Impact of Inclined Magnetic Field on Peristaltic Flow of Ellis Fluid in a Curved Channel

BY

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Candidate of **Master of Science in Mathematics** at the National University of ModernLanguages do hereby declare that the thesis **Impact of Inclined Magnetic Field on Peristaltic Flow of Ellis Fluid in a Curved Channel** submitted by me in partial fulfillment of MS Mathdegree, is my original work, and has not been submitted or published earlier. I also solemnly declare that it shall not, in future, be submitted by me for obtaining any other degree from thisor any other university or institution. I also understand that if evidence of plagiarism is found in my thesis/dissertation at any stage, even after the award of a degree, the work may be cancelled and the degree revoked.

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ABSTRACT

Title: Impact of Inclined Magnetic Field on Peristaltic Flow of Ellis Fluid in a Curved Channel.

The primary goal of this thesis is to investigate the impact of inclined magnetic field and partial slip on the peristaltic flow of Ellis fluid. A mathematical formulation of the problem for the peristaltic flow of MHD Ellis fluid has been created. Furthermore, the walls of the channel are considered to be curved. The perturbation technique is used to address the problem that has been simulated. The challenge is made simpler by employing the low Reynolds numbers and long wavelength approach. The effects of diverse parameter on streamlines, longitudinal velocity, and pressure are investigated. The graphs for longitudinal velocity, stream function, pressure gradient are achieved using Mathematica software.

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TITLE

LIST OF SYMBOLS

S	Extra Stress Tensor
ρ	Density
p	Hydrostatic Pressure
A_1	First Rivilin Ericksen Tensor
τ_0	Shear Stress Corresponding to the half Dynamic Viscosity
μ	Dynamic Viscosity
Π	Second Order Invariant of Stress Tensor
R^*	Circle's Radius
b	Half Width of Channel
R	Radial Direction
X	Axial Direction
a	Amplitude
c	Wave Speed
λ	Wavelength
$\mathbf{B_0}$	Inclined Magnetic Field
ψ	Stream Function
β	Slip Parameter

U	Velocity in axial Direction
V	Velocity in Radial Direction
Re	Reynold Number
η	Peristaltic Wall
k	Curvature
ϕ	Inclination of Magnetic Field
F	Flow rate in the Waveframe
Δp_{λ}	Pressure rise per Wavelength
$\mathbf{J}$	Current Density

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I want to thank and honour Allah Ta'ala for making this study possible and fruitful. Without the sincere support provided by numerous sources for which I would want to sincerely thank you this project could not have been completed. However, a lot of people helped me succeed, and I will always be grateful for their support. I owe a debt of gratitude to Dr. Hadia Tariq, whose counsel, insight, and steadfast support have been invaluable to me during this study process. I consider myself extremely fortunate to have had her as my mentor because her knowledge and guidance have been helpful.

DEDICATION

This thesis work is dedicated to my parents and my teachers, whose steadfast encouragement and support have propelled me to pursue my academic goals. I am incredibly appreciative to my parents for their traits and contributions that have inspired and pushed me during this journey. Your advice and kindness have been a consistent source of support.

CHAPTER 1

INTRODUCTION

The area of physical science that studies how fluids behave both at rest and in motion is known as fluid mechanics [1].The movement of fluids is the subject of fluid mechanics. For physicists, who are primarily interested in comprehending phenomena, its study is crucial. For instance, they might be curious to know why certain wave patterns occur in the water and the atmosphere, or the reason why a fluid layer got hot from below separates into cellular structures. The reasons behind why a tennis ball is hit with "top spin" drops fairly rapidly, how birds soar and how fish swim are all examples of these. Engineers, whose highest attention is in using fluid mechanics to address industrial complications, find the study of fluid mechanics to be equally vital. Designing aircraft with minimal resistance and strong "lift" force to support the aircraft's weight may be of interest to aerospace engineers. Designing water supply systems, irrigation canals, and dams may be of interest to civil engineers. Engineers in pollution control may be motivated to protect our world from the ongoing discharge of industrial effluent into the atmosphere and seas. The design of fluidcouplings, heat exchangers, and turbines may be of interest to mechanical engineers. Designing effective machinery to mix industrial chemicals may be of interest to chemical engineers. Like thestudy of any other field of science, fluid mechanics requires both mathematical analysis and experimental. Finding resolutions to some idealized and basic difficulties and seeing the unity behind seemingly unrelated occurrences are both facilitated by analytical approaches [2].

Fluid mechanics is not only one of the oldest physical sciences, but also one of the most significant in engineering. Aristotle first discussed density in his writings in the fourth century B.C. In comparison to uncommon, dense has more substance packed into a given cubic area. The theory of uniform acceleration, which is frequently credited to Galileo (1564), was also developed by Aristotle. Following Aristotle, the Greek scientist and philosopher Archimedes (287–212 B.C.) proposed a calculated solution to the similar physical issues that preoccupied Aristotle. For instance, Aristotle had previously expressed the first quantitative law of the lever as follows: Justas the weight moving it is to the weight being moved.

The length of the arm supporting the weightis inversely proportional to the length of the arm closest to the poser. Archimedes codified this statement. Numerous fluid static principles were developed by Archimedes, among them the buoyancy principle, which bears his name. Then, for almost 18 centuries, nothing important happened before Galileo began the current investigation into fluids. Galileo saw bodies falling through fluids using mathematical and inductive reasoning rather than direct physical measurement. He opened the path for the greatest scientific awakening humanity had ever knownby holding that the easiest way to represent change in nature mathematically was because mathematics was the language of motion. Despite the fact that calculus hadn't been developed yet. Galileo had the insight to understand that geometric arguments contained conceptions of change. Then, during the reign of Isaac Newton (1642–1726), science blossomed. James Bernoulli and Isaac Newton are credited with the invention of rational mechanics. The former developed the concepts of continuous motion and the relationship between dynamics and statics, and the seconddescribed motion as the effect of outward forces. Leibniz (1646–1716) was likely the earliest to formulate schemes of linear momentum conservation to characterize motion as energy. The modernization of mechanics is credited to L. Euler (1707), a famous Swiss mathematician who treated mechanics as logical and turned Newton's scientific principles into exact calculations.

During Newton's time, mathematical experts and physicists who developed fluid mechanics are credited with the following notable accomplishments:

1. John Bernoulli's hydrostatics.
2. Pressure idea and integral form of linear momentum by Daniel Bernoulli.
3. The hydraulics equation for optimum fluids by John Bernoulli.
4. Mechanics and force ideas by Isaac Newton.
5. Energy and linear momentum by Gottfried Leibniz.
6. Kinematics, the equation of motion for an ideal fluid, the deformable bodies theory,theoretical hydrodynamics and the theory of stress by Leonhard Euler.
7. The idea of stream function and potential velocity by Joe-Louis Lagrange.
8. Concept of vorticity and Potential theory were developed by Augustin Louis Cauchy.
9. Applied hydrodynamics by Jean Le Rond d'Alembert.

The age of 19th century thinkers followed as the next stage in the development of fluid mechanics. During this time, hydrodynamics underwent mathematical developments and modern aerodynamics was born. Here are some theories that have been developed, along with the great mindswho came up with them:

1. Using a complicated variable method of two-dimensional potential flow, was developed by HermannHelmholtz and Gustav Kirchhoff.
2. Viscosity and three-dimensional potential flow was developed by Simeon Denis Poisson.
3. Singularity approach, gas dynamics, and wave theory by Lord Kelvin and William J. Rancine.
4. Rotational motion by Lord Kelvin and H. Helmhotz.
5. Unstable laminal flow and vortex flow by Gromeka.
6. Gelatinous flow by George G. Stokes and Claude L.M.H. Naiver [3].

1.2 Newtonian Fluid

These fluids, which adhere to Newton's equation of viscosity, shows the amount of shear stressand the amount of angular deformation follow a path that is linear. These fluids have an even viscosity regardless of the rate of shear. Some examples of these Newtonian fluids are alcohol,water, petrol, air and glycerin.

1.3 Non-Newtonian Fluid

These fluids defy Newton's rule of viscosity, and depending on the type of fluid being sheared, either have a lower or higher viscosity. The following explanations provide an overview of the five characteristics of non-Newtonian fluids.

1.3.1 Dilatant

A dilatant is a non-Newtonian fluid in which shear stress is applied and the fluid's shear viscosity rises. For example, quicksand and mud slur.

1.3.2 Pseudoplastic

Paint and ketchup are examples of pseudoplastic fluids having shear thinning qualities, in which the viscosity reduces as the rate of applied shear stress increases.

1.3.3 Bingham Plastic

A viscoplastic material known as a Bingham plastic exhibits rigid body behavior at low stresses and viscous fluid behavior at high stresses.

1.3.4 Rheopectic

The fluid's viscosity rises as shear stress rises, and this relationship varies throughout time. Gypsum paste can be the example of such fluids.

1.3.5 Thixotropic

With a rise in shear stress, the fluid's viscosity drops, and the relationship between the two changes throughout time. Two examples of thixotropic fluids are paint and glue.

1.4 Peristalsis

The Greek word peristalikos, which means to clutch and compress, is the source of the English word peristalsis. It refers to a wave of gradual contraction that moves through a channel or tube, causing the cross-sectional area to change. The term "peristalsis" refers to the continual wavelike muscular contraction and relaxation of blood vessels (capillaries, arteries, veins, etc.), other void tubes, other hollow tubes, and physiological vessels like the esophagus, stomach, intestines, and rarely the ureters. Numerous biological organs have been found to be impacted by vasomotion, including the flow of blood in arteries, the movement of ovaries in the tubes known as fallopian pipes, and the transportation of the spermatozoa in the ductus efferentesof male propagative systems and in the canal of the cervical. Peristalsis is a form of locomotion used by some worms. On the same concept, finger pumps and roller for sticky fluids work [4]. Due to the importance of peristaltic transport many researchers studied it [5- 6].

1.5 Magnetohydrodynamics

Research in the area of magnetohydrodynamics, also referred to as MHD looks at the fusion of the electromagnetic and fluid dynamics fields. When a fluid passes through a fieldof magnets, an electric current is produced. The temperature of the fluid as well as the fluid's flowbehavior may be impacted by this electric current [7]. The study of dynamics bio magnetic fluid is another crucial area with several applications. Because blood is made up of cells suspended in plasma that contains cell to cell proteins, hemoglobin, and cell membranes, blood is a biomagnetic fluid. With the addition of a sufficient magnetical field, it is now thinkable to regulateflow and blood pressure behaviour founded on the magnetohydrodynamics appearances of blood.As a result, research into MHD flows may be advantageous for magnetotherapy. When low frequency and intensity pulsating magnetics field are administered in a regulated way, the behavior of the cells and tissues is altered. The combination of a field of magnet and peristaltical movement improves the efficacy of cancer treatment while minimising bleeding during procedures [8].

1.6 Application of Magnetohydrodynamics

We can enumerate some significant cosmic circumstances where magnetohydrodynamics is used.

1.6.1 The electromagnetic pump

A conducting fluid inside a pipe moves due to the Lorentz force, which is created when identical electric current and magnetic field are applied perpendicular to the pipe. Such a mechanism has been used to transfer heat-carrying sodium liquid from the fission reactor's core to the external heat exchanger.

1.7 Ellis Fluid

Water, caustic or abrasive substances, shear thickening agents like oxidizers and gels are all included in the carrying liquid. Agents that thin by shear include paints and polymer solutions.The Power-law model of flow mimics the behavior of fluids that thin under shear and/or thicken under shear. It is suitable to employ the Ellis model when model deviations are clearly proven at low shear rates. In order to account for the restrictive value behavior of the power law liquid model under attention. Ellis provided a three parameters fluid models to demonstrate the flow characteristic over a narrow range of shear rates range.

The generalised Newtonian fluid (GNF) model includes the Ellis model as a subtype. Among the typical GNF models are the Herschel Bulkley model, Carreau fluid model and the power law model. The advantage of the Ellis model is that it can predict power-law role at high shear stresses and Newtonian role at low shear stresses. The Ellis fluid model's primary advantage is that it produces power-law behavior under large shear stress and Newtonian behavior under small shear stress. Thanks to its value, the Ellis fluid model can study the transfer of mass and heat on peristaltic movements with chemical changes in an asymmetrical curve [9-10].

