and its velocity

acts on a moving charge in the field known as magnetic. Furthermore, the magnetic field aligns with the positioning of the curled fingers. A vital area of the science of magneto is the use of the magnetic field in medicine. By revealing a person to a field known as magnetic, it is possible to enhance blood flow throughout their body as a prospective alternative to the previously available medicine with its negative side effects. Electrohydrodynamics (EHD), which discusses the effects of electric forces, and (MHD) magnetohydrodynamics which discusses collaboration between fields known as magnetic and fluid transmitters of electricity, are the two main categories into which the study of various magnetic fields and fluid interactions can be separated. Tariq *et al.* [1] studied the MHD supports to recognition of the movement of electrically conducting fluids. MHD has evolved in a broad field of engineering and physical research that includes anything from fluid metal movements in the metallurgic sector to astronomical solar and planetary flows. The effects of magnetic fields on liquid movement and heat transmission and their use in the health and business fields have not been the subject of scientific research that has been duplicated.

#### 1.4 Peristalsis

The term peristalsis comes from the Greek word peristalikos, which means clasping and compressing. In 1966, Peristalsis was initially described by Latham [2]. Shapiro *et al.* [3] conducted more research on this piece. The peristaltic flow of second-grade dusty fluid has attracted great interest and study. So, the attention of scientific and mathematical communities towards this is quite reasonable. Many numerical and analytical techniques were used for the study. It has been broadly studied in the papers and literature and it explains many complex and nonlinear wave phenomena. Therefore, it is now becoming more and more effective in the field of fluid mechanics. These flows also offer efficient methods for sanitary fluid transportation, and as a result, they are used in industrial peristaltic pumping and medical devices. For example, the printing industry uses mechanical roller pumps to move viscous fluids, and the nuclear industry uses peristaltic pumping to move toxic fluids. Many researchers have taken into account the theoretical and real-world aspects of peristaltic transportation since this process has become an important research subject. Polluted water contains dust granules, water purification plants purify such water to make it reusable. Peristaltic pumps are mounted in such plants and magnetic effect

may be utilized to make the sedimentation process more efficient. This study may contribute to such a situation.

#### 1.5 Non-Newtonian Fluid

Such fluids are known as non-Newtonian fluids that do not act upon Newton's law of viscosity. Second-grade fluid is a sub-class of non-Newtonian fluid. Following is the description of second-grade fluid.

#### 1.5.1 Second Grade Fluid

Non-Newtonian fluids are a diverse family of fluids that relate the connection between shear rate and shear stress relative to non-linearity; hence, there is no one general governing model that encompasses all of the non-Newtonian fluid's traits and characteristics. In addition, to account for the additional non-linear factors, the mathematical theories and formulas become more complicated. Here the most recent researches on fluids that is non-Newtonian has been discussed. They are primarily divided into fluids of the integral, rate, and differentiation types. According to the fluid classifications above, the rate type of fluids falls into the second-grade fluid category and has memory-related impacts (the retarded phenomena) and elastic characteristics (the relaxing time). Due to its numerous uses in science and technology, it is clear from the previous works that the dynamics of maximum movements for rate-type fluids/systems have received significant attention by using second-grade fluid has a velocity field with up to second order derivatives in stress-strain tensor connection. Because of the specific explanation of the structure connection, second-grade fluid follows have distinctive features.

#### 1.6 Dusty Fluid

Fluid and tiny dust particles are combined to form a dusty fluid. A dispersion of solid elements in fluids (liquid or gas) fluxes causes the phenomena to occur presented by Wei et al. [4]. A couple of examples are the way dusty air moves in fluidization issues and the way little dust particles condense to make rainfall. Recent research has shown a particular interest in the flow patterns of dusty fluid models. Numerous kinds of mechanical applications use fluid flows and dust particles, including transportation activities, the making of cement and steel, flying ash from heating plants, and the cooling effects of air conditioners. There are several important applications for two-phase flows when solid elements are dispersed in a second-class fluid. In recent years, a number of studies examined the flow of fluid with suspended dust particles. Environmental deterioration, combustion, petroleum, polymer and geophysical processes, refrigeration, contaminated soil, air, and water, dust or fumes in the gas cooling system, cultivation, fossil oil purification, polymer technology, and tint systems are all instances where studies about the flow of dusty fluids are very useful. Dusty fluid flows are two phases since the fluid is mixed with the particles. The combination of airborne dust particles with water when it rains and the earth's withdrawal of oil and gas is a great instance of fluids with dust. The phenomena of dust particles in fluids that are Newtonian and others which is non-Newtonian have been considered by many academics in recent years. The work is relevant in air pollution, automobile emissions of smoke and exhaust, industrial effluent emissions, air conditioner cooling effects, flying ash produced by thermal reactors, and raindrop formation, among others. Additionally, it aids in understanding lunar ash flow, which defines many characteristics of lunar soil. Numerous scholars used the dusty fluid model to study different flow configurations and boundary conditions while maintaining an interest in two-phase flows.

#### **1.7** Asymmetric Channel

In recent years, the asymmetric channel has attracted the interest of several mathematical and mathematical and industrial investigations. This flow is well recognized to be very important in many industrial and biochemical processes. Numerous of its uses have been discovered through contemporary investigations. The asymmetric channel is important in several studies that included second-grade fluid. Thus researchers are urged to conduct more research in the area of asymmetric channels. The peristaltic movement path on the substrate is selected to have varying hefts and phases in order to create path asymmetry. In an oscillating frame referencing that moves at the speed of the wave, the flow is examined. The majority of research efforts in the literature have been directed toward creating fluid rectifiers that increase rates of flow in a certain path because of resistance to flow depending on the position. The nozzle-diffuser and Tesla valves are two common asymmetric designs used by conventional fluidic rectifiers, and they can individually perform movement rectifying at high Reynolds numbers or by employing non-Newtonian fluids. The fluidic rectifier's main issue is that its efficiency valving is subpar, which causes flow drip. In recent years, certain synthetic fluidic devices have been proven by applying a material with a hardness, rigidity, or bending differential to a material to create a wettability slope. The effect of quantity, temperature, and generated field known as magnetic on the peristaltic movement of Prandtl nanofluid technology were all attempted to be jointly correlated in the present work. It was carried out in an asymmetrical channel.

#### **1.8** Thesis Contribution

This dissertation comprises a detailed study on MHD peristaltic second-grade fluid along with slip conditions through an asymmetric channel. The methodology contains a continuity equation with stream functions and uses the perturbation technique to develop the solution. Graphs are obtained by using the Mathematica.

# 1.9 Thesis Organization

The leftover thesis is distributed into the following chapters:

Chapter 2 covers the related literature work that has already been done by the researchers.

Chapter 3 deals with the basic concepts that are essential to understand the research work covered in this thesis.

Chapter 4 represents the review of the work done by Khan et al. [37].

**Chapter 5** presents the extension work presented in chapter 4. We have added MHD in this work. Also the channel is considered to be inclined.

**Chapter 6** is the conclusion of the research work presented in chapter 5 along with the future work.

In the end, the references list comprises all the sources that have been utilized in the whole research work.

#### **CHAPTER 2**

#### LITERATURE REVIEW

#### 2.1 Overview of Related Literature

Fluid mechanics has seen a rise in passion for the study of fluid flow under slip conditions. The existence of slip has previously been documented and crucial for years meanwhile the Fluid Mechanics assumption of a no-slip barrier is not at all times acceptable for different circumstances in complex fluids. Researchers have paid very close attention to slip conditions. Several investigations have focused on the fluid's slip effects at the boundary. Here is an overview of current studies carried out within this particular field.

Jamalabadi *et al.* [5] propose the use of an entropy generation minimization approach for the best possible arrangement of magnetohydrodynamic convection flows that are mixed in a channel that is vertical with slip boundary conditions and radiation from heat effects. Singh et al. [6] investigated mass allocation in a two-dimensional (MHD) slip flow of an electrically leading, incompressible, viscid, and steady flow of alumina water nanofluid across a flat plate. An analysis has been done to explore the issue of the fully established flow of a fourth-grade non-Newtonian fluid within two motionless plates in the context of an outwardly supplied uniform vertical magnetic field, according to Moakher *et al.* [7] research. At the channel wall, slip conditions are considered. The study of a conducting fluid's dynamics while a magnetic field is present is known as magnetohydrodynamics (MHD). A significant role MHD plays in a variety of fields, including fluid engineering, astronomy, technology, and geophysics. For instance, the production of MHD influence, MHD pumps, utilization in the movement of liquid metals and amalgams, , petroleum sector, cardiology, and also implication in the flow of mercury amalgams. Due to its numerous applications in astrophysics, geophysics, and fluid engineering, the study of MHD rotational flow has drawn a lot of attention from researchers. This investigation focused on the fractional secondgrade fluid's peristaltic flow within a cylindrical tube. When heat transmission is present, the effects of the magnetic field are considered. Equations of motion, energy, and continuity form the

9

foundation of mathematical modeling. The limitations of low Reynolds number and long wavelength have been applied to this analysis. For the temperature field, pressure gradient, and velocity, closed-form solutions are found. The new properties of friction force and pressure rise were analyzed using numerical integration by Hameed *et al.* [8].