1.8 Slip Boundary Conditions

The no-slip boundary condition, which asserts that the fluid towards the boundaries adheres at the wall, is typically enforced at the fluid solid connections. The no-slip boundary criterion, however, fails in a variety of circumstances. For particle fluids such emulsions, foaming agents, polymeric solutions and suspensions, the boundary condition with no-slip is incorrect. The no-slip boundary requirement is also applicable when the surface is rough to some extent. The no-slip boundary requirement is insufficient for suitably smooth surfaces. In such cases, the boundary with no slip condition is switched by the boundary condition with linear slip, which declares that the tangential segment of surface velocities is proportional to the wall shear stress. On his own, Naiver put forth the slip boundary condition as well as Maxwell [11].

1.9 Thesis Contributions

In this thesis, a detailed review of work by Ali *et al*. [31] has been included. The impact of inclined magnetic field and partial slip on the peristaltic flow of Ellis fluid are discussed. Some important parameters, MHD effect and partial slip parameter are discussed. For computational work we used Mathematica software. Obtained results will be presented graphically.

1.10 Thesis Organization

The following chapters, which make up this thesis, are further divided into:

Chapter 2 includes the details of the relevant literature.

Chapter 3 includes basic definitions and descriptions of concepts used in this study.

Chapter 4 provides a review work of Ali *et al*. [31].

Chapter 5 is the extension work of Ali *et al*. [31].

Chapter 6 contains the conclusion drawn in chapter 5.

In the end, all the references utilized in this research are listed.

CHAPTER 2

LITERATURE REVIEW

The movement of fluid is made possible by a phenomenon called peristalsis, which occurs when waves move along the walls of a tube or duct. Biological organs like the oesophagus, bile duct, arteries and intestines exhibit this phenomenon. Peristaltic motion is a common method for transporting industrial fluids, including corrosive, poisonous, and hygienic fluids [11]. Latham [12] used analytical and experimental methods to pioneer the study of peristaltic flows in his thesis. Over the past half-century, several researchers have examined peristaltic flow issues in various geometrical structures. Shapiro *et al*. [13] studied the peristalsis of viscous solution under conditions of low Reynolds number and long wavelength. Ebaid [14] discussed analytically and statistically, in an asymmetric curve, peristaltical movement of a Newtonian fluid in relation to magnetic field and wall slip circumstances. In a wave frames of reference, the flow is studied while moving at the speed of the wave. The flow is exposed to a uniformly strong magnetical field that is applied in the direction that is transverse. In a wave frames of references that moves with the tendency, the flow is examined. For axial pressure gradients, axial velocity and force of friction, equations have been discovered. Kumari *et al*. [15] discussed the impacts of different factors on the properties of the flow.

As a model for impeded digestive (intestinal) transport, Tripathi *et al*. [16] investigated non-Newtonian liquid flowing peristaltically in a two dimensional, asymmetrical tube with porous medium. Javid *et al*. [17] were inspired by this fact to combine the classic difficulty of peristaltic flow in the channel with complicated wavy curves due to magnetical influences. We can use a low Reynolds number and the lubrication assumption because the complicated wave in a channel with few gaps causes creeping flow. A MHD fluid was reported for peristaltic flow in an asymmetric passage by Sinha *et al*. [18]. In this work, the effects of velocityslip, viscosity variation and thermal slip have were been addressed. For low Reynolds model viscosity parameters, the combined nonlinear differential calculations were analytically explained using the perturbation method.

Ali *et al*. [19] concentrated on the investigation of non-Newtonian liquid peristaltic flows in a curved path. For a third-grade liquid that was non-Newtonian the constitutive relationship between shear rate and stress was employed. In a work by Nadeem and Akram [20], it was examined that, how a magnetic field had slanted effects on the peristaltical movement of a Williamson fluid in a disposed symmetric or asymmetric conduit. The lubrication technique simplifies the highly nonlinear equations. The proposed problem's analytical and numerical explanations had been calculatedand examined. The graphical findings were shown to show the effects of several physical parameters of relevance.

The area of continuum mechanics known as magnetohydrodynamics focuses on how a conducting electrical power fluid moves when a magnetical field is present. Other names for the topic include "hydromagnetics" and "magneto-fluid dynamics." Potential differences are produced when steering material moves across forces along magnetic lines; these potential differences, in turn, generate electric flows. The Lorentz force, is connected to the movementof a power current via a field of magnetism and affects the direction of fluid flow. What really defines and distinguishes magnetohydrodynamics is this close relationship betweenhydrodynamics and electrodynamics. In their study, Srinivas *et al*. [21] evaluated the combinedimpacts of slip condition and the handover of heat on MHD peristaltic movement in a porous path with the influence of boundary characteristics. They were able to solve the issueanalytically for both non-uniform and uniform networks using approximations with long wavelengths and low Reynolds numbers. Mekheimer *et al*. [22] presented a research on the assumption of layer of boundaries concept, the impacts of the slip and Hall current state on the movement of MHD caused by the sinusoidal peristaltical waves in curved walls througha porous material with a relatively high Reynolds number were taken into account. The amplitude of the waving wall is smaller than the boundary layer. By using a regular perturbation method, answers were produced in form of a series expansion with regard to limited amplitude. The Jeffrey liquid model's peristaltical blood movement in an inconsistent porous tube was examined by Bhatti and Abbas [23]. This study shows how slip and magnetichydrodynamics (MHD) interacted.

Yasmeen *et al*. [24] investigated how a field of magnets affects the peristaltic movement of a liquid that is viscous via a curved, 3-dimensionals tube. Investigating the impact of the tube's curvature is of special interest. For a small curvature parameter, a regular perturbationexpansion is sought as a solution.

Jha and Aina [25] studied the influence of MHD on the liquid flowing past a vertical annulus. Slip boundary conditions were also studied along with the temperature jump in the fluid. Bhatti *et al*. [26] observed the collective effects of partial slip and magnetohydrodynamics on Ree-Eyring liquid flow through an absorbent media. The barriers of the irregular porous passage were regarded as amenable. Lubrication approach wasused to study the Ree-Eyring fluid governing equation for blood movement. Hafez *et al*. [27] examined the power of the transfer of mass and heat on the hydro magnetic peristaltical movement of a liquid called Casson in an inclined asymmetry configuration. By using long wavelength assumptions and low Reynolds number the main calculations of the model weregenerated, simplified, and then addressed analyticall. This model may be critical in the realmsof crude oil and biomedical engineering processing. In the presence of a chemical reaction, Srinivas and Muthuraj [28] described in a vertically porous domain, MHD combined convective transfer of mass and heat through peristaltic movement. A reference frame for waves that moved with the wave's speed was used to evaluate the flow. To produce the channel asymmetry, the peristaltic waves pattern on wall surfaces is selected to have a distinctive phaseas well as amplitude.

The Bingham and power law models are combined to one model by the Ellis model. The Ellisliquid model illustrates the nonlinear connection between strain rate amd shear stress. Water,caustic or abrasive substances, and shear-thickening components like oxidizers and gels are allpresent in the moving liquid, paints and polymer solutions are examples of shear-thinning substances. The Power-law framework is a motion characteristic that mimics liquids that are thickening or thinning under shear. The Ellis model is applicable when large variations from the framework are seen at low shear charges. Due to the restricted rate behaviour of the powerlaw liquid model, Ellis liquid model developed a 3 parameter fluids system that flipped the responsibilities of strain rates and shear stress to demonstrate transport components across tinyshear rate ranges discussed by Beleri and Kotnurkar [29]. In 1966, Longwell [30] investigated how an Ellis fluid moved through a pipe with a solid wall. In a planar channel, Ali *et al*. [31] looked at the peristaltic pumping of a non-Newtonian Ellis liquid.In the commonly accepted norms of long wavelength and low Reynolds numbers, differential equations linked nonlinear partial equation leading the difficult question into simpler model. The formulae for the Ellis fluid's wavelength-specific pressure rise, longitudinal movement, pressure change, and streamfunction were obtained using a semi-analytical method. Ellis fluid's peristaltic motion througha compliant channel had been studied by Abbasi *et al*. [32]. The long wavelength estimate is helpful to solve the Ellis fluid model's major equation of motion and equation of continuity while disregarding motional forces.

Sajid *et al.* [33] looked into the renal tubule's Ellis liquid in motion. The pressure differentialacross the wall and the permeability of the wall both have an effect on the liquid absorption atthe wall. If the tubule radius is assumed to be significantly lower than the tubular length, the foremost equations are greatly simplified. Taking into account the low Reynold numbers and long wavelength assumptions, the consequence of perfect slip in velocity on the axissymmetrical peristaltic passage of Ellis liquid through a constant movable pipe was examinedby Saravana *et al*. [34]. It has been explored how emergent parameters affect the rise in pressurewith stream line and time-averaged flux behaviors, as well as analytical formulas for axial speed, stream operation, the fluctuation in pressure for shear thinning and shear thickening solutions, and more. Analytical analysis of the movement of two water electro-osmoticperistaltic movement via a container was conducted by Ali *et al*. [35]. The fluid in the center core, sometimes referred to as the interior section or central section, is described by the Ellis equation in terms of its rheology. The vicinity of the wall, also known as the outside region orexterior zone, contains a Newtonian fluid. The proper expectations of a low Reynold numbers and long wavelength are used to simulate the calculations driving the flow in each location. In their study, Kumar *et al*. [36] used low Reynolds number and long wavelength rules to examine the peristaltic movement of an Ellis liquid system in a tilted regular pipe with a barrier's parameters. The mathematical formulas for the distribution of temperatures, velocity,and function of stream had been found. The outcomes for the shear thinning fluid and shear thickening fluid scenarios were covered in depth.

According to Bhatti *et al*. [37] peristaltic Ellis fluid (blood) flow over a no uniform network was explored to determine the generation of entropy through mass and heat transfer. The channel's walls were regarded as compliant. The closeness of creeping flow and longwavelength conditions were used to simplify the energy, entropy and concentration equationsas well as the equations that regulate for the Ellis fluid model. I thad been found that the fluid characteristics significantly reduced the fluid's velocity. The results of the current study can also be used to address numerous diagnostic issues and to develop new drug supply systems in the fields of pharmacological, biomedical and thermal engineering as well. Javed *et al*. [38] conducted a fictitious examination into the calendering of Ellis fluid using a lubrication assumption. The flow's governing equations were nondimensionalized, solved, and converted into closed form terminologies for velocity and gradient in stress. The pressure distribution was computed using the Runge-Kutta technique. Ali *et al*. [39] proposed a bifurcation analysis for the Ellis liquid model's transportation caused by peristaltic movement.

By the use of the dynamical system approach, the location, qualitativemakeup, and bifurcations of stagnation-points were discussed. The lubricating approach yieldsa precise equation for stream function. There are three different types of flows that are seen: enhanced, trapping, and backward flow. A bifurcation occurs when one flow changes into another. It has been discovered that viscous fluids experience bifurcations earlier than shear- thinning fluids. Particularly when the shear forces are extremely minor, this model closely resembles Newtonian behaviour. When a horizontal gradient of pressure is dictated in a horizontal porous level, resulting in a basic through flow, the formation of the Rayleigh-Bénardinstabilities is explored. Both theoretically and statistically, the threshold conditions for this system's linear instabilities are determined. The onset of the uncertainty happens under the identical parameter situations described in the works for Newtonian fluids saturated a porous media in the case of a negligibly low flow rate. Celli *et al*. [40] discussed this in their work. Inthe stable state of minimal level movement of a viscoelastic Ellis fluid that comes into touch with a vertical cylinder, Alam *et al*. [41] investigated drainage and lift problems. Thedifferential equation that was solved in a closed format using the commonly used binomial series method. Furthermore, unique examples including Newtonian fluid, Bingham plastic, and power law film had been recovered during the investigation of high shear viscoelastic Ellis fluid layer. For both the lift and draining phenomena, physical parameters such as the vorticityvector, liquid film width, average velocity, and flow rate had been studied. On cylindrical surfaces, the Ellis fluid film thickness had been estimated. In their study, Goud and Reddy [42] regulated the peristaltic movement of an Ellis liquid system in a vertical, symmetrical pipe with the boundary characteristics by approximating them with low Reynolds numbers and longwavelengths. The calculated formulae for temperature dispersion, stream function, and velocity had been developed.