Thermal radiation, mixed convection, and magnetohydrodynamics (MHD) are studied and equations for temperature, mass, and momentum were developed at long wavelengths and low Reynolds numbers by Tanveer et al. [9]. The pair stress fluid peristaltic motion in a twodimensional inclined path was examined by Rathod et al. [10]. Numerous physical factors' impacts on speed, pressure gradient, and frictional force have been addressed and quantitatively calculated. Graphs are used to discuss how certain critical parameters affect the results. Munawar et al. [11] studied the peristaltic motion of a fluid that's viscous with changing viscosity in a symmetric channel has been thoroughly thermodynamically analysed. In the existence of heat transfer and wall slide, the magnetic and slip effects of this channel were investigated by Sankad et al. [12]. A magnetic field that is external is meant to affect the system, and Joule heating is supposed to occur. The Debye-Hückel approximation has been used to compute the fluid's speed and temperature. Mallick et al. [13] taking into account low Reynolds numbers and long wavelength approximations. Effects of fluid pseudoelasticity and dilatancy on non-Newtonian fluid heat transfer and peristaltic flow in a non-uniform asymmetric channel has been examined by Tahir et al. [14]. The flow of silver-water nanofluid and silicon dioxide-water nanofluid via a porous media is compared using an inquiry that compares the combination of peristalsis and electroosmosisdriven flow of each fluid.By a symmetric flow channel, the fluid is moved. The flow problems are mathematically defined under the impact of a convection. Crossing channel walls requires no-slip conditions by Akram et al. [15]. When a hybrid nanofluid with single-walled and multi-walled carbon nanotubes is flowing peristaltically in a wavy rectangular duct, the eigenfunction expansion approach is used by Nadeem et al. [16].

Numerical analyses for the peristaltic movement of dusty nanofluids in a curve channel are carried out by Rashied *et al.* [17]. The impact of homogeneous-heterogeneous responses on the MHD peristaltic wave of Ellis fluid in curved channels are examined in the current inquiry using a model, and the results are discussed graphically through the ND Solve Mathematics programme.

Javed *et al.* [18] in their research work used fundamental relations for the Ellis fluid framework in the formulation of the problem. Also, use the perturbation technique. It is investigated that peristaltic flow in the duodenum, the first segment of the small intestine, is modelled as a C-shaped tube. The sculpture was built on a benchtop using a silicone tube full with a glycerol-water solution and bent through a spinning roller. Using wave speeds of 13 *mm/s* and contraction amplitudes of 34%, element image velocimetry was employed to image movement shapes for deformities simulating circumstances in the duodenum. Under the roller, fluid had reversed flow, travelling in the opposite direction of how peristaltic waves propagate, according to Palmada *et al.* [19]. The peristaltic process of Eyring Powell fluid through a non-uniform conduit has been studied. The study is done with wall qualities present and changing liquid properties influencing it, and the flow issue mathematically constructed by Gudekote *et al.* [20]. The mathematical simulation of the peristaltic movement for incompressible Sutter by fluid in the space among coaxial tubes, where the inner duct is stationary and the outer duct exhibits sinusoidal rhythmic oscillations along the channel walls, is provided in the paper by Ammar *et al.* [21].

Few related investigations were provided based on heat transfer, fractional fluid models, magnetized fluids, and a few other topics, even if the study on the second-grade model can still be pursued. Ramesh *et al.* [22] examined how heat transfer affects the peristaltic flow of a second-grade incompressible magnetohydrodynamic fluid in vertical symmetric and asymmetric channels. Selecting distinct amplitudes and phases for the peristaltic waves on the walls results in channel asymmetry. They concluded that the flow while moving at the wave's velocity occurs in the wave frame of reference. Under the long wavelength assumption, perturbation solutions are found for the temperature, pressure gradient, and stream function. Numerical integration is used to discuss variations in pressure and frictional force. In addition, it discussed how different relevant parameters affect the flow.

Tariq *et al.* [23] studied the effect of numerous peristaltic flow parameters for dusty fluid that is also second-grade, using a rounded design. To simulate independent mathematical equations for the dust particles and fluid, stream function conversions are utilized. Graphs are used to validate the analytical answers obtained through the application of the perturbation approach.

Numerous engineering applications include the use of two-phase particulate suspension flows that contain a continuous fluid phase and a discrete particle phase. Applications including gas masks, turbine blade erosion, dust collecting equipment, and aircraft icing all make use of these types of flow fields, which are of interest. MHD peristaltic transport of a dusty fluid in a uniform channel having elastic wall characteristics via a porous medium has been stuided. A system of differential equations that are partial controlling the movement of the fluid and solid elements are consumed to model this phenomenon by Parthasarathy et al. [24]. Using the perturbation technique, the system of Navier-Stokes equations is transformed into linear ones under the long wavelength assumptions and solved with suitable boundary conditions. Under various conditions, the effects of particle concentration and slip condition are examined on the suspended fluid's peristaltic flow down a channel. An external uniform magnetic field is provided crosswise to the walls of the channel, with the walls being assumed to be flexible. The pressure, streamlines, and velocity distributions for both fluid and dusty particles are obtained by Eldesoky et al. [25] through the solution of the equations of motion that is analytical, using the perturbation method. Under the long wavelength approximation, the impact of flexible wall elasticity on the peristaltic movement of a dusty fluid with mass and heat transfer in a horizontal channel during a chemical reaction has been studied by Muthura et al. [26]. Khan et al. [27] examined how mass transfer affects a dusty fluid's peristaltic flow in a curved configuration with elastic wall features. Hafeze et al. [28] studied the peristaltic transport characteristics of a second-grade dusty fluid flow with heat transfer through a tube revisited.

The use of asymmetric channels in technical occurrences at the level of manufacturing gives the study of these channels a large amount of significance. To forecast and comprehend the flow behaviour in asymmetric channels, numerical approaches like computational fluid dynamics are frequently used. In microfluidics, where small-scale devices exhibit intricate and irregular channel topologies, asymmetric channels are frequently seen. Under outlooks of long wavelengths and low Reynolds numbers, the topic of the peristaltic movement of fluid that is incompressible and non-Newtonian in a tapering channel that is asymmetric is discussed. By use of a transverse magnetic field, the fluid is thought to be fourth-order and conductor of electricity. The boundary walls that are non-uniform are subjected to peristaltic waves that have various amplitudes and phases, as done by Kothandapani *et al.* [29], in order to create the flow's tapering asymmetry. By

deciding to have a peristaltic wave train with varying amplitudes and phases on the thin walls, the tapering asymmetric channel was created. It has been used to combine a long wavelength and a low Reynolds number by Prakash *et al.* [30]. In order to better understand how heat radiation and entropy formation affect the peristaltic blood movement of such fluid that is magneto-micro polar in a tapering channel, Asha *et al.* [31] conducted research on the subject.

Reddy *et al.* [32] examined the impacts of chemical changes and thermal radiation on the transmission of mass and heat of a peristaltic electro-osmotic flow of a pair stress fluid down in a channel that is inclined asymmetric. Abd-Alla *et al.* [33] investigated that how the Newtonian fluid model behaves when it flows peristaltically over an incline channel with asymmetrical flow. It has been mathematically analyzed how an inclined magnetic field interacts with the peristaltic flow of blood in an inclined asymmetric channel, as well as how heat and mass are transferred. In this investigation of the peristaltic movement of Prandtl nanofluids in a passage that is inclined asymmetric, Akram *et al.* [34] explains the impacts of the induced field that is magnetic, temperature, and concentration convection. The relationship between double-diffusivity convection and an induced magnetic field in Prandtl nanofluids is explained mathematically in great depth. The work by Abbasi *et al.* [35] describes the significance of peristaltic occurrences in biology and medical technology, which has recently attracted a lot of interest. The peristaltic structure of Ellis fluid in a passageway has been examined in the current work after a wave train of infinite size has propagated across it. Due to the asymmetric channel, the limited flow regime is presumptive.

# **CHAPTER 3**

# **BASIC DEFINITIONS**

## 3.1 Fluid Mechanics [36]

It deals with fluids, whether they are in flow or at rest and the impact the fluid has on the boundaries as a result.