An asymmetric compliant channel's peristaltic movement of an incompressible viscid fluid was explored by Haroun *et al*. [43]. The peristaltic transport pattern on the walls was chosen tohave different phases and amplitudes in order to create the station asymmetry. By taking into account the equations for motion for both the fluid and the deformed barriers, the problem of fluid-solid interaction was investigated. By supposing that the channel walls are conform, the driving mechanism of the muscle was described. On reversal flow and mean velocity, the influence of wave magnitude ratio, channels length, phase difference, wall damping, and wall tension had been examined. The findings show that reversal flow happenedclose to the margins, which was not feasible in the context of a flexible symmetric network. Under low-Reynolds and long-wavelength figure norms, the peristaltical motion of an incompressible glutinous fluid in an asymmetrical curve was investigated by Mishri and Rao [44]. In a wave framework of position that moves at the rapidity of the waveform its movement was examined. On the streamline structure, pumping features, entrapment, and reflux occurrences, the impacts of amplitudes of waves, altering channel width, and phase differenceswere explored. They established the time averaged flow limitations for reflux and trapping. It had been noted that the reflux layer, pumping against trapping, and pressure rise only appear when the channel's cross section changes. Due to the constant cross-section of the channel, zero flux rate was manufactured by peristaltical transport on the barriers with the same phase and amplitude propagation. Srinivas and Pushparaj [45] investigated the issue of peristaltical movement of a hydromagnetic viscid incompressible liquid in a tilted path with electrical boundaries that were insulated under long wave length and low-Reynolds number predictions.The peristaltical wave pattern on the partitions was chosen to have varying phases and amplitudes in order to create the channel asymmetry. In the form of a reference point that moves at the wave's speed, the flow was examined. We find terms for the pressure gradient, the velocity field, and the shera stress on the wall. The peristaltic motion of an asymmetrical channel using Oldroyd fluid was examined in the work by Khan *et al*. [46]. An angled magneticfield had been considered during the mathematical evaluation. Along with heat transfer. The methodical results of coupled equations were initially established by modelling the underlyingphysical issue using the regular perturbation approach. Long wavelength approximationassumptions were made.

On the temperature and axial velocity, the effect of an inclined magnetical field are discussed. Kothandapani and Srinivas [47] investigated a Jeffrey fluid's peristaltic movement in an asymmetrical path. Through the action of a magnetical field that was transverse the fluid conducts electricity. It had been possible to find the equations for the gradient of axial pressure, stream function, and the gradient of axial velocity. They demonstrated and discussed how different emergent parameters affect the flow characteristics.It is addressed how different wave types, such as trapezoidal, square, triangular, and sinusoidal,compare to the flow. The mutual effects of the wall characteristics and mass or heat transmission on the peristalsis phenomena of a hyperbolic-tangent fluids in a curving pipe were discussed by Arooj *et al*. [48]. Using the perturbation process, it was easy to explain the temperature, heat transmission coefficient for tiny Weissenberg numbers, speed, stream function and concentration. The equations that govern for a hyperbolic tangent in two dimensions fluids model were modelled by Nadeem and Akram [49]. The regular perturbationapproach has been used to solve the calculations governing the process of a hyperbolic tangentfluid for an asymmetrical tunnel under low Reynolds number and long wavelength conditions.Numerical analysis had been used to derive the pressure rise equation. Several physical variables had been depicted in the conclusion.

It was discovered that the contracted portion of the channel needs a high pressure gradient, and that the gradient of pressure in the area that was slight diminishes as the Weissenberg number and channel width grow. Hayat *et al*. [50] observed the peristaltical movement of a pseudoplastic fluids in a truncated hose. Additionally studied the impact of magnetohydrodynamics (MHD). Asymmetrical channel was taken into account. First, the pertinent problem was developed, andafter that, it was non-dimensionalized. The solution to the lubricant approach-subjected nonlinear difference system. They presented the graphs that showed how various parameters vary with velocity and pressure rise. The phenomena of pumping and trapping was also investigated. Lachiheb [51] provided a thorough analysis of how viscosity variations affect fluid velocity, reflux limits, and trapping. It had been noted that changes in the viscosity parameter have an impact on the pressure increase, pumping area, trapping limit, and friction force, but not on the reflux limit or free pumping.

Kothandapani and Srinivas [52] demonstrated and explored the power of numerous new factorson the flow features. According to the numerical results, the size and amount of confined boluses rised with increasing flexibility, elastic stress and weight characterizing variables, but decreased for high Hartmann number values. According to Khan *et al*. [53] the slip condition influences the peristaltically movement of a non- Newtonian liquid that was incompressible andhas different viscosity via porous media in an inclined symmetrical passage. Using the standard perturbation technique, the leading nonlinear partial differential equation system was resolved, and analytical results in a format of stream functions were produced for both pressure and velocity rise. The combined impact of convection and free magnetohydrodynamic (MHD) flowvia a plate that is vertically embedded within a porous medium were developed by Vijayalakshmi *et al*. [54]. Skin lubrication, the Sherwood number, and the Nusselt number aredetermined along with the wave speed for all feasible values of parameters.

Due to its significance, peristalsis has recently become a major issue in biological science and biomedical engineering. Abd-Alla *et al*. [55] explored the behavior of fractional liquid with MHD effects flowing past a non-uniform path caused by the peristaltic waves. In mathematicalmodelling, a modified version of the process by which fluid fills a porous space was describedby the law of Darcy. To answer the leading equations, the obtained equations were statistically solved using the long-wavelength hypothesis. Limited solutions were used to resolve problemsincluding temperature, speed, pressure rise, pressure gradient, and friction forces. The acquired results were confirmed by literature studies on the current level of threats.

The interplay between mass and heat transmission in the peristaltic movement of a second grade liquid with a magnetic field via a tube was theoretically investigated by A. M.Abd-Allae [56]. The impact of several physical characteristics, including material constants, magnetic fields,and fraction variables, on temperatures, concentrations, and axial. Particular attention was placed on the discussion of pressure gradient, velocity, friction forces, pressure rise, and the coefficient of mass and heat transfer. This was due to the oscillating character of the mass andheat transfer coefficient, which was in line with what is expected physically given the oscillatory behavior of the conduit wall. It was observed that the velocity reduces as the Hartmann quantity rised.

Williamson nanofluid flow through an endoscope was studied by Hayat *et al*. [57]. Although the external tube was sensitive to sinusoidal peristaltic waves, the interior pipe was stiff. Whilethe internal hose had no slip condition, the outside tube satisfied the partial slip criteria. Theytook complaint wall characteristics into account. In blood vessels, where the internal radius ofthe tube is relatively narrow, these preferences are practicable. The resulting problem was studied numerically. Through graphs the speed, temperature, and the effects of physical quantities were examined. It had been determined that slide contributes to decrease temperatureand velocity. While thermophoresis and Brownian diffusion respond in different ways to change in concentration and temperature. Shahzad *et al*. [58] analyzed the consequences of a field of magnets with radial variation on the peristaltic movement of a Carreau-Yasuda fluid through a curve-shaped path with no slip conditions. The resulting nonlinear solution subject to complex boundary circumstances was numerically solved using numerical shooting and Runge-Kutta method. As a result, the innermost layer of a curved channel experienced more stress from the fluid running through it. Ali *et al*. [59] debated about the properties of slip circumstances on the peristaltic examination of a magneto hydro dynamic liquid that was viscous in a two-dimensional tube with varying viscosity. For the situation of a hydrodynamicfluid, an accurate solution was given; however, a series solution was found for a magnetic hydrodynamic fluid in the tiny range of the viscosity component.

CHAPTER 3

BASIC DEFINITIONS

3.1 Fluid Mechanics

Understanding, forecasting, and manipulating a fluid's behavior are all part of fluid mechanics. A basic understanding of fluid mechanics is necessary for daily living because we live in a dense gas environment on a planet that is primarily covered in liquid. Fluid dynamics is a crucial area ofthe applied sciences for engineers with a wide range of intriguing applications. Fluid statics and fluid dynamics are the two branches of the field of fluid mechanics that have historically existed.

3.2 Fluid Statics

Fluid statics, often known as hydrostatics, is the study of a fluid's behavior when it is at rest or nearly so.

3.3 Fluid Dynamics

Fluid dynamics is the field that focuses on fluids in motion.

3.4 Shear Strain

Stress results in displacement relative to an object's original dimensions, which is known as shear strain. When deformation happens perpendicular to a line rather than parallel to it, it is referred to as shear strain. It is denoted by γ.

$$\gamma = \frac{displacement\ of\ plane\ in\ direction\ of\ shear\ force}{prependicular\ distance\ from\ opposite\ plane}.$$

3.5 Shear Stress

In contrast to normal stress, which is applied perpendicularly, shear stress (tau) is defined as a stress that is delivered parallel or tangentially to a surface of an object. It is denoted by r.

$$\tau = \frac{F}{A},$$

where,

F is applied force and A is the portion of a material's cross section that is parallel to the direction of the applied force.

3.6 Shear Modulus

The ratio of shear stress to shear strain at the elastic limit is known as the shear modulus or modulus of rigidity. According to Hooke's law in the form of shear stress, the shear stress (r) is directly proportional to the shear strain (γ) within the elastic limit.

Mathematically

$$\tau \propto \gamma,$$

$$\tau = G\gamma,$$

where G is the elastic constant known as shear modulus, that is given by

$$G = \frac{\tau}{\gamma}.$$

3.7 Newtonian Fluid

Shear stress and shear strain rate for a fluid have the following relationship

$$\tau \propto \frac{d\gamma}{dt},$$

this relationship is linear and can be expressed as follows for all gases and many common liquids:

$$\tau = \mu \frac{d\gamma}{dt},$$

where the absolute or dynamic viscosity of the fluid, ‘μ’ is the proportionality constant. This is referred to as Newton's law of viscosity, and a fluid is considered to be Newtonian if it follows this equation.

3.8 Non-Newtonian Fluid

A fluid is said to be non-Newtonian if it deviates from Newton's viscosity law or if its viscosity is variable and dependent on stress. Non-Newtonian fluids' viscosities can alter in response to force, becoming either more liquid or more solid. Examples of non-Newtonian fluids include several salt solutions, molten polymers, custard, toothpaste, starch suspensions and paint.

3.9 Viscosity

Viscosity is a term used in informal language to describe the "thickness" of fluids; syrup, for instance, has more viscosity than water.

3.10 Mass

A capacity of an object's resistance to acceleration, or a variation in velocity, is said to be that object's mass Newton's second law explains the connection between acceleration (*a*), force (*F*), and mass (*M*) for an object of fixed mass:

$$F = Ma.$$

Engineers working in the aircraft industry and other interconnected sectors also use the slug to measure mass, but kilograms (kg) and pounds mass (lbm) are the units used most frequently.

3.11 Weight

The strength of the force used on a body by the gravity field of the Earth is measured by its weight, or W. Thus, the above equation provides a definition of weight. If g is the acceleration caused by gravity, then

$$W = mg,$$

it is measured in newtons (N).

3.12 Density

According to their density, fluids exhibit varying degrees of acceleration resistance. The mass of a fluid sample is calculated by dividing it by its volume, V. Mathematically

$$\rho = \frac{M}{V}.$$

3.13 Reynolds Number

The ratio of inertial forces to viscous forces in a fluid flow is termed as Reynolds number.
The most important dimensionless group in fluid mechanics, the Reynolds number, is defined as

$$Re = \frac{\rho VL}{\mu},$$

where μ is the fluid viscosity, $'L'$ is the length scale, $'V'$is the fluid velocity scale, and $'\rho'$ is the fluid density. Osborne Reynolds (1842–1912), a renowned pioneer in the study of turbulence and pipe flow, is honored by having this dimensionless group bear his name.

3.14 Laminar Flow

Laminar flow, which is often referred to as streamline flow, is a kind of liquid (which can be liquid or gas) movements in which the fluid travels easily or in expected directions. In a laminar flow, velocity, the fluid's pressure, and other flow factors are all consistent throughout the fluid. Laminar flow can be thought of as a series of thin, parallel layers, or laminae, that move over a horizontal surface. The fluid that comes into contact with the horizontal surface is stationary while all other layers glide over one another.

Laminar flow doesn't happen frequently unless the flow route is comparatively minor, the liquid moves slowly, and the viscosity is moderately high. Oil moving through a thin tube or blood moving through capillaries is referred to as laminar flow.

3.15 Turbulent Flow

Contrary to laminar flow, in which the fluid moves in uniform layers or channels, turbulent flow is the process of a liquid or gas flowing in which the fluid experiences unexpected changes or mixing. In a turbulent flow, the rate of the fluid at a single area is constantly changing in both amplitude and direction.. River currents and wind are frequently turbulent. Examples of turbulent flow include lava flowing , the flow in boat wakes, blood flowing via arteries, the flow through pumps and turbines , air and ocean currents, the flow through pumps and turbines, oil moving through pipelines, and the flow over aircraft wing tips.

3.16 Compressible Fluids

Real fluids undergo volume changes when they are compressed or stretched by an outside force, as well as when pressure or temperature changes. A fluid is considered to be compressible if its volume changes, which is a property of volume change.

3.17 Incompressible Fluids

A fluid that cannot be compressed or expanded and whose volume remains constant is said to be incompressible. There isn't a rigidly incompressible fluid in nature. However, when compressibility is less of a concern, such as when air or water is moving around us, a flow can be conceived of as an incompressible fluid flow.

3.18 Ideal Fluid

Ideal or perfect fluids are incompressible fluids that don't have viscosity. There isn't an ideal fluid in the world. However, an ideal fluid plays an important part for the fundamentals of fluid dynamics since it is simple to treat theoretically.

3.19 Pressure

Pressure is the term used to describe the physical power applied on a body. A perpendicular forces utilized to the surfaces of the objects per unit area. The simple calculation for pressure is forceper unit area. Pascal (Pa) are used to measure pressure. Different types of pressure include gauge, atmospheric, differential, and absolute pressures.

3.20 Surface Force

The force acting across an internal or exterior surface element in a material body is known as the surface force, indicated by the symbol f_s. Shear forces and normal forces are two perpendicular parts that make up surface force. Shear forces act tangentially across an area while normal forces act normally over a region.

3.21 Body Force

In physics, a body force is a force that permeates the entire volume of a body. Electric fields, magnetic fields, and Gravitational forces are a few examples of body forces. Contrasted with surface or contact forces, which are applied to an object's surface, are body forces.

3.22 Inertial Force

An accelerating force operates on a body in one direction, whereas an inertial force acts in the opposite direction, being equal to the product of the accelerating force and the body's mass.

3.23 Uniform Flow

In a zone of uniform flow, the magnitude and direction of the velocity vectors remain constant. The pilot of a vehicle flying through a motionless atmosphere at a constant speed perceives the air as moving towards the vehicle at a steady pace. As a result, the upstream flow field provides an example of a uniform flow from the pilot's point of view.

3.24 Steady State

A flow is deemed constant when the Eulerian velocity field is unaffected by time. Thus,

$$u = u(x),$$

describes a continuous flow. If a velocity field is known in functional form, the lack of time in the function used to express the velocity field denotes a continuous flow. Steady flows are typical in engineering since many equipment and systems are built to function in a constant state manner. For instance, a wind tunnel will acquire a stable working condition after several minutes of operation, until the settings are specifically changed, the speed control will maintain the air speed in the test portion at this point.

3.25 No Slip Condition

It is required to describe circumstances that simulate the behavior of the flow and fluid characteristics at various sorts of boundaries when utilizing the governing equations to solve a flow problem. The terms "no-penetration " and "no-slip" refer to the boundary conditions that solely hold for the velocity field. Observation demonstrates that a fluid does not move in the tangential direction with respect to a solid surface in almost all engineering flows. Instead, a condition known as no-slip occurs where the fluid adheres to the surface. The boundary's tangential component velocity UT, is equal to the tangential component of velocity uT, we infer from this. In fluid dynamics, this boundary condition,

$$u_T = U_T,$$

is known as the no-slip condition.

3.26 Convective Derivative

The convective derivative is a derivative taken with regard to a coordinate system moving with velocity u, and it is frequently employed in fluid mechanics and classical mechanics. It is also sometimes pointed as the advective derivative, substantive derivative, or the material derivative.

3.27 Local Derivative

The rate at which a quantity changes over time at a fixed position in a fluid, expressed as $\frac{\partial f}{\partial t}$.

3.28 Cauchy Stress Tensor

The second order tensor that bears Augustin-Louis Cauchy's name is the Cauchy stress tensor, also mentioned to as the true stress tensor or just the stress tensor in continuum mechanics. The situation of stress at a position or configuration inside a deformed objects is defined by the tensor's nine constituent elements, or σ_{ij}.

3.29 Law of Inertia

An object at rest remains at rest, while an object in motion continues to move while maintaining a constant speed and a straight path, unless affected by an uneven force.

3.30 Wave Speed

The distance a wave travels in a certain amount of time, such as the distance it travels in one second, is measured as its wave speed. The following is the equation that represents the wave speed:

$$\text{Speed} = \frac{Distance}{Time}.$$

3.31 Wave Length

The wavelength of a wave is used to describe its length. The distance between the crest (top) of one wave and the crest wave of the next is known as the wavelength. By taking a measurement from the "trough" (bottom) of one wave to the "trough" of the next, the wavelength can also be ascertained.

3.32 Amplitude

The largest point displacement on a wave is its amplitude.

3.33 Curvature

The rate at which a curve's direction varies in relation to its length is referred to as its curvature in mathematics.

CHAPTER 4

CHANNEL FLOW OF ELLIS FLUID DUE TO PERISTALSIS

4.1 Introduction

In this chapter, we reviewed the work of Ali *et al*. [31]. The author studies non-Newtonian Ellis fluid peristaltic pumping in a planar path. Under the commonly accepted norms of a long wavelength and low Reynolds number, a pair of partial differential calculations that are nonlinear governing the issue are made simpler. The formulas for longitudinal velocity, stream function, pressure rise per wavelength, and pressure gradient are achieved using a semi-analytical approach. For numerous standards of the material factor of the Ellis liquid, the key features of peristaltic motion are visually explained.

4.2 Physical Model

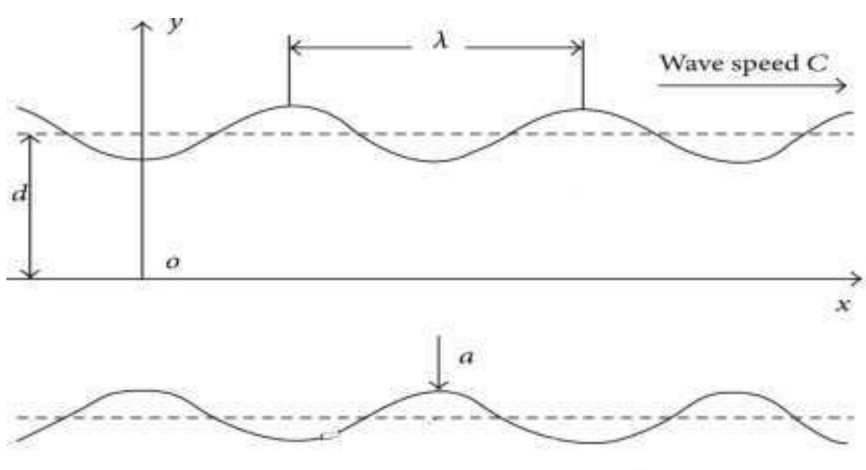


Figure 4.1 Geometry of the problem

Taking a non-compactible Ellis liquid flows as a result of an endless wave train propagating along the channel edges. System of Cartesian coordinates (X, Y), where the x-axis runs parallel to the flow direction and the y-axis runs perpendicular to it are taken. The following equation provides a mathematical definition of the channel's walls:

$$\mathrm{h}(X, t) = a + b sin\left[\frac{2\pi}{\lambda}(X - ct)\right], \tag{4.1}$$

where a is half width of channel, b is amplitude, c is wave speed and λ is wavelength d is the channel half width and t is the time. Unsteady and two-dimensional models are possible for the flow under investigation. Due to this, we define

$$\boldsymbol{V} = [(X, Y, t), (X, Y, t), 0], \tag{4.2}$$

where $U\ and\ V$ are velocity components in X and Y directions.

4.3 Mathematical Formulation

The ruling equations to describe the flow of the fluid model are:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \tag{4.3}$$

$$\rho\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial X} + V\frac{\partial}{\partial Y}\right)U = -\frac{\partial p}{\partial X} + \frac{\partial S_{XX}}{\partial X} + \frac{\partial S_{XY}}{\partial Y}, \tag{4.4}$$

$$\rho\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial X} + V\frac{\partial}{\partial Y}\right)V = -\frac{\partial p}{\partial Y} + \frac{\partial S_{XY}}{\partial X} + \frac{\partial S_{YY}}{\partial Y}. \tag{4.5}$$

The flow in the laboratory frame $(X,)$ is inherently erratic. In a frame travelling at the speed of a wave, it can be considered steady. The term "wave frame" refers to such a frame. The following changes are made in between the two frames:

$$x = X - ct,\ y = Y,\ u = U - c,\ v = V, \tag{4.6}$$

where v is component of velocity in y direction and u is component of velocity in x direction.

The dimensionless variables taken into account for the above equations are

$$x = \frac{\lambda x^*}{2\pi},\ y = ay^*, u = cu^*,\ v = cv^*,\ S = \frac{\mu c}{a}S^*,\ p = \frac{\lambda\mu c p^*}{2\pi a^2},\ h = ah^*. \tag{4.7}$$

After removing the asterisks, equations (4.3) through (4.5) assume the following form.

$$\delta\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0\,, \tag{4.8}$$

$$Re\left[\left(\delta u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)u\right]=-\frac{\partial p}{\partial x}+\delta\frac{\partial S_{xx}}{\partial x}+\frac{\partial S_{xy}}{\partial y}, \tag{4.9}$$

$$\delta Re\left[\left(\delta u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)v\right]=-\frac{\partial p}{\partial y}+\delta^2\frac{\partial S_{xy}}{\partial x}+\delta\frac{\partial S_{yy}}{\partial y}, \tag{4.10}$$

where the components of Ellis fluid model are:

$$S_{xx}=\frac{2\delta\frac{\partial u}{\partial x}}{1+(\beta\mathbb{X})^{\alpha-1}}\,, \tag{4.11}$$

$$S_{xy}=\frac{(\frac{\partial u}{\partial y}+\delta\frac{\partial v}{\partial x})}{1+(\beta\mathbb{X})^{\alpha-1}}\,, \tag{4.12}$$

$$S_{yy}=\frac{2\frac{\partial v}{\partial y}}{1+(\beta\mathbb{X})^{\alpha-1}}\,, \tag{4.13}$$

with

$$\mathbb{X}=(\tfrac{1}{2}(S_{xx})^2+2\left(S_{xy}\right)^2+(S_{yy})^2)^{\frac{1}{2}}\,.$$

The dimensionless numbers are

$Re=\frac{\rho ca}{\mu}$ is the Reynold number,

$\beta=\frac{c}{a{\tau_0}^2}$ is the dimensionless material factors,

$\delta=\frac{2\pi a}{\lambda}$ is the wave number.

Defining the stream function (x,y) by the relation $u=\frac{\partial\psi}{\partial y}, v=-\delta\frac{\partial\psi}{\partial x}$.

The continuity equation (4.11) is satisfied identically and equation (4.9)-(4.13) become:

$$Re\delta\left[\left(\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}\right)\frac{\partial\psi}{\partial y}\right]=-\frac{\partial p}{\partial x}+\delta\frac{\partial S_{xx}}{\partial x}+\frac{\partial S_{xy}}{\partial y}, \tag{4.15}$$

$$\delta^3 Re\left[\left(\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}\right)\frac{\partial\psi}{\partial x}\right]=-\frac{\partial p}{\partial y}+\delta^2\frac{\partial S_{xy}}{\partial x}+\delta\frac{\partial S_{xy}}{\partial y}, \tag{4.16}$$

$$S_{xx}=\frac{2\delta\frac{\partial^2\psi}{\partial y^2}}{1+(\beta\mathbb{x})^{\alpha-1}}, \tag{4.17}$$

$$S_{xy}=\frac{\left(\frac{\partial^2\psi}{\partial y^2}-\delta^2\frac{\partial^2\psi}{\partial x^2}\right)}{1+(\beta\mathbb{x})^{\alpha-1}}, \tag{4.18}$$

$$S_{yy}=\frac{-\delta\frac{\partial^2\psi}{\partial y\partial x}}{1+(\beta\mathbb{x})^{\alpha-1}}. \tag{4.19}$$

The symmetry condition at the centerline and the no slip condition at the channel wall apply to Equations (4.15) and (4.16). In terms of stream function, these criteria are mathematically stated as

$$\frac{\partial^2\psi}{\partial y^2}=0\ \ at\ \ y=0\ \ and\ \ \frac{\partial\psi}{\partial y}=-1,\ at\ \ h=1+\phi sinx, \tag{4.20}$$

where $\phi=\frac{b}{a}$ is the amplitude ratio.