# 3.2 Fluid [36]

Any liquid, gas, or material, in general, that cannot bear a tangential or shearing force at rest and that continuously changes form in the presence of such a stress. Examples include water, air, honey, paint, etc.

#### 3.3 Types of Fluid

The numerous types of fluid namely

- Newtonian fluid,
- Non-Newtonian fluid,
- Incompressible fluid,
- Compressible fluid,
- Ideal fluid,

• Real fluid etc.

#### 3.3.1 Newtonian Fluid [36]

Newtonian fluids are defined as those fluids that follow the Newton's law of constant viscosity. These fluids show zero shear rate and maintain a constant viscosity under shear stress. Water, air, alcohol and glycerol are familiar examples.

#### 3.3.2 Non-Newtonian Fluid

Some fluids are referred to as non-Newtonian fluids because they do not adhere to the Newton's law of constant viscosity. Paint, food, and honey are a few examples.

#### 3.3.3 Ideal Fluid

Fluid which exhibits zero viscosity is known as ideal fluid. There is no examples of ideal fluid in real life these only exist in theory.

#### 3.3.4 Real Fluid

Such fluid that has viscosity more than zero i.e.  $(\mu > 0)$  is called real fluid. All fluids in daily life practice are real fluid.

## 3.3.5 Compressible Fluid

A compressible fluid is one where the substantial density fluctuations that take place throughout its flow must be taken into account, as generally the case with vapours and gases. Its examples are air, oxygen and steam.

#### 3.3.6 Incompressible Fluid

A fluid that does not vary in density or volume with pressure is said to be incompressible. Its common examples are water and oil.

#### 3.4 Newton's Law of Constant Viscosity

The viscous stresses are proportional to the coefficient of viscosity and element strain rates,

$$\tau = \mu \frac{du}{dy}.$$

#### 3.5 Types of Flow

Following are some types of flows

#### **3.5.1 Steady and Unsteady Flow**

When conditions (for example velocity, pressure, and cross-section) are different from one location to other but persist constant over time, the situation is called steady flow. If conditions in the fluid change over a period of time at any point in the flow, it is said to be unstable.

#### 3.5.2 Uniform and Non-Uniform Flow [36]

A common instance of uniform flow is the steady flow along a long, straight conduit with a fixed diameter. A flow is known as to non-uniform if its features and properties fluctuate and vary at various points throughout the flow path.

#### 3.5.3 One, Two and Three-Dimensional Flow

The number of spatial coordinates necessary to describe a flow is referred to as the term one, two, or three dimensional flow. Every physical flow seems to be three-dimensional in nature. However, these are challenging to compute and need as much oversimplification as you can.

#### **3.5.4 Rotational or Irrotational Fluid**

In contrast to irrotational flow, which includes fluid particles moving without rotating, rotational flow contains fluid particles moving in a swirling or circular fashion around a central axis. Due to the rotation, there are areas with varying levels of velocity, which results in complex and dynamic flow patterns.

#### 3.5.5 Laminar or Turbulent Flow

Laminar flow, is a form of fluid movement that occurs when the fluid flows easily or through predictable paths, as opposed to flow that is turbulent, in where the fluid undergo irregularity in oscillations and mixing.

#### 3.5.6 Creeping Flow

Fluid flow with little inertia is referred to as creeping flow. The fluid is subjected to forces that are larger than inertia, including viscous and pressure forces. High-viscosity fluids often move in a creeping manner because they have trouble flowing. The settling of dust grains and the floating of microbes are two examples of extremely tiny things moving in a fluid that exhibit creeping flow.

#### **3.6** Viscosity [36]

The inertia of a fluid to a variation in the form or motion of neighboring segments comparative to one another is identified as viscosity. Viscosity is an indication of flow resistance.

#### 3.7 Shear Thinning

Fluids that become pseudo-plastic (shear-thinning) as shear rate is increased are those whose viscosity falls. Examples of shear-thinning materials include glue, shampoo, egg, and blood.

#### 3.8 Shear Thickning

A fluid that exhibits an increase in viscosity with an increase in shear rate. A combination of corn flour and water is an instance of a dilatant fluid.

#### 3.9 Shear Stress

It's a force that tends to distort a material by causing it to slide along a plane or planes parallel to the applied stress.

#### 3.10 Shear Strain

By dividing the measure of deformation the body experiences in the direction that of the applied force by the body's starting dimensions Shear force Strain is calculated. Forces parallel to and lying in planes or cross-sectional regions generate a shear strain.

#### 3.11 Density [36]

A liquid's mass per unit volume is indicated by the Greek letter (lowercase rho). Gases have a significant degree of density variability, and it rises almost directly in direct proportion to pressure. Liquids have a relatively constant density.

# 3.12 No Slip Condition [36]

When flow is constrained by a solid surface, interactions between molecules cause the fluid that comes into contact with that surface to try to find momentum and energy equilibrium with it,

# $V_{fluid} = V_{wall}$ .

#### 3.13 MHD

It examines how magnetic fields and fluid conductors of electricity interact. Plasma, liquid metals, salt water, and electrolytes are a few examples.

#### 3.14 Continuity Equation

The rate at which mass leaves a structure equals the rate at which it enters a structure regardless of the time this process, based on the continuity equation for fluid dynamics equation.

$$\frac{\partial \rho}{\partial t} + \nabla . \left( \rho . V \right) = 0,$$

where,

t = time,  $\rho$  = density of the fluid and V = flow velocity vector field.

#### 3.15 Newton's Second Law of Motion

Newton's second law, indicates that a body's temporal rate of change in momentum is proportionate to the total of the forces acting on it.

Newton's second law can be written as:

$$\mathbf{F} = \frac{d(mv)}{dt}.$$

### 3.16 Mass [36]

As a way to gauge how much matter is there in a body, we might use the term mass. Kilogram (kg), which is represented by the m symbol, is the SI unit of mass.

#### 3.17 Body Force

A force that acts over the volume of a body. Which consist of forces resulting from gravity, magnetic fields and electric fields.

# 3.18 Surface Force

Surface forces, or the forces created when two bodies come into contact, act on surfaces and are responsible for the stress distributions that result from this interaction. Interior surfaces of materials are likewise subject to surface forces.

#### 3.19 Pressure [36]

A distinct number known as the fluid pressure or p is equal to the normal stress on a plane via fluid element when it is at rest.

#### **3.20** Dimension [36]

The unit of measurement used to express a physical variable is called a dimension. A specific method of connecting an amount to the quantitative dimension is via using units.

#### 3.21 Reynolds Number [36]

The relationship among the forces that are inertial and viscous is expressed through the Reynolds number.

Given by the ratio,

 $Re = \frac{inertial forces}{viscous forces}.$ 

#### 3.22 Streamline [36]

A streamline is a line that it's all point are perpendicular to the velocity vector at any given time.

#### 3.23 Stream Function [36]

Stream functionality is possible to estimate the components of velocity by differentiation of the stream function with respect to the provided coordinates since the stream function is a threedimensional feature of the hydrodynamics of an inviscid liquid and a function of coordinates and time.
### 3.24 Perturbation

Finding estimated analytical solutions for nonlinear problems is a common use of perturbation techniques. This method, however, is based on the presumption that a small parameter exists in the differential equation that governs the nonlinear physical phenomena as well as the approximations obtained by perturbation methods, and, in most instances, is only applicable for small values of the small parameter.

#### **CHAPTER 4**

## PERISTALTIC FLOW OF SECOND-GRADE DUSTY FLUID THROUGH A POROUS MEDIUM IN AN ASYMMETRIC CHANNEL

#### 4.1 Introduction

The impact of a porous medium on a fluid that is second-grade and dusty along with a slip factor within an asymmetric passage has been investigated by Khan *et al.* [37]. Nonlinear coupled equations are used to illustrate the scenario mathematically. By using a standard perturbation procedure, the problem is analytically resolved. Graphs are also included of the pressure gradient and the stream functions together with solid and liquid particles.

#### 4.2 Mathematical Formulation

Supposing the second-grade dusty fluid obeying peristaltic motion with magnetic field and slip condition through an asymmetric channel having breadth  $(d_1 + d_2)$ , waves that are peristaltic presumed to travel beside the walls of the passage. Equations that describe the channel walls are

$$H_1(X,t) = d_1 + a_1 \sin\left[\frac{2\pi}{\lambda}(X - ct)\right],$$
(4.1)

$$H_{2}(X,t) = -d_{2} + a_{2} \sin\left[\frac{2\pi}{\lambda}(X - ct) + \phi\right],$$
(4.2)



**Figure 4.1: Geometry of the problem** 

In the above equations  $a_1$  and  $a_2$  representing the amplitudes of the waves,  $\lambda$  stands for the wavelength, c is the speed of the wave propagation beside the walls of the passage, and  $\emptyset$  is the phase angle.