4.4 Solution Methodology:

Equations (4.15)–(4.19) are higher-order nonlinear partial differential equations. Getting a closed form from the solution of these equations is challenging. But in many real-world peristalsis-related physical issues, the wavelength is far larger than the channel's breadth (radius). The channel width to wave wavelength ratio is represented by the parameter δ in our problem. Therefore, if δ is small, equations (4.15) to (4.19) reduce to:

$$\frac{\partial p}{\partial x}=\frac{\partial S_{xy}}{\partial x}, \tag{4.21}$$

$$\frac{\partial p}{\partial y}=0, \tag{4.22}$$

$$S_{xx} = S_{xy} = 0, \tag{4.23}$$

$$S_{xy} = \frac{\frac{\partial^2\psi}{\partial y^2}}{1+(\beta S_{xy})^{\alpha-1}}. \tag{4.24}$$

According to Equation (4.22) p is not a function of y. As a result, the only explanation is that p is a function of x. This indicates that $\frac{dp}{dx}$ can be treated as a constant when integrating equation (4.21) as it is just a function of x.

Using the boundary constraint $\frac{\partial^2\psi}{\partial y^2}$ = 0, we may integrate equation (4.21) with respect to y and obtain

$$S_{xy} = \frac{dp}{dx}y. \tag{4.25}$$

Substituting equation (4.25) into equation (4.24) and integrating twice one finds

$$\psi = \frac{y^3}{6}\frac{dp}{dx} + \frac{\beta^{\alpha-1}(\frac{dp}{dx})^{\alpha}}{(\alpha+1)(\alpha+2)}y^{\alpha+2} + C_3 y + C_4, \tag{4.26}$$

where the integration constants C_3 and C_4 are used. The expression (4.26) contains three unknowns because $\frac{dp}{dx}$ is unknown. The boundary condition $\frac{\partial\psi}{\partial y}$ = -1 at $y = h$ is insufficient to determine the value of each of these unknowns in a single way. By requiring a constant flow rate

for each cross-section, an additional boundary requirement on stream function can be imposed. The following additional boundary conditions are produced by this assumption.

$$\psi = 0, at\ y = 0, and\ \psi = F, at\ y = h. \tag{4.27}$$

where F denotes the wave frame-required rate of flow. The stream function in the following expression can be determined in light of the no-slip requirement and the first condition (4.27)

$$\psi = \frac{y^3}{6}\frac{dp}{dx} - \frac{\beta^{\alpha-1}\left(\frac{dp}{dx}\right)^{\alpha}}{(\alpha+1)(\alpha+2)} y^{\alpha+2} + \left[-1 - \frac{h^2}{2}\frac{dp}{dx} - \frac{\beta^{\alpha-1}\left(\frac{dp}{dx}\right)^{\alpha}}{(\alpha+1)} h^{\alpha+1}\right] y. \qquad (4.28)$$

$\frac{dp}{dx}$ in the expression above is still undefined. The following non-linear algebraic equation can be generated by using the second boundary condition (4.27)

$$\psi = \frac{h^3}{6}\frac{dp}{dx} + \frac{\beta^{\alpha-1}\left(\frac{dp}{dx}\right)^{\alpha}}{(\alpha+1)(\alpha+2)} h^{\alpha+2} + \left[-1 - \frac{h^2}{2} - \frac{\beta^{\alpha-1}\left(\frac{dp}{dx}\right)^{\alpha}}{(\alpha+1)} h^{\alpha+1}\right] h = F. \qquad (4.29)$$

Any computing software can be used to calculate the aforementioned equation for $\frac{dp}{dx}$ for each the cross-sections x for a particular set of parameters. The solution is finished once the $\frac{dp}{dx}$ value is known. The amount of pressure increase across one wavelength is calculated by integrating $\frac{dp}{dx}$ over a single wavelength. Making use of the formula

$$\Delta p_{\lambda} = \int_0^{2\pi} \frac{dp}{dx} dx.$$

4.5 Results and Disscusion:

The impacts of material constant α on rise in pressure per wavelength are shown in Fig. 4.2. This figure demonstrates three distinct zones. Peristaltic pumping region is the area where Q > 0, $\Delta P_\lambda >$ 0 occurs peristalsis must effort against the pressure increase in this area to move the fluids. The free pumping region is the area where $Q > 0$, $\Delta P_\lambda = 0$. Free pumping flux is the appropriate value of Q for $\Delta P_\lambda = 0$. The free pumping movement is entirely initiated by peristaltic waves since $\Delta P_\lambda =$ 0.The final zone, known as the increased pumping region, is where Q > 0, $\Delta P_\lambda < 0$.

Due to peristalsis, the pressure helps the flow in this area. It should be noticed that an increase in α reduces the pressure increase per wavelength in the peristaltic pumping zone for a settled rate of the specified flow rate. It is almost discovered that the free pumping flux exists independently of α. Even so, with a fixed amount of mean flow rate Q, the pressure's help in augmented pumping regions reduces as pressure increases α. The Ellis model's second defining material constant is the symbol β. It is discovered that the effects of β on the ΔP_λ are comparable to those of α.
Fig.4.3 further demonstrates that as one switches from a Newtonian to an Ellis fluid, ΔP_λ declines.

In Figs.4.4 and 4.5, the longitudinal velocities at a cross-section of $x = -\pi$ is depicted for various values of α and β. These graphs do show that the material constants of the Ellis fluid have a considerable impact on longitudinal velocity. According to Figs. 4.4 and 4.5, increasing either of α or β results in a decrease in the magnitude of longitudinal speed close to the channels centre. Furthermore, Fig. 4.5 shows that the longitudinal rate at the canal's centre falls when the fluid transitions from Newtonian to Ellis.

Figs.4.6 and 4.7 depict the streamline form for various values α of and β. Again, it is shown that raising either α or β has a comparable effect on the streamlines of the flow. In fact, it has been found that by increasing α or β, the strong point of the recirculating zone that appears in the larger area of the channel reduces. Though, it is noted that growing β makes such a drop occur more quickly.

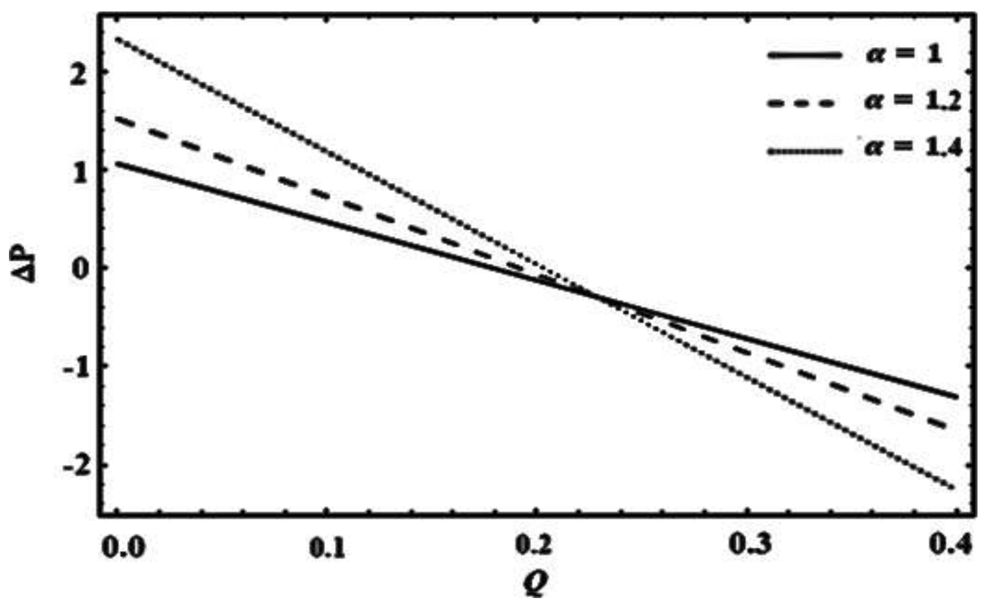


Figure 4.2 Plot showing the relationship between flow rate Q and pressure rise per wavelength ΔP_λ for several values of the material parameter for $\phi = 0.4, F = -0.8, and\ \beta = 20$.

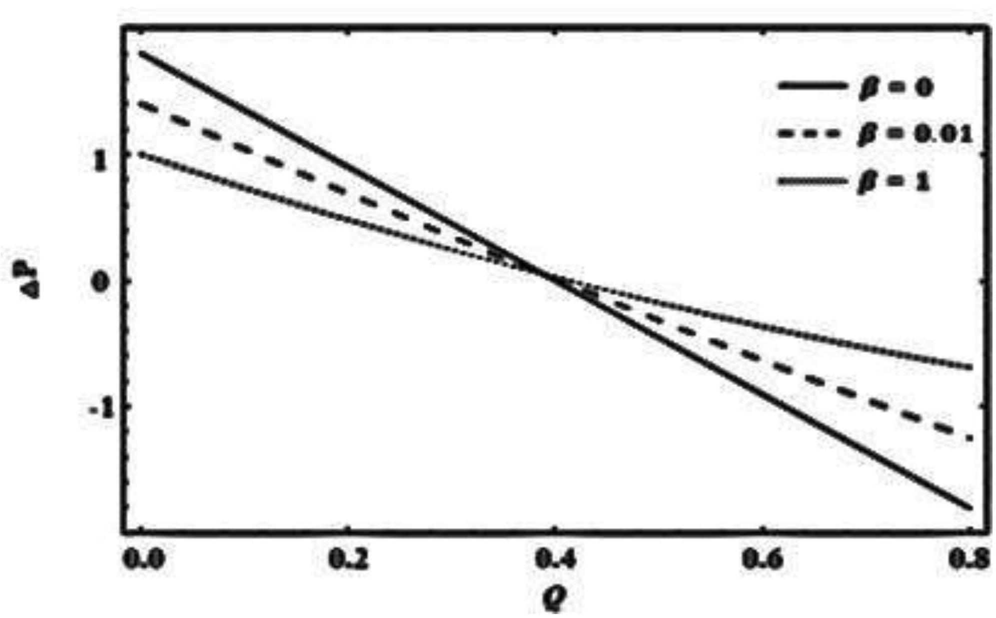


Figure 4.3 Plot showing the relationship between flow rate Q and pressure rise per wavelength ΔP_λ for several values of β when $\phi = 0.4, F = -0.8, and\ \alpha = 1.5$.

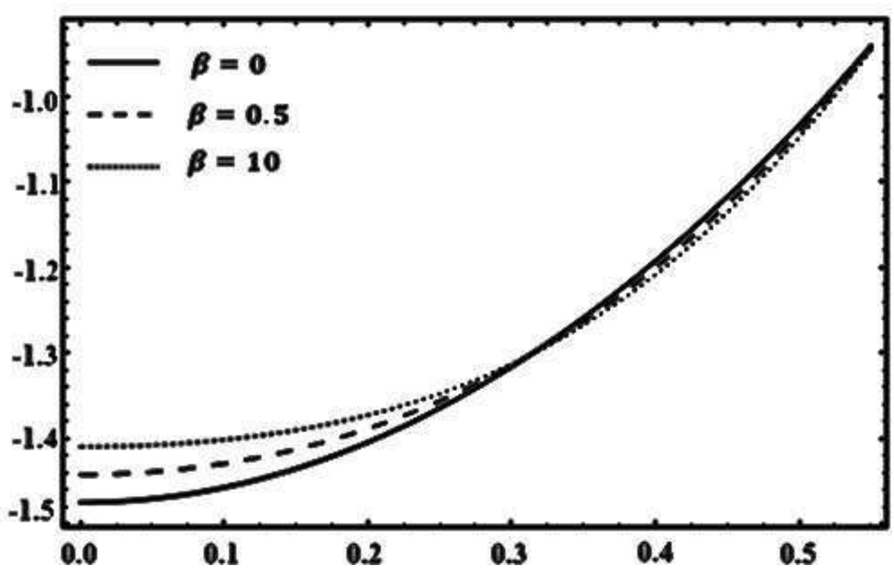


Figure 4.4 Plot of longitudinal velocity (u) for several values β when $\phi = 0.4, F = -0.8, and\ \alpha = 2.$

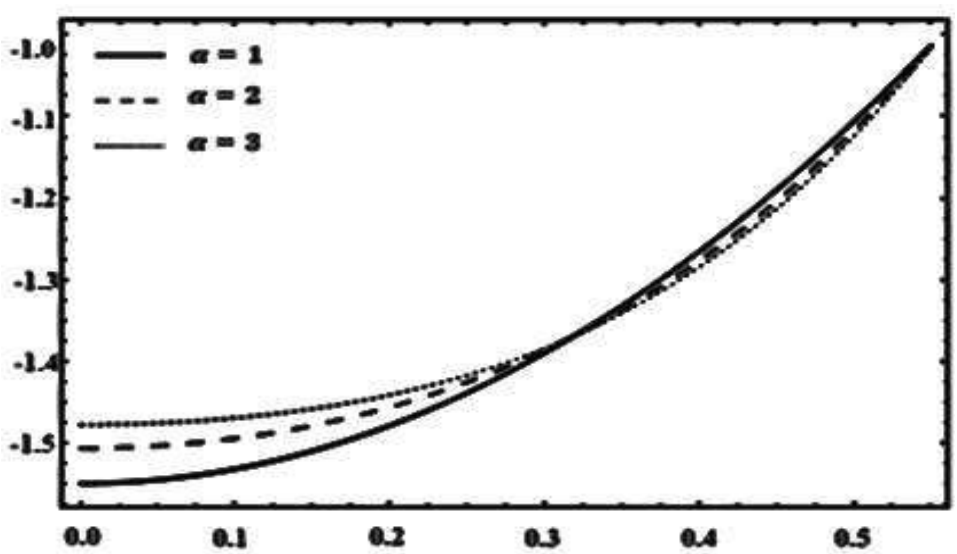
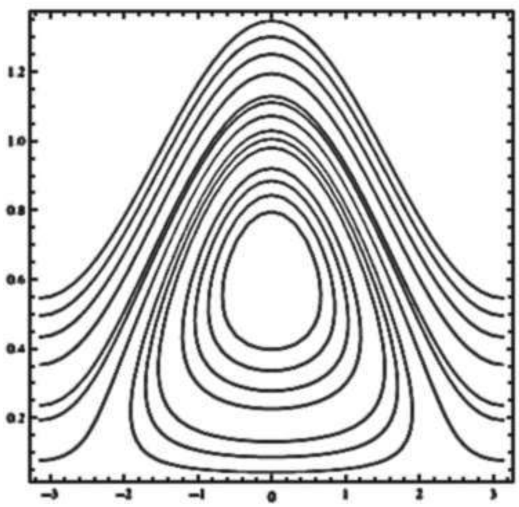
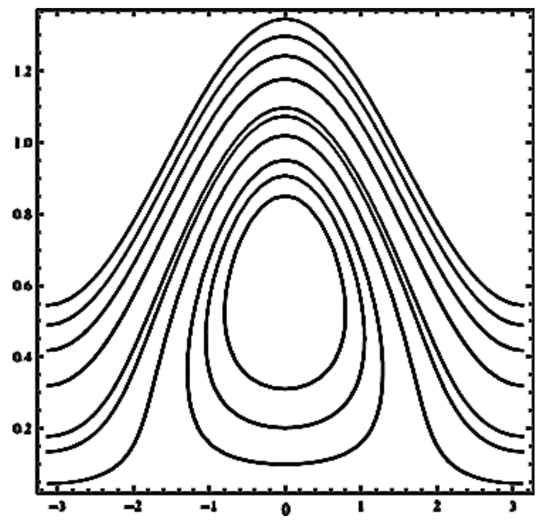


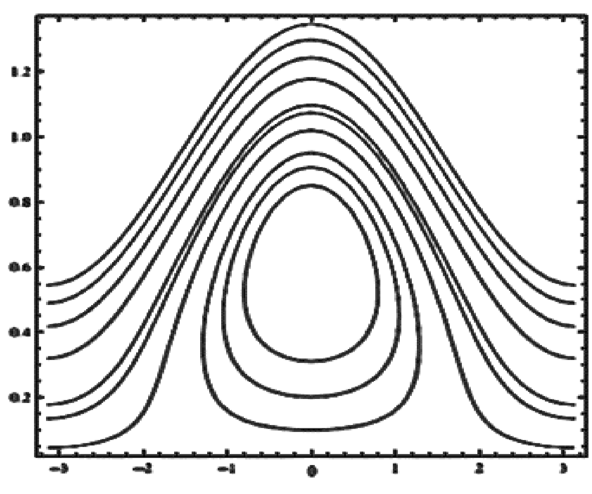
Figure 4.5 Plot of longitudinal velocity (u) for several values α when $\phi = 0.4, F = -0.8, and\ \beta = 2.$



(a)

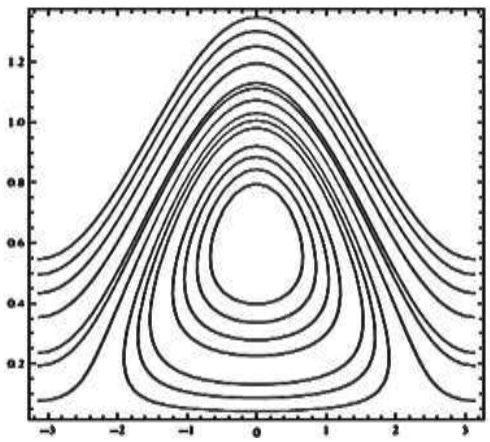


(b)

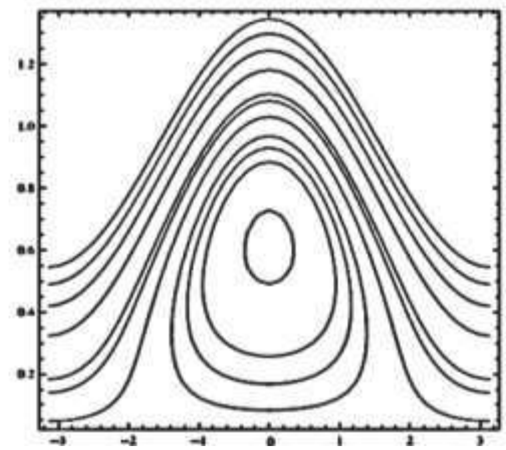


(c)

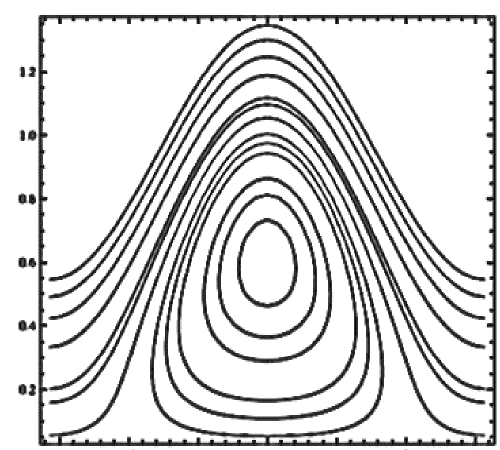
Figure 4.6 Plot of streamlines for (a) $\beta = 0$ (b) $\beta = 2$ (c) $\beta = 10$ when $\phi = 0.4$, $\alpha = 2$ and $F = -0.25$.



(a)



(b)



(c)

Figure 4.7 Plot of streamlines for (a) $\alpha = 1$ (b) $\alpha = 2$ (c) $\alpha = 3$ when ϕ=0.4, F=-0.25 and β=0.5.

CHAPTER 5

Impact of Inclined Magnetic Field on Peristaltic Flow of Ellis Fluid in a Curved Channel

5.1 Introduction

This chapter will present and discuss the effects of magnetic field that is inclined on the Ellis fluid's peristaltic movement in a curved channel along with slip conditions at the boundary. The perturbation technique is helped to obtain the solutions of the modelled problem. It is a very effective tool in actual fluid mechanics and contemporary mathematical physics. To represent the effects of different factors on the velocity, pressure increases and contours of the fluid, MATHEMATICA software has been utilized.

5.2 Problem Formulation

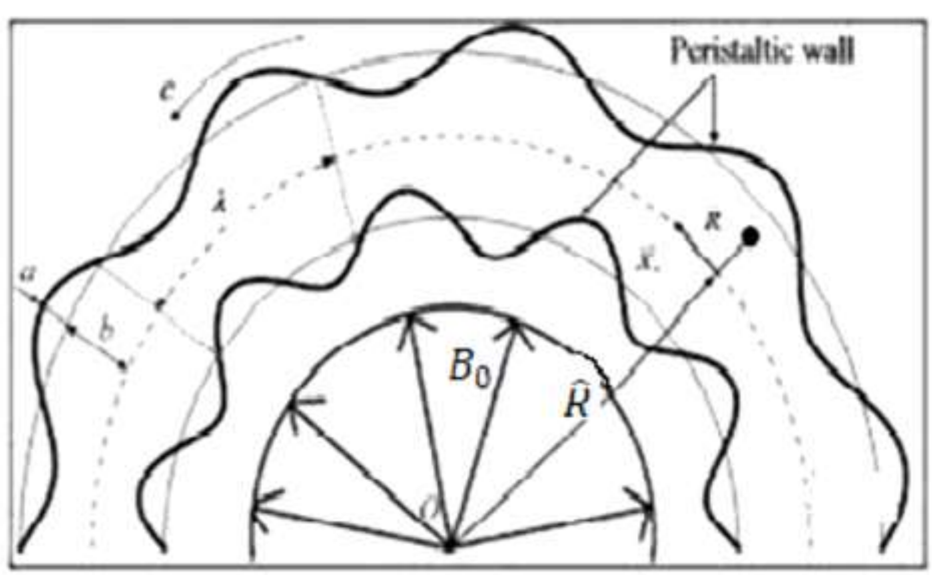


Figure 5.1 Geometry of the problem

The peristaltic process of magnetohydrodynamics incompressible Ellis fluid has been researchedin curved geometry. A sinusoidal wave of amplitude is being displayed in wall channels. Channelwalls have a complaint-like appearance. $\hat{R}$ stands for the circle's radius, and b is the channel's half-width, with O as its center. In a system of coordinate with curved axes *(R, X)*, where X stands for axial direction and R stands for radiated direction, flow has been studied.

$$r = \eta(x,t) = b + a\, sin \frac{2\pi}{\lambda}(\mathrm{X} - ct), \tag{5.1}$$

where a is the amplitude of sinusoidal wave, λ is wavelength and c is wave speed , η represent the displacement of lower wall is equal to the displacement of upper wall respectively.

The mass and linear momentum balances for the flow under study, with body forces present, are

$$div\boldsymbol{V} = 0, \tag{5.2}$$

$$\rho\frac{d\boldsymbol{V}}{dt} = -\boldsymbol{\nabla}p + div\boldsymbol{S} + \rho\bar{\boldsymbol{b}}. \tag{5.3}$$

where S is the extra stress tensor, p is the hydrostatic pressure, the material derivative is $\frac{d}{dt}$, ρ is the density, V is the velocity and $\bar{b}$ is the body force.

For an Ellis liquid the extra stress S is

$$S = \frac{\mu}{1+(\frac{\Pi}{\tau_0})^{\alpha-1}}A_1\ . \tag{5.4}$$

In the above equation τ_0 *and* α are material constant. A_1 is the first Rivilin–Ericksen tensor, second order invariant of stress tensor and Π is second order invariant of stress tensor. The constant τ_0 is commonly defined as the shear stress corresponding to the half dynamic viscosity. μ is the dynamic viscosity.

The components of stress tensor are:

$$S_{RR} = \frac{2\mu\frac{\partial V}{\partial R}}{1+(AX)^{\alpha-1}}, \tag{5.5}$$

$$S_{RX} = \frac{\mu\left(\frac{\partial U}{\partial R}+\frac{\hat{R}}{\hat{R}+R}\frac{\partial V}{\partial X}-\frac{U}{\hat{R}+R}\right)}{1+(AX)^{\alpha-1}}, \tag{5.6}$$

$$S_{XX} = \frac{-2\mu\frac{\partial V}{\partial R}}{1+(AX)^{\alpha-1}}, \tag{5.7}$$

With

$$AX = \left[\frac{8\left(\frac{\partial V}{\partial R}\right)^2+2\left(\frac{\partial U}{\partial R}\right)+\frac{R}{R+r}\left(\frac{\partial V}{\partial X}\right)-\frac{(U+c)^2}{r+R}}{\overline{\tau_0}^2}\right]^{\alpha-1}. \tag{5.8}$$

The field of inclined geometry in curved geometry is represented as:

$$\boldsymbol{B}_0 = \left(\frac{\hat{R}B_0}{\hat{R}+R}sin\phi_0, \frac{\hat{R}B_0}{\hat{R}+R}cos\phi_0, 0\right). \tag{5.9}$$

Due to the low magnetic Reynolds number, it should be noted that induced magnetic field impacts are disregarded. Here $'\phi_0'$ is indicating the inclination of the magnetic field.