The defining equations for fluid flow are given by

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \tag{4.3}$$

$$\rho\left(\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + V\frac{\partial V}{\partial y}\right) = -\frac{\partial P}{\partial x} + \frac{\partial S_{XX}}{\partial x} + \frac{\partial S_{XY}}{\partial y} + r\widetilde{N}(U_{S} - U) - \frac{\mu}{k_{1}}(U),$$
(4.4)

$$\rho\left(\frac{\partial V}{\partial t} + U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y}\right) = -\frac{\partial P}{\partial Y} + \frac{\partial S_{XY}}{\partial X} + \frac{\partial S_{YY}}{\partial Y} + r\widetilde{N}(V_{S} - V),$$
(4.5)

$$\frac{\partial U_s}{\partial X} + \frac{\partial V_s}{\partial Y} = 0, \tag{4.6}$$

$$U_{s}\frac{\partial U_{s}}{\partial X} + V_{s}\frac{\partial U_{s}}{\partial Y} = \frac{r}{m}(U - U_{s}), \qquad (4.7)$$

$$U_{s}\frac{\partial V_{s}}{\partial X} + V_{s}\frac{\partial V_{s}}{\partial Y} = \frac{r}{m}(V - V_{s}).$$
(4.8)

In the above equations, U and V are the velocities of fluid particles along the x-axis and yaxis respectively. While  $U_s$  and  $V_s$  are the velocities of solid particles along the x-axis and y-axis respectively. In equations r is the resistance, m is the mass of the dust particles,  $k_1$  represents the permeability of the porous medium, and  $\tilde{N}$  is the amount density of solid particles, which is taken as a constant.

The connection between fixed and moving frames is given as follows:

$$p(x) = P(X,t), x = X - ct, v = V, u = U - c, y = Y, u_S = U_S - c, v_S = V_S,$$
 (4.9)

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \tag{4.10}$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial x} + \frac{\partial S_{xx}}{\partial x} + \frac{\partial S_{xy}}{\partial y} + r\widetilde{N}(u_s - u) - \frac{\mu}{k_1}(u + c), \tag{4.11}$$

$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \frac{\partial S_{xy}}{\partial x} + \frac{\partial S_{yy}}{\partial y} + r\widetilde{N}(v_s - v) - \frac{\mu}{k_1}(v).$$
(4.12)

Similarly,

$$\frac{\partial u_s}{\partial x} + \frac{\partial v_s}{\partial y} = 0, \tag{4.13}$$

$$u_{s}\frac{\partial u_{s}}{\partial x} + v_{s}\frac{\partial u_{s}}{\partial y} = \frac{r}{m}(u - u_{s}), \qquad (4.14)$$

$$u_{s}\frac{\partial v_{s}}{\partial x} + v_{s}\frac{\partial v_{s}}{\partial y} = \frac{r}{m}(u - v_{s}), \qquad (4.15)$$

where,

$$S_{xx} = 2\mu \frac{\partial u}{\partial x} + \alpha_1 \left[ 2\left(u \frac{\partial^2 u}{\partial x^2} + v \frac{\partial^2 u}{\partial x \partial y}\right) + 4\left(\frac{\partial u}{\partial x}\right)^2 + 2\frac{\partial u}{\partial x}\frac{\partial v}{\partial x} + 2\left(\frac{\partial u}{\partial x}\right)^2 \right] + \alpha_2 \left[ 4\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 \right],$$
(4.16)

$$S_{xy} = \mu \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) + \alpha_1 \left[ u \left( \frac{\partial^2 u}{\partial x \partial y} \right) + v \frac{\partial^2 u}{\partial x^2} + u \frac{\partial^2 v}{\partial x^2} - 2 \frac{\partial v}{\partial x} \frac{\partial u}{\partial x} + 2 \frac{\partial v}{\partial x} \frac{\partial v}{\partial x} \right],$$
(4.17)

$$S_{yy} = 2\mu \frac{\partial v}{\partial y} + \alpha_1 \left[ 2 \left( v \frac{\partial^2 v}{\partial y^2} + u \frac{\partial^2 v}{\partial x \partial y} \right) + 4 \left( \frac{\partial v}{\partial y} \right)^2 + 2 \frac{\partial u}{\partial y} \frac{\partial v}{\partial x} + 2 \left( \frac{\partial u}{\partial x} \right)^2 \right] + \alpha_2 \left[ 4 \left( \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \right].$$
(4.18)

Presenting the stream functions and dimensionless variables for dust and fluid particles as:

$$u = \frac{\partial \psi}{\partial y}, u_{s} = \frac{\partial \varphi}{\partial y}, v = -\delta \frac{\partial \psi}{\partial x}, v_{s} = -\delta \frac{\partial \varphi}{\partial x}, p^{*} = \frac{pd_{1}^{2}}{\mu c\lambda}, x^{*} = \frac{x}{\lambda}, y^{*} = \frac{y}{d_{1}}, \psi^{*} = \frac{\psi}{cd_{1}}, \varphi^{*} = \frac{\varphi}{cd_{1}}, \alpha_{1}^{*} = \frac{c\alpha_{1}}{\mu d_{1}}, \alpha_{2}^{*} = \frac{c\alpha_{2}}{\mu d_{1}}, s^{*} = \frac{sd_{1}}{\mu c}, a = \frac{a_{1}}{d_{1}}, b = \frac{a_{2}}{d_{1}}, d = \frac{d_{2}}{d_{1}}, Re = \frac{\rho cd_{2}}{\mu}, \delta = \frac{d_{1}}{\lambda}.$$
(4.19)

The equations in dimensionless form are given as:

$$\operatorname{Re}\delta\left[\frac{\partial^{2}\psi}{\partial x \partial y}\frac{\partial \psi}{\partial y} - \frac{\partial \psi}{\partial x}\frac{\partial^{2}\psi}{\partial y^{2}}\right] = -\frac{\partial p}{\partial x} + \delta\frac{\partial S_{xx}}{\partial x} + \frac{\partial S_{xy}}{\partial y} + A\left(\frac{\partial \varphi}{\partial y} - \frac{\partial \psi}{\partial y}\right) - \frac{d_{1}^{2}}{k_{1}}\left(\frac{\partial \psi}{\partial y} + 1\right), \quad (4.20)$$

$$\operatorname{Re}\delta^{3}\left[\frac{\partial \psi}{\partial x}\frac{\partial^{2}\psi}{\partial x \partial y} - \frac{\partial \psi}{\partial y}\frac{\partial^{2}\psi}{\partial x^{2}}\right] = -\frac{\partial p}{\partial y} + \delta^{2}\frac{\partial S_{xy}}{\partial x} + \delta\frac{\partial S_{yy}}{\partial y} + \delta^{2}A\left(\frac{\partial \psi}{\partial x} - \frac{\partial \varphi}{\partial x}\right) + \delta^{2}\frac{d_{1}^{2}}{k_{1}}\left(\frac{\partial \psi}{\partial x}\right). \quad (4.21)$$

For solid particles

$$\frac{\partial^2 \varphi}{\partial y \partial x} \frac{\partial \varphi}{\partial y} - \delta \frac{\partial^2 \varphi}{\partial y^2} \frac{\partial \varphi}{\partial x} = \frac{r}{m} \left( \frac{\partial \psi}{\partial y} - \frac{\partial \varphi}{\partial y} \right), \tag{4.22}$$

$$\delta \frac{\partial^2 \varphi}{\partial x \partial y} \frac{\partial \varphi}{\partial x} - \frac{\partial^2 \varphi}{\partial x^2} \frac{\partial \varphi}{\partial y} = \frac{r}{m} \left( \frac{\partial \varphi}{\partial x} - \frac{\partial \psi}{\partial x} \right). \tag{4.23}$$