By Ohm's law

$$\boldsymbol{J} = [V \times \boldsymbol{B}_0], \tag{5.10}$$

and by Lorentz force

$$F = \boldsymbol{J} \times \boldsymbol{B}_0, \tag{5.11}$$

in which $\boldsymbol{J}$ shows current density and σ shows electrical conductivity

$$F = \sigma^2\left(\frac{\hat{R}B_0}{\hat{R}+R}\right)^2 (Ucos\phi_0 - Vsin\phi_0)^2. \tag{5.12}$$

The governing equations are:

$$\hat{R}\frac{\partial U}{\partial X} + \frac{\partial}{\partial R}\left(\left(\hat{R}+R\right)V\right) = 0, \tag{5.13}$$

$$\rho\left(\frac{dU}{dt}+\frac{UV}{\hat{R}+R}\right) = \frac{\hat{R}}{(\hat{R}+R)}\frac{\partial P}{\partial X} + \frac{1}{(\hat{R}+R)^2}\frac{\partial}{\partial R}\{\left(\hat{R}+R\right)^2 S_{RX}\} + \frac{\hat{R}}{\hat{R}+R}\frac{\partial S_{RR}}{\partial X} + \sigma\left(\frac{\hat{R}B_0}{\hat{R}+R}\right)^2 (Usin\phi_0 cos\phi_0 - Vsin^2\phi_0), \tag{5.14}$$

$$\rho\left(\frac{dV}{dt}-\frac{U^2}{\hat{R}+R}\right)=-\frac{\partial P}{\partial R}+\frac{1}{\hat{R}+R}\frac{\partial}{\partial R}\{(\hat{R}+R)S_{XX}\}-\frac{S_{RR}}{(\hat{R}+R)}+\hat{R}\frac{\partial S_{RX}}{\partial X}-\sigma\left(\frac{\hat{R}B_0}{\hat{R}+R}\right)^2(Ucos^2\phi_0-Vsin\phi_0cos\phi_0)\ . \tag{5.15}$$

Using the transformation defined below, weshall non-dimensionalize our modelled equations.

$$x^*=\frac{x}{\lambda},\ t^*=\frac{t}{\lambda},\ \epsilon=\frac{a}{d},\ \eta^*=\frac{\eta}{d},\ h^*=\frac{b}{a_0},\ k^*=\frac{\hat{R}}{d},$$

$$Re=\frac{pcd}{\mu},\ p^*=\frac{pd^2}{\mu c\lambda},\ r^*=R,\ \psi^*=\frac{\psi}{cd},\ B_0=\sqrt{\frac{\sigma^2{B_0}^2{d_0}^2}{\mu}},$$

$$\delta=\frac{d}{\lambda},\ X=x^*+ct,\ B=\frac{c}{d{\tau_0}^2},\ U=u^*+c,\ V=v^*,\ u=\frac{-k\delta}{r+k}\frac{\partial\psi}{\partial x},\ v=\frac{\partial\psi}{\partial r},$$

where

p is pressure, δ is a wave number, is peristaltic wall, Re is Reynolds number, k is curvature, ϵ is amplitude and B is Ellis fluid parameter.

After dropping the * the component of the extra stress tensor become:

$$S_{xx}=\frac{2\mu\frac{\partial v}{\partial r}}{1+(A\mathbb{X})^{\alpha-1}}, \tag{5.16}$$

$$S_{rx}=\frac{\mu\left(\frac{\partial u}{\partial r}+\frac{k}{r+k}\frac{\partial v}{\partial x}-\frac{\left(\frac{\partial\psi}{\partial r}+1\right)}{r+k}\right)}{1+(A\mathbb{X})^{\alpha-1}}, \tag{5.17}$$

$$S_{rr}=\frac{-2\mu\frac{\partial v}{\partial r}}{1+(A\mathbb{X})^{\alpha-1}}, \tag{5.18}$$

where

$$(A\mathbb{X})^{\alpha-1}=\left[\frac{8\left(\frac{\partial v}{\partial r}\right)^2+2\left(\left(\frac{\partial u}{\partial r}\right)+\frac{k}{r+k}\left(\frac{\partial v}{\partial x}\right)-\left(\frac{\partial\psi}{\partial r}+1\right)\right.}{{\tau_0}^2}\right]^{\alpha-1}. \tag{5.19}$$

Assuming the long wavelength and low Reynolds numbers approximation the governing equations are reduced as following:

$$\frac{k}{(k+r)}\frac{\partial p}{\partial x} = \frac{1}{(k+r)^2}\frac{\partial}{\partial r}\left((k+r)^2 S_{rx}\right) - \left(\frac{kB_0}{k+r}\right)^2 \left(1+\frac{\partial \psi}{\partial r}\right) sin^2\phi_0, \tag{5.20}$$

$$\frac{\partial p}{\partial r} = 0, \tag{5.21}$$

$$S_{xx} = 0, \tag{5.22}$$

$$S_{rx} = -\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r+k}\frac{\partial \psi}{\partial r} - B\left(-\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r+k}\frac{\partial \psi}{\partial r}\right), \tag{5.23}$$

$$S_{rr} = 0. \tag{5.24}$$

The corresponding boundary condition are:

$$\psi = -\frac{F}{2}, \qquad \frac{\partial \psi}{\partial r} + \beta S_{rx} = -1 \qquad at \quad r = -\eta\ , \tag{5.25}$$

$$\psi = \frac{F}{2}, \qquad \frac{\partial \psi}{\partial r} - \beta S_{rx} = -1 \qquad at \quad r = \eta, \tag{5.26}$$

where $\eta = 1 + dsin(2\pi x)\ and\ F = \int_{-\eta}^{\eta} \frac{\partial \psi}{\partial r} dr = \psi(\eta) - \psi(-\eta)$.

5.3 Solution Methodology

The solution series regarding the perturbation parameter B has been expanded and represented using the standard perturbation technique.

$$\psi = \psi_0 + B\psi_1 + O(B^2)\ , \tag{5.27}$$

$$\mathrm{F} = F_0 + BF_1 + O(B^2)\ , \tag{5.28}$$

$$S_{rx} = S_{0rx} + BS_{1rx} + (B^2). \tag{5.29}$$

5.3.1 System of Zero Order

$$\frac{\partial}{\partial r}\left[\frac{1}{(r+k)}\frac{\partial}{\partial r}\left((r+k)^2 S_{0rx}\right) - \frac{(k\boldsymbol{B}_0 sin\phi_0)^2}{k+r}\left(\frac{\partial \psi_0}{\partial r} + 1\right)\right] = 0, \tag{5.30}$$

with boundary conditions:

$$\psi_0 = -\frac{F_0}{2}, \quad \frac{\partial \psi_0}{\partial r} + \beta S_{0rx} = -1, \quad at \quad r = -\eta,$$

(5.31)

$$\psi_0 = \frac{F_0}{2}, \quad \frac{\partial \psi_0}{\partial r} - \beta S_{0rx} = -1, \quad at \quad r = \eta,$$

and

$$\frac{\partial p_0}{\partial x} = \frac{1}{k(r+k)}\frac{\partial}{\partial r}\left((r+k)^2 S_{0rx}\right) - \frac{(k\boldsymbol{B}_0 sin\phi_0)^2}{k+r}\left(\frac{\partial \psi_0}{\partial r} + 1\right), \tag{5.32}$$

in which

$$S_{0rx} = -\frac{\partial^2 \psi_0}{\partial r^2} + \frac{1}{r+k}\frac{\partial \psi_0}{\partial r}. \tag{5.33}$$

5.3.2 First Order System:

$$\frac{\partial}{\partial r}\left[\frac{1}{(r+k)}\frac{\partial}{\partial r}\left((r+k)^2 S_{1rx}\right) - \frac{(k\boldsymbol{B}_0 sin\phi_0)^2}{k+r}\frac{\partial \psi_1}{\partial r}\right] = 0, \tag{5.34}$$

with boundary conditions

$$\psi_1 = -\frac{F_1}{2}, \quad \frac{\partial \psi_1}{\partial r} + \beta S_{1rx} = 0, \quad at \quad r = -\eta,$$

(5.35)

$$\psi_1 = \frac{F_1}{2}, \quad \frac{\partial \psi_1}{\partial r} - \beta S_{1rx} = 0, \quad at \quad r = \eta,$$

and

$$\frac{\partial p_1}{\partial x} = \frac{1}{k(r+k)}\frac{\partial}{\partial r}\left((k+r)^2 S_{1rx}\right) - \frac{(k\boldsymbol{B}_0 sin\phi_0)^2}{k+r}\frac{\partial \psi_1}{\partial r}, \tag{5.36}$$

where

$$S_{1rx} = -\frac{\partial^2 \psi_1}{\partial r^2} + \frac{1}{r+k}\frac{\partial \psi_1}{\partial r} - \beta\left(\frac{1}{r+k}\frac{\partial \psi_0}{\partial r} - \frac{(k\boldsymbol{B}_0 sin\phi_0)^2}{k+r}\frac{\partial^2 \psi_0}{\partial r^2}\right). \tag{5.37}$$

The solutions of ψ_0 and ψ_1 are obtained from Mathematica software and the resultsare demonstrated graphically.

5.4 Results and Disscusion

5.4.1 Pressure Profile

Peristaltic pumping zone is defined as the area where $Q > 0, \Delta P_\lambda > 0$. To move the fluid forward in this area, peristalsis must struggle against the pressure increase. The free pumping area is definedas the area where $Q > 0, \Delta P_\lambda = 0$. For $\Delta P_\lambda = 0$, the matching value of Q is known as flux for free pumping. The free pumping flux can only be attributed to peristaltic waves as $\Delta P_\lambda = 0$. The increased pumping region is the final area when $Q > 0, \Delta P_\lambda < 0$.

In fig. 5.2 as we increases the value of curvature the peristatic pumping area increases. However in augmented pumping area the pressure decrease with the increasing value of curvature. In fig. 5.3 the peristaltic pumping region increases as rises the value of slip parameter. In fig. 5.4 the effect of Ellis fluid parameter on the ΔP_λ are similar to the result of slip parameter. In fig. 5.5 when we increase the value of inclination of magnetic field, the overall peristaltic pumping region increases. There is no free pumping region, in this case.

5.4.2 Velocity Profile

Fig. 5.6 and fig. 5.7 are plotted to demonstrate the behavior of slip parameter and the Ellis fluid parameter. The impact of β on the velocity profile has been investigated in fig.5.6. It is found that increasing the slip parameter β enhances the velocity profile, with minimal behaviors occurring close to the curved channel's centerline. Fig. 5.7 is plotted to examine the effects of Ellis fluid parameter B on the velocity profile of the fluid. The increase in Ellis parameter promotes the velocity near the boundary region.

5.4.3 Stream Functions Profile

Figs 5.8 - 5.11 are arranged to show streamlines nature for different values measured in this work. These figures depict that with growing Ellis fluid parameter the bolus size expands in upper half of curved channel. In lower half the bolus size decreases. Streamlines paths for k is depicted in fig. 5.9 that bolus size grows. In fig 5.10, when slip parameter increase the bolus reduces in the upper half of a curved channel and expands in lower half of a channel. In fig. 5.11,when inclination of magnetic field increases the bolus decreases in lower half of a curved path.

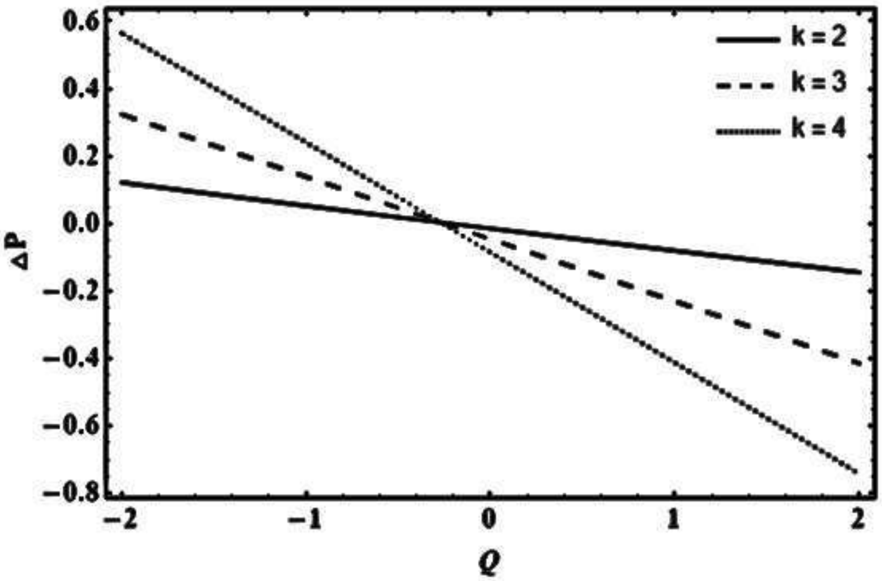


Figure 5.2 Plot of pressure versus flow rate for various value of curvature (k).