Compatibility equation for the fluid and solid particles are

$$\operatorname{Re}\left[\delta\left(\frac{\partial\psi}{\partial y}\frac{\partial^{3}\psi}{\partial x\partial y^{2}}-\frac{\partial\psi}{\partial x}\frac{\partial^{3}\psi}{\partial y^{3}}\right)-\delta^{3}\left(\frac{\partial\psi}{\partial x}\frac{\partial^{3}\psi}{\partial y\partial x^{2}}-\frac{\partial\psi}{\partial y}\frac{\partial^{3}\psi}{\partial x^{3}}\right)\right] = \left(\frac{\partial^{2}}{\partial y^{2}}-\delta^{2}\frac{\partial^{2}}{\partial x^{2}}\right)S_{xy}+\delta\frac{\partial^{2}}{\partial x\partial y}\left(S_{xx}-S_{yy}\right)+A\left(\nabla_{1}^{2}\varphi-\nabla_{1}^{2}\psi\right)-\frac{d_{1}^{2}}{k_{1}}\left(\nabla_{1}^{2}\psi\right),$$

$$(4.24)$$

$$\delta\left(\frac{\partial \varphi}{\partial y}\frac{\partial}{\partial x}\nabla_{1}^{2}\varphi - \frac{\partial \varphi}{\partial x}\frac{\partial}{\partial y}\nabla_{1}^{2}\varphi\right) = R(\nabla_{1}^{2}\psi - \nabla_{1}^{2}\varphi), \qquad (4.25)$$

where

$$\nabla_1^2 = \left[\frac{\partial^2}{\partial y^2} + \delta^2\left(\frac{\partial^2}{\partial x^2}\right)\right]$$
,  $A = \frac{rNd_1^2}{\mu}$  and  $R = \frac{rd_1}{cm}$  are nondimensionlized parameters and  $k = \frac{d_1^2}{k_1}$ 

is the permeability coefficient of the porous medium

The walls equations in nondimensional form are:

$$h_1(x) = 1 + a\sin(2\pi x),$$
 (4.26)

$$h_2(x) = -d - b\sin(2\pi x + \emptyset),$$
 (4.27)

and the nondimensional boundary condition are:

$$\psi = \frac{F}{2}, \frac{\partial \psi}{\partial y} + \beta S_{xy} = -1, \phi = \frac{N}{2} \text{ at } y = h_1(x),$$
(4.28)

$$\psi = -\frac{F}{2}, \frac{\partial \psi}{\partial y} - \beta S_{xy} = -1, \varphi = -\frac{N}{2} \text{ at } y = h_2(x).$$
(4.29)

$$Q = F + 1 + d,$$
 (4.30)

where

$$F = \int_{h_2(x)}^{h_1(x)} \frac{\partial \Psi}{\partial y} dy.$$
(4.31)

The time flow rate of the dust particles in dimensionaless form is

$$N = \int_{h_2(x)}^{h_1(x)} \frac{\partial \varphi}{\partial y} dy.$$
(4.32)

The nondimensional time flow in the fixed frame as

$$Q_s = N + 1 + d.$$
 (4.33)

$$\Delta P = \int_{0}^{1} \frac{\partial p}{\partial x} dx.$$
(4.34)

### 4.3 Method of Solution

The perturbation method has been chosen to obtain the analytical solution of the modeled problem.

$$\psi = \psi_0 + \delta \psi_1 + O(\delta^2), \tag{4.35}$$

$$\varphi = \varphi_0 + \delta \varphi_1 + O(\delta^2), \tag{4.36}$$

$$F = F_0 + \delta F_1 + O(\delta^2), \tag{4.37}$$

$$N = N_0 + \delta N_1 + O(\delta^2), \tag{4.38}$$

$$p = p_0 + \delta p_1 + O(\delta^2).$$
(4.39)

## 4.3.1 Zeroth –Order System

$$\frac{\partial^2 S_{0xy}}{\partial y^2} + A\left(\frac{\partial^2 \varphi_0}{\partial y^2} - \frac{\partial^2 \psi_0}{\partial y^2}\right) - k \frac{\partial^2 \psi_0}{\partial y^2} = 0, \qquad (4.40)$$

$$R\left(\frac{\partial^2 \psi_0}{\partial y^2} - \frac{\partial^2 \varphi_0}{\partial y^2}\right) = 0, \qquad (4.41)$$

$$p_{0x} = \frac{\partial S_{0xy}}{\partial y} + A\left(\frac{\partial \varphi_0}{\partial y} - \frac{\partial \psi_0}{\partial y}\right) - k\left(\frac{\partial \psi_0}{\partial y} + 1\right), \qquad (4.42)$$

where,

$$S_{0xy} = \frac{\partial^2 \psi_0}{\partial y^2},$$

$$\psi_0 = \frac{F_0}{2}, \phi_0 = \frac{N_0}{2}, \frac{\partial \psi_0}{\partial y} + \beta S_{0xy} = -1 \quad at \ y = h_1(x),$$
(4.43)

$$\psi_0 = -\frac{F_0}{2}, \phi_0 = -\frac{N_0}{2}, \frac{\partial \psi_0}{\partial y} - \beta S_{0xy} = -1 \quad at \ y = h_2(x).$$
(4.44)

# 4.3.2 First Order System

$$Re\left(\frac{\partial\psi_{0}}{\partial y}\frac{\partial^{3}\psi_{0}}{\partial x\partial y^{2}} - \frac{\partial\psi_{0}}{\partial x}\frac{\partial^{3}\psi_{0}}{\partial y^{3}}\right) = \frac{\partial^{2}}{\partial x\partial y}\left(S_{0xx} - S_{0yy}\right) + \frac{\partial^{2}}{\partial y^{2}}S_{1xy} + A\left(\frac{\partial^{2}\varphi_{1}}{\partial y^{2}} - \frac{\partial^{2}\psi_{1}}{\partial y^{2}}\right) - k\left(\frac{\partial^{2}\psi_{1}}{\partial y^{2}}\right),$$

$$(4.45)$$

$$R\left(\frac{\partial^2 \psi_1}{\partial y^2} - \frac{\partial^2 \varphi_1}{\partial y^2}\right) = \frac{\partial \varphi_0}{\partial y} \frac{\partial^3 \varphi_0}{\partial x \partial y^2} - \frac{\partial \varphi_0}{\partial x} \frac{\partial^3 \varphi_0}{\partial y^3},\tag{4.46}$$

$$p_{1x} = \operatorname{Re}\left(\frac{\partial\psi_0}{\partial y}\frac{\partial^2\psi_0}{\partial x\partial y} - \frac{\partial\psi_0}{\partial x}\frac{\partial^2\psi_0}{\partial y^2}\right) - \frac{\partial}{\partial y}\left(S_{1xy}\right) + A\left(\frac{\partial\varphi_1}{\partial y} - \frac{\partial\psi_1}{\partial y}\right) - k \frac{\partial\psi_1}{\partial y},\tag{4.47}$$

Where,

$$S_{1xy} = \frac{\partial^2 \psi_1}{\partial y^2} + \alpha_1 \left( 2 \frac{\partial^2 \psi_0}{\partial y \partial x} \frac{\partial^2 \psi_0}{\partial y^2} + \frac{\partial \psi_0}{\partial y} \frac{\partial^3 \psi_0}{\partial x \partial y^2} - \frac{\partial \psi_0}{\partial x} \frac{\partial^3 \psi_0}{\partial y^3} \right).$$
  
$$\psi_1 = \frac{F_1}{2}, \ \phi_1 = \frac{N_1}{2}, \ \frac{\partial \psi_1}{\partial y} + \beta S_{1xy} = 0 \qquad at \ y = h_1(x), \tag{4.48}$$

$$\psi_1 = -\frac{F_1}{2}, \ \varphi_1 = -\frac{N_1}{2}, \ \frac{\partial \psi_1}{\partial y} - \beta S_{1xy} = 0 \qquad at \ y = h_2(x).$$
(4.49)

To solve the above mentioned system of equations Mathematica software has been used. Also the graphs obtained by the same software.

#### 4.4 **Results and Discussion**

The topic of pressure rise and the contour graphs relating to liquid and solid particles is covered in this section. The impacts of various factors on fluid particles are shown in Figures 4.2–4.6. The effects of factors upon the dust particles embedded in the fluid are shown in Figures 4.7–4.11. Graphical depictions of pressure change for particles of fluid are shown in Figures 4.12–4.16.

The expansion of the fluid particle bolus is demonstrated by Figure 4.2 when  $\alpha_1$  values grow. We note that the bolus grows along with the values of  $\delta$ , as Figure 4.3 illustrates. Although Figure 4.4(b) and 4.4(c) demonstrate that the bolus grows with an increase in slip parameter, Figure 4.4(a) depicts the impact of no slip upon the fluid particles. Figure 4.5 illustrates how porosity works. See Figure 4.6(a) for an illustration of the minimal porosity quantity and the modest influence of porosity on the fluid profile. Expanding bolus is a result of higher porosity. A higher Reynolds number (*Re*) causes the fluid particle bolus to constrict, as seen in Figure 4.6As  $\alpha_1$  increases, the bolus of the solid particles grows, as shown in Figure 4.7. As shown in Figure 4.8, the bolus extends by increasing  $\delta$  for solid particles. The solid particles are shown to have noslip effects in Figure 4.9(a), but Figure 4.9(b) and 4.9(c) demonstrate how the bolus begins to expand when the slip parameter increases. Figure 4.10 offers an examination of the effects for the porosity factor on particles that are solid. We see a small impact of reduced porosity on the solid particle flow. Effects on the particles that are solid are shown to increase with porosity in Figure 4.10(b) and 4.10(c). *Re* and bolus grow in tandem, as Figure 4.11 illustrates. Figure 4.12 shows the deviation of the pressure rise for various amounts of Reynolds number (*Re*). As *Re* increases, the pressure increases as well because a higher Reynolds number indicates that inertial forces become stronger and viscous forces become weaker, widening the retrograde region for fluid pumping. The impact of  $\delta$  upon the rate of pressure rise is depicted in Figure 4.13. The pumping rate in the retrograde zone falls as  $\delta$  grows, whereas in the areas with free pumping and co-pumping regions, the effect is the opposite. Figure 4.14 provides an analysis of the effects of slip factor  $\beta$  on the rise in pressure. As the slip coefficient increases, there is a modest increase in pressure rise because pumping the fluid requires more pressure. Pressure increases significantly in response to increases in the porosity parameter, *k*. When the *k* approaches 0, we can see in Figure 4.15 that pressure has all but disappeared. The pressure rise increases noticeably as the porosity variables increases because it would take more pressure to pump the fluid through the porous passage. The impacts of  $\alpha_1$  on the increase in pressure are graphically depicted in Figure 4.16. In the retrograde region ( $\Delta P > 0$ ), we observe that the pumping rate increases with an increase in  $\alpha_1$ , whereas in the co-pumping area ( $\Delta P < 0$ ), they decrease.





**Figure 4.2:** Portrays the contour patterns for fluid particles for diverse values of  $\alpha_1$ .





Figure 4.3: Portrays the contour patterns for fluid particles for diverse values of  $\delta$ .





**Figure 4.4:** Portrays the contour patterns for fluid particles for diverse values of  $\beta$ .



Figure 4.5: Portrays the contour patterns for fluid particles for diverse values of *k*.



**Figure 4.6:** Portrays the contour patterns for fluid particles for diverse values of Reynolds number (*Re*).

0.1

0.2

(c) Re = 1

0.3

0.4

0.5

-0.1

0.0





**Figure 4.7:** Portrays the contour patterns for dust particles for diverse values of  $\alpha_1$ .





**Figure 4.8:** Portrays the contour patterns for dust particles for diverse values of  $\delta$ .





**Figure 4.9:** Portrays the contour patterns for dust particles for diverse values of  $\beta$ .





Figure 4.10: Portrays the contour patterns for dust particles for diverse values of *k*.



**Figure 4.11:** Portrays the contour patterns for dust particles for diverse values of Reynolds number (*Re*).



**Figure 4.12:** Deviation of  $\Delta p(P)$  with Q for diverse values of *Re*.



**Figure 4.13:** Deviation of  $\Delta p(P)$  with Q for diverse values of  $\delta$ .



**Figure 4.14:** Deviation of  $\Delta p(P)$  with Q for diverse values of  $\beta$ .



**Figure 4.15:** Deviation of  $\Delta p(P)$  with Q for diverse values of *k*.



**Figure 4.16:** Deviation of  $\Delta p(P)$  with Q for diverse values of  $\alpha_1$ .

#### **CHAPTER 5**

# EFFECTS OF MAGNETIC FIELD ON PERISTALTIC FLOW OF SECOND-GRADE DUSTY FLUID IN AN INCLINDED ASYMMETRIC CHANNEL

#### 5.1 Introduction

This chapter deals with the peristaltic flow of second-grade dusty fluid with the effects of magnetic field flowing past and slip condition by an inclined asymmetric passage. Equations are solved for both fluid particles and solid particles separately. The solution of the system of equations is found by using DSolver in Mathematica.

#### 5.2 Mathematical Formulation

Supposing the second-grade dusty fluid following peristaltic motion with magnetic field and slip condition in an inclined asymmetric channel having width $(d_1 + d_2)$ , waves that are peristaltic presumed to travel beside the walls of the passage with speed c. The asymmetric passage is considered to be inclined, making the angle  $\gamma$  with the x-axis. Equations that describe the channel walls are

$$H_{1}(X,t) = d_{1} + a_{1} \cos\left[\frac{2\pi}{\lambda}(X - ct)\right],$$
(5.1)

$$H_{2}(X,t) = -d_{2} + a_{2} \cos\left[\frac{2\pi}{\lambda}(X - ct) + \phi\right].$$
(5.2)



Figure 5.1: Geometry of the problem

The ruling equations designed for fluid flow are given by:

$$\frac{\partial \hat{\mathbf{0}}}{\partial \hat{\mathbf{X}}} + \frac{\partial \hat{\mathbf{V}}}{\partial \hat{\mathbf{Y}}} = \mathbf{0},\tag{5.3}$$

$$\rho\left(\frac{\partial \hat{U}}{\partial \hat{t}} + \hat{U}\frac{\partial \hat{U}}{\partial \hat{X}} + \hat{V}\frac{\partial \hat{V}}{\partial \hat{Y}}\right) = -\frac{\partial \hat{P}}{\partial \hat{X}} + \frac{\partial \hat{S}_{\hat{X}\hat{X}}}{\partial \hat{X}} + \frac{\partial \hat{S}_{\hat{X}\hat{Y}}}{\partial \hat{Y}} + KL(\hat{U}_{S} - \hat{U}) + \rho g \sin \gamma - \sigma B_{0}^{2}\hat{U},$$
(5.4)

$$\rho\left(\frac{\partial \hat{V}}{\partial \hat{t}} + \hat{U}\frac{\partial \hat{V}}{\partial \hat{X}} + \hat{V}\frac{\partial \hat{V}}{\partial \hat{Y}}\right) = -\frac{\partial \hat{P}}{\partial \hat{Y}} + \frac{\partial \hat{S}_{\hat{X}\hat{Y}}}{\partial \hat{X}} + \frac{\partial \hat{S}_{\hat{Y}\hat{Y}}}{\partial \hat{X}} + KL(\hat{V}_{S} - \hat{V}) + \rho g \cos \gamma.$$
(5.5)

For solid particles, the governing equation is

$$\frac{\partial \hat{U}_{S}}{\partial \hat{X}} + \frac{\partial \hat{V}_{S}}{\partial \hat{Y}} = 0, \tag{5.6}$$

$$\frac{\partial \hat{U}_{S}}{\partial \hat{t}} + \hat{U}_{S} \frac{\partial \hat{U}_{S}}{\partial \hat{X}} + \hat{V}_{S} \frac{\partial \hat{U}_{S}}{\partial \hat{Y}} = KL(\hat{U} - \hat{U}_{S})$$
(5.7)

$$\frac{\partial \hat{V}_{S}}{\partial \hat{t}} + \hat{U}_{S} \frac{\partial \hat{V}_{S}}{\partial \hat{X}} + \hat{V}_{S} \frac{\partial \hat{V}_{S}}{\partial \hat{Y}} = KL(\hat{V} - \hat{V}_{S})$$
(5.8)

In the above equations, U and V are the velocities of fluid particles along the x-axis and y-axis respectively. While  $U_s$  and  $V_s$  are the velocities of solid particles with the x-axis and y-axis correspondingly. L is the resistance, K is the amount density of solid particles, which is taken as a constant.

The association between moving and fixed structures is given below and presenting the stream functions and nondimensional variables for fluid and dust particles

$$p(\hat{\mathbf{x}}) = P(\hat{\mathbf{X}}, \hat{\mathbf{t}}), \ \hat{\mathbf{v}} = \hat{\mathbf{V}}, \ \hat{\mathbf{u}} = \hat{\mathbf{U}} - c, \ \hat{\mathbf{u}}_{s} = \hat{\mathbf{U}}_{s} - c, \ \hat{\mathbf{y}} = \hat{\mathbf{Y}}, \ \hat{\mathbf{v}}_{s} = \hat{\mathbf{V}}_{s}, \ \hat{\mathbf{x}} = \hat{\mathbf{X}} - c\hat{\mathbf{t}}$$
(5.9)  
$$u = \frac{\partial \psi}{\partial y}, \ u_{s} = \frac{\partial \varphi}{\partial y}, \ v = -\delta \frac{\partial \psi}{\partial x}, \ v_{s} = -\delta \frac{\partial \varphi}{\partial x}, \ p^{*} = \frac{pd_{1}^{2}}{\mu c\lambda}, \ x^{*} = \frac{x}{\lambda}, \ y^{*} = \frac{y}{d_{1}}, \ \psi^{*} = \frac{\psi}{cd_{1}}, \ \phi^{*} = \frac{\varphi}{cd_{1}},$$
$$\alpha_{1}^{*} = \frac{c\alpha_{1}}{\mu d_{1}}, \ \alpha_{2}^{*} = \frac{c\alpha_{2}}{\mu d_{1}}, \ s^{*} = \frac{sd_{1}}{\mu c}, \ a = \frac{a_{1}}{d_{1}}, \ b = \frac{a_{2}}{d_{1}}, \ d = \frac{d_{2}}{d_{1}}, \ M^{2} = \frac{\sigma B_{0}^{2} d_{1}}{\mu}, \ Re = \frac{\rho cd_{2}}{\mu},$$
$$\delta = \frac{d_{1}}{\lambda}, \ Fr = \frac{c^{2}}{gd_{1}}.$$
(5.10)

Equations (5.4), (5.5) are transformed into the following equations:

$$\operatorname{Re}\delta\left[\frac{\partial^{2}\psi}{\partial x\,\partial y}\,\frac{\partial\psi}{\partial y} - \,\frac{\partial\psi}{\partial x}\frac{\partial^{2}\psi}{\partial y^{2}}\right] = -\frac{\partial P}{\partial x} + \delta\,\frac{\partial S_{XX}}{\partial X} + \frac{\partial S_{XY}}{\partial Y} + A\left(\frac{\partial\phi}{\partial y} - \frac{\partial\psi}{\partial y}\right) + \frac{\operatorname{Re}}{\operatorname{Fr}}\sin\gamma - M^{2}\left(\frac{\partial\psi}{\partial y} + 1\right),$$
(5.11)

$$\operatorname{Re}\delta^{3}\left[\frac{\partial^{2}\psi}{\partial x^{2}}\frac{\partial\psi}{\partial y} - \frac{\partial\psi}{\partial x}\frac{\partial^{2}\psi}{\partial x\partial y}\right] = -\frac{\partial P}{\partial y} + \delta^{2}\frac{\partial S_{XY}}{\partial X} + \delta\frac{\partial S_{YY}}{\partial Y} + A\left(\frac{\partial\psi}{\partial x} - \frac{\partial\varphi}{\partial x}\right) - \frac{\operatorname{Re}}{\operatorname{Fr}}\cos\gamma,$$
(5.12)

Equations (5.7), (5.8) are transformed into the following equations:

$$\frac{\partial^2 \varphi}{\partial y \partial x} \frac{\partial \varphi}{\partial y} - \delta \frac{\partial^2 \varphi}{\partial y^2} \frac{\partial \varphi}{\partial x} = B\left(\frac{\partial \psi}{\partial y} - \frac{\partial \varphi}{\partial y}\right),\tag{5.13}$$

$$-\frac{\partial^2 \varphi}{\partial x^2} \frac{\partial \varphi}{\partial y} + \delta \frac{\partial \varphi}{\partial x} \frac{\partial^2 \varphi}{\partial x \partial y} = B\left(\frac{\partial \varphi}{\partial y} - \frac{\partial \psi}{\partial x}\right).$$
(5.14)

Compatibility equation for the fluid and dust particles are

$$\delta \operatorname{Re}\left[\frac{\partial \psi}{\partial y}\frac{\partial}{\partial x}\left(\nabla_{1}^{2}\psi\right)-\frac{\partial \psi}{\partial x}\frac{\partial}{\partial y}\left(\nabla_{1}^{2}\psi\right)\right] = \left(\frac{\partial^{2}}{\partial y^{2}}-\delta^{2}\frac{\partial^{2}}{\partial x^{2}}\right)S_{xy}+\delta\left(\frac{\partial^{2}}{\partial x\partial y}\left\{S_{xx}-S_{yy}\right\}\right)-M^{2}\frac{\partial^{2}\psi}{\partial y^{2}}+A\left[\left[\nabla_{1}^{2}\varphi-\nabla_{1}^{2}\psi\right]\right].$$
(5.15)

$$\delta\left(\frac{\partial\varphi}{\partial y}\frac{\partial}{\partial x}\nabla_{1}^{2}\varphi - \frac{\partial\varphi}{\partial x}\frac{\partial}{\partial y}\nabla_{1}^{2}\varphi\right) = B(\nabla_{1}^{2}\psi - \nabla_{1}^{2}\varphi), \tag{4.16}$$

Where

$$\nabla_1^2 = \left[\frac{\partial^2}{\partial y^2} + \delta^2\left(\frac{\partial^2}{\partial x^2}\right)\right]$$
,  $A = \frac{rKd_1^2}{\mu}$  and  $B = \frac{Ld_1}{cm}$  are nondimensionlized parameters

The dimensionless form of walls are:

$$h_1(x) = 1 + a\cos(2\pi x), \tag{5.17}$$

$$h_2(x) = -d - b\cos(2\pi x + \emptyset),$$
 (5.18)

and the dimensionless boundary condition are

$$\psi = \frac{F}{2}, \ \frac{\partial \psi}{\partial y} + \beta S_{xy} = -1, \ \varphi = \frac{N}{2}, \qquad \text{at } y = h_1(x),$$
(5.19)

$$\psi = -\frac{F}{2}, \ \frac{\partial \psi}{\partial y} - \beta S_{xy} = -1, \ \varphi = -\frac{N}{2}, \quad \text{at } y = h_2(x).$$
 (5.20)

By using the same approximation and assumptions as discussed in chapter 4.

# 5.3 Method of Solution

The perturbation method has been implemented to get the analytical solution of the modeled problem.

$$\psi = \psi_0 + \delta \psi_1 + O(\delta^2), \tag{5.21}$$

$$\varphi = \varphi_0 + \delta \varphi_1 + O(\delta^2), \tag{5.22}$$

$$F = F_0 + \delta F_1 + O(\delta^2),$$
(5.23)

$$N = N_0 + \delta N_1 + O(\delta^2),$$
 (5.24)

$$p = p_0 + \delta p_1 + O(\delta^2).$$
(5.25)

# 5.3.1 Zeroth –Order System

$$\frac{\partial^2 S_{0xy}}{\partial y^2} + A\left(\frac{\partial^2 \varphi_0}{\partial y^2} - \frac{\partial^2 \psi_0}{\partial y^2}\right) - M^2 \frac{\partial^2 \psi_0}{\partial y^2} = 0,$$
(5.26)

$$B\left(\frac{\partial^2 \psi_0}{\partial y^2} - \frac{\partial^2 \varphi_0}{\partial y^2}\right) = 0,$$
(5.27)

$$p_{0x} = \frac{\partial S_{0xy}}{\partial y} + A\left(\frac{\partial \varphi_0}{\partial y} - \frac{\partial \psi_0}{\partial y}\right) - M^2 \frac{\partial^2 \psi_0}{\partial y^2} + \frac{Re}{F_r} \sin\gamma , \qquad (5.28)$$

with

$$S_{0xy} = \frac{\partial^2 \psi_0}{\partial y^2},$$
  
$$\psi_0 = \frac{F_0}{2}, \ \phi_0 = \frac{N_0}{2}, \ \frac{\partial \psi_0}{\partial y} + \beta S_{0xy} = -1 \quad \text{at } y = h_1(x),$$
 (5.29)

$$\psi_0 = -\frac{F_0}{2}, \ \phi_0 = -\frac{N_0}{2}, \ \frac{\partial \psi_0}{\partial y} - \beta S_{0xy} = -1 \quad \text{at } y = h_2(x).$$
(5.30)

#### 5.3.2 First Order System

$$\operatorname{Re}\left(\frac{\partial\psi_{0}}{\partial y}\frac{\partial^{3}\psi_{0}}{\partial y^{3}} - \frac{\partial\psi_{0}}{\partial y}\frac{\partial^{3}\psi_{0}}{\partial y^{3}}\right) = \frac{\partial^{2}}{\partial x \, \partial y}\left(S_{0xx} - S_{0yy}\right) + \frac{\partial^{2}}{\partial y^{2}}S_{1xy} + A\left(\frac{\partial^{2}\varphi_{1}}{\partial y^{2}} - \frac{\partial^{2}\psi_{1}}{\partial y^{2}}\right) - M^{2}\left(\frac{\partial^{2}\psi_{1}}{\partial y^{2}}\right),$$
(5.31)

$$R\left(\frac{\partial^2 \psi_1}{\partial y^2} - \frac{\partial^2 \varphi_1}{\partial y^2}\right) = \frac{\partial \varphi_0}{\partial y} \frac{\partial^3 \varphi_0}{\partial x \partial y^2} - \frac{\partial \varphi_0}{\partial x} \frac{\partial^3 \varphi_0}{\partial y^3},$$
(5.32)

$$p_{1x} = \frac{\partial}{\partial x} \left( S_{0xy} \right) + \frac{\partial}{\partial y} \left( S_{1xy} \right) + A \left( \frac{\partial \varphi_1}{\partial y} - \frac{\partial \psi_1}{\partial y} \right) - M^2 \frac{\partial^2 \psi_1}{\partial y^2} - \operatorname{Re} \left( \frac{\partial \psi_0}{\partial y} \frac{\partial^2 \psi_0}{\partial x \partial y} - \frac{\partial \psi_0}{\partial x \partial y^2} \right),$$
(5.33)

with

$$S_{0xx} = -\alpha_1 \left(\frac{\partial^2 \psi_0}{\partial y^2}\right)^2,$$

$$S_{0yy} = \left[2\frac{\partial^2 \psi_0}{\partial y^2} - \alpha_1^* \left(\frac{\partial^2 \psi_0}{\partial y^2}\right)^2\right],$$

$$S_{1xy} = \frac{\partial^2 \psi_1}{\partial y^2} + \alpha \left(2\frac{\partial^2 \psi_0}{\partial y \partial x}\frac{\partial^2 \psi_0}{\partial y^2} + \frac{\partial \psi_0}{\partial y}\frac{\partial^3 \phi_0}{\partial x \partial y^2} + \frac{\partial \psi_0}{\partial x}\frac{\partial^3 \psi_0}{\partial y^3}\right).$$

$$\psi_1 = \frac{F_1}{2}, \phi_1 = \frac{N_1}{2}, \frac{\partial \psi_1}{\partial y} + \beta S_{1xy} = 0 \qquad \text{aty} = h_1(x),$$

$$\psi_1 = -\frac{F_1}{2}, \phi_1 = -\frac{N_1}{2}, \frac{\partial \psi_1}{\partial y} - \beta S_{1xy} = 0 \qquad \text{aty} = h_2(x).$$
(5.35)

## 5.4 Results and Discussion

Effects of slip and magnetic field on peristaltic flow of second-grade dusty fluid has been considered here. The governing equations are reduced into non dimensional form then use compatibility transformation and solved by the regular perturbation method the computational results are demonstrated in graphical form. The graphs demonstrating numerous constraint for the fluid's velocity profile and for pressure are deliberate in this unit. Figures 5.2 to 5.9 represent the

51

graphs of velocity profile of fluid and dust particles under the effects of several parameters. While figures 5.10 to 5.14 are the graphical representation of pressure with impacts of different parameters.

The influence of magnetic field (*M*) on the velocity of fluid particles is captured in figure 5.2.As magnetic force is known as resistive force so it causes the reduction in the fluid movement. Hence it is found that by enhancing the values of *M* the velocity of fluids particles starts decreasing. Its observed that figure 5.3 illustrate the effects of the slip perimeter on the velocity of fluid particle. Fluid flow patterns are influenced by slip conditions that are imposed on a cavity's side wall. The quantity of slip at the boundary is measured by the velocity slip parameters. The distance between the surface where the velocity of fluid changes from the no-slip to the slip state is known as the slip length, and it is represented by this number. So by enhancing the values of  $\beta$ , velocity of fluid decreased. The number of repeating units of a propagating wave that is, the number of times a wave has the same phase per unit of space is known as the wavenumber, which is the spatial equivalent of frequency. From figure 5.4 it is seen that by increasing the wave number  $\delta$  velocity of fluid models usually results in a decrease in velocity. This is because a fluid's resistance to deformation is implied by a greater second-grade value. This results in a decrease in velocity for whatever applied force since the fluid needs greater force for maintaining a given velocity.

The behavior of the velocity of fluid for various values of  $\alpha_1$  is shown in figure 5.5 which displays that velocity of fluid decreases as 1 increases while at some values velocity of fluid remains constant. It can be seen in figure 5.6 that magnetic field *M* effects the velocity of dust particles in a similar way as effects the velocity of fluid particles i.e. when M increases the velocity of dust particles decreases. It's revealed in figure 5.7 that the velocity of solid particles has a slightly different behavior at the edges of the channel. By enhancing the values of  $\beta$  firstly the velocity of the solid particle decreases but for some time it remains constant and then starts increasing. Furthermore, figure 5.8 demonstrates the totally different behavior of the velocity of dust particles. The velocity of dust particles increases with an increase in wave number  $\delta$  which is a second-grade fluid parameter that also affects the solid's particle velocity. Here a similar phenomenon is observed for the fluid particle velocity. In figure 5.9 velocity of the dust particles decreases due to a rise in the values of  $\alpha_1$ . Pressure plays a key role in the flow of fluid. Many parameters affect the pressure. It is aimed to show the change in pressure due to the magnetic field in figure 5.10. As the magnetic field decreases the velocity which leads to leads to increase in pressure. So by exceeding the values of M at the start pressure increases but after some time pressure decreases. For the slip parameter pressure has the same behavior as on magnetic field. Both figures 5.10 and 5.11 show the same results. The effects of wavenumber  $\delta$  on the pressure are plotted in figure 5.12 which indicates that by enhancing the values of  $\delta$  then pressure decreases. In addition, figure 5.13 interprets the relation between the second-grade fluid parameter  $\alpha_1$  and the pressure. There is a slight change in pressure or can say that there is no change in pressure due to a change in  $\alpha_1$  it almost remain same. It's observed by the figure 5.14 that Reynolds number Re also affects the pressure. As Reynolds number is relationship among inertial forces and viscous forces therefore by increasing the values of Re pressure starts decreasing. Figure 5.15 exhibits the relationship between the pressure and the inclination of the channel it displays the declining pressure is because of the inclining in firstly pressure declined slowly but after sometime it declined fastly.



Figure 5.2: Velocity distribution of the fluid for diverse values of *M*.



**Figure 5.3:** Velocity distribution of the fluid for diverse values of  $\beta$ 



Figure 5.4: Velocity distribution of the fluid for diverse values of  $\delta$ .



**Figure 5.5:** Velocity distribution of the fluid for diverse values of  $\alpha_1$ .



Figure 5.6: Velocity distribution of the dust particles for diverse values of *M*.



Figure 5.7: Velocity distribution of the dust particles for diverse values of  $\beta$ .



Figure 5.8: Velocity distribution of the dust particles for diverse values of  $\delta$ 



**Figure 5.9:** Velocity distribution of the dust particles for diverse values of  $\alpha_1$ .



Figure 5.10: Effects of M on pressure.



**Figure 5.11:** Effects of  $\beta$  on pressure.


**Figure 5.12:** Effects of  $\delta$  on pressure.



**Figure 5.13:** Effects of  $\alpha_1$  on pressure.



Figure 5.14: Effects of *Re* on pressure.



**Figure 5.15:** Effects of  $\gamma$  on pressure.

## **CHAPTER 6**

# CONCLUSION

In this research investigation about the second-grade dusty fluid has been taken into account, while the effects of peristalsis, magnetic field and slip boundary conditions are also considered. The governing equations are reduced into non-dimensional form then a compatibility equation has been derived for both fluid and solid particles. Due to the non-linear nature of the equation, a regular perturbation technique was employed and the obtained results were demonstrated through graphs.

#### 6.1 Significant Results

It's observed that the magnetic field (*M*) influenced the velocity of the fluid particles and solid particles as well. As force known as magnetic is also known as resistive force it causes the reduction among the fluid movement. Hence it is found that by enhancing the values of *M* the velocity of fluids and solid particles starts decreasing. Similarly, by enhancing the values of slip perimeter  $\beta$  velocity of fluid and solid particles decreased. It is seen that by increasing the wave number  $\delta$  velocity of fluid and solid particles declined. The behavior of the velocity of fluid for various values of  $\alpha_1$  is also shown in graphs which display that the velocity of fluid decreases as  $\alpha_1$  increases while at some values velocity of fluid remains constant.

The flow of fluid is significantly influenced by pressure. Numerous factors influence the pressure. Pressure rises when the initial values of M are exceeded, but eventually drops. Pressure behaves the same way in a magnetic field as it does for the slip parameter. The relationship between pressure and the second-grade fluid parameter  $\alpha_1$ . Because of a change in  $\alpha_1$ , the pressure changes very slightly, or rather, it practically stays the same. It's observed by the graphs that Reynolds number *Re* also affects the pressure. As Reynolds number is the relationship among inertial forces and viscous forces therefore by increasing the values of *Re* pressure start decreasing. The

relationship between the pressure and the inclination of the channel displays that the declining pressure is because of the inclining in  $\gamma$  firstly pressure declined slowly but after some time it declined quickly.

#### 6.3 Future Work

The given research work can be extended for the different fluid models in the future. This work can be extended for third-grade or fourth-grade dusty fluid. Other fluid models like the Williamson fluid model or Ellis fluid models can also be deliberated to investigate the impact of slip and field known as magnetic with dust particles suspended in them.

Moreover, this work can be extended by adding more body forces to the model. The porous medium can be added to the suggested model or the impact of convective boundary conditions can also be investigated. Geometries like endoscopes, ducts and tubes can also be taken into account to expand the scope of investigation of dusty fluids.

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