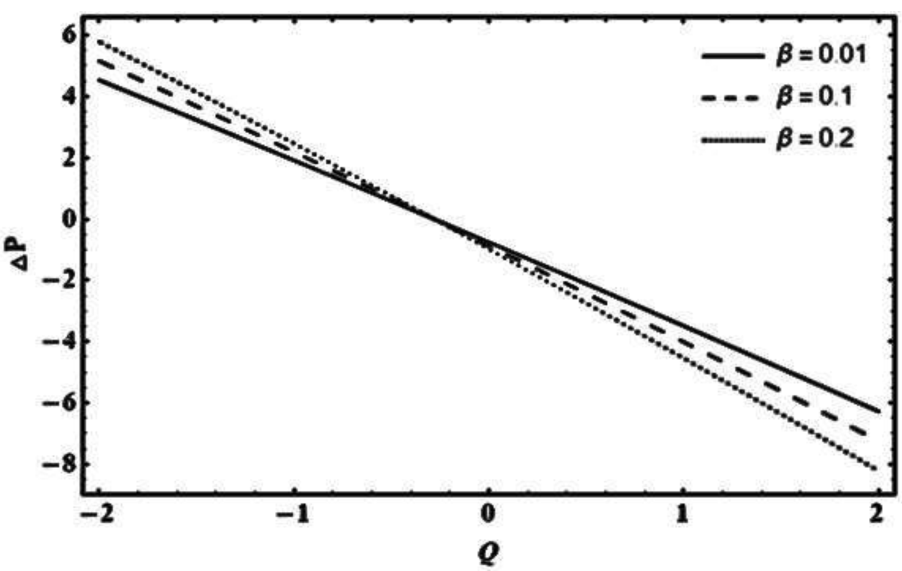


Figure 5.3 Plot of pressure versus flow rate for various values of (β).

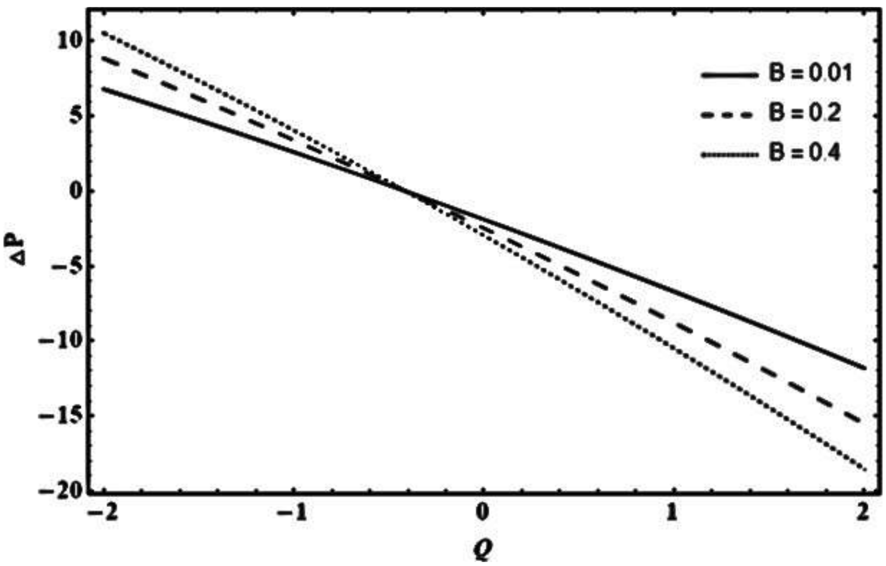


Figure 5.4 Plot of pressure versus flow rate for various of Ellis fluid parameter (B).

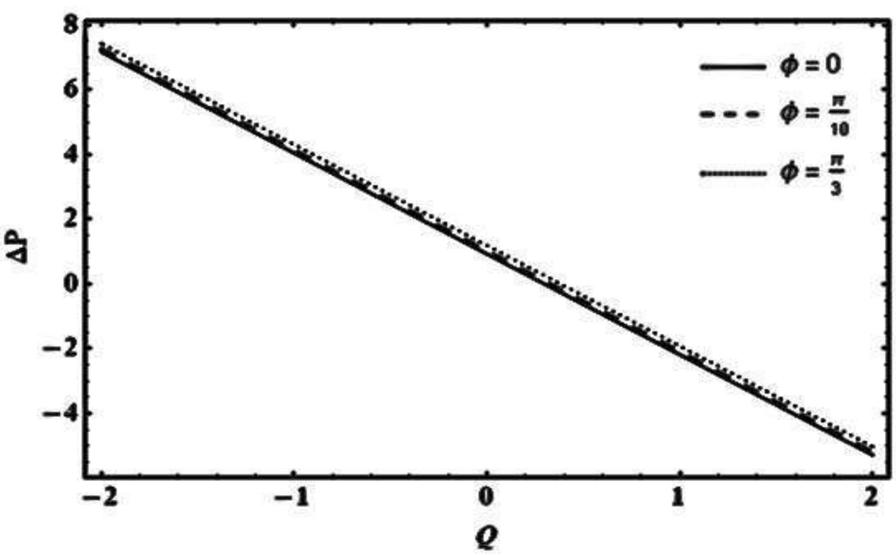


Figure 5.5 Plot of pressure versus flow rate for various values of inclination of magneticfield (ϕ).

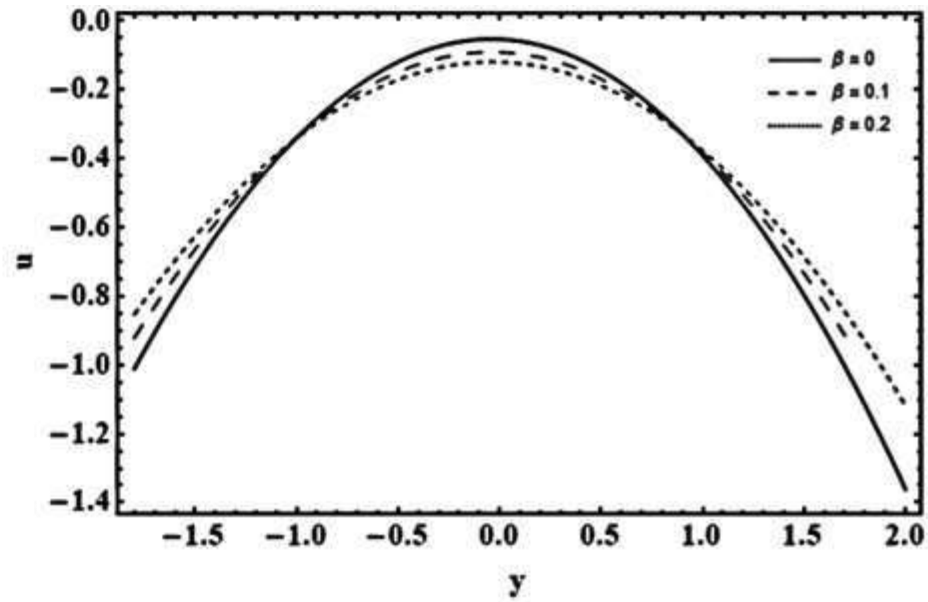


Figure 5.6 Variation in velocity profile for slip parameter (β).

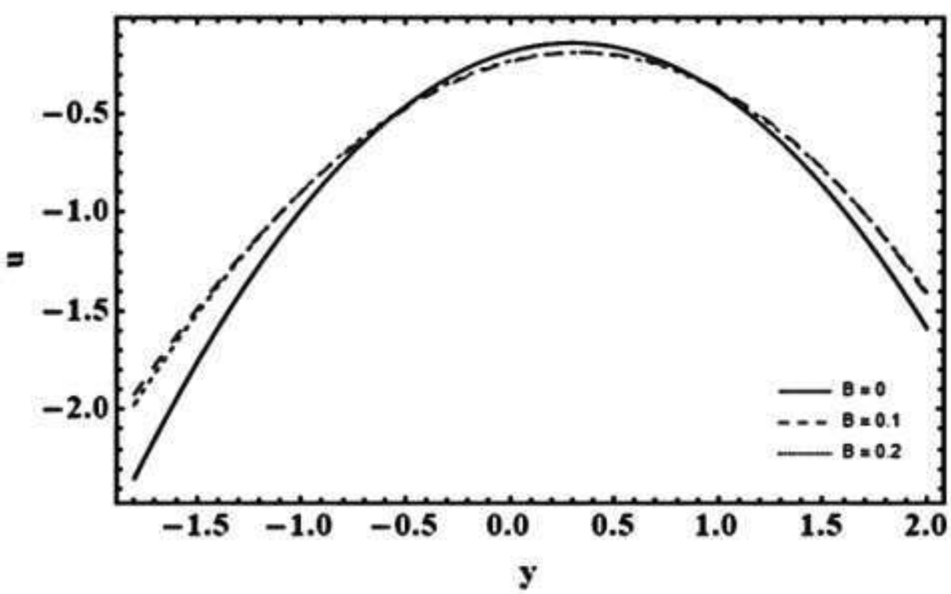
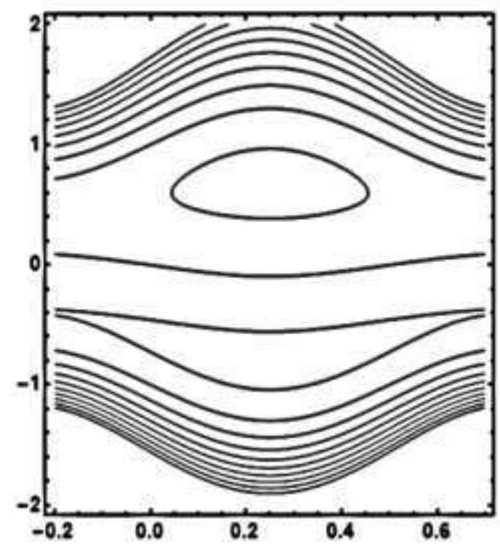
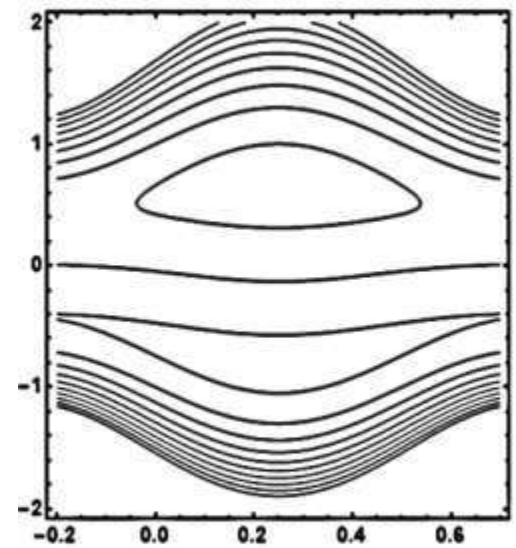


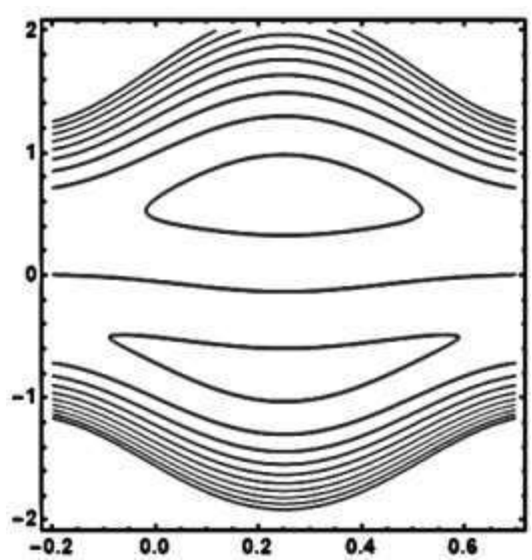
Figure 5.7 Variation in velocity profile for Ellis fluid parameter (B).



$B = 0.1$

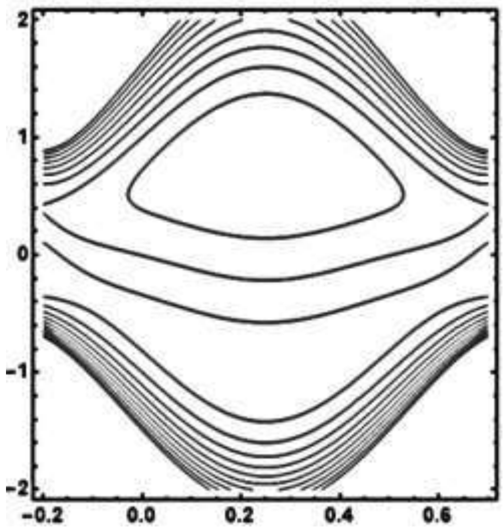


$B = 0.5$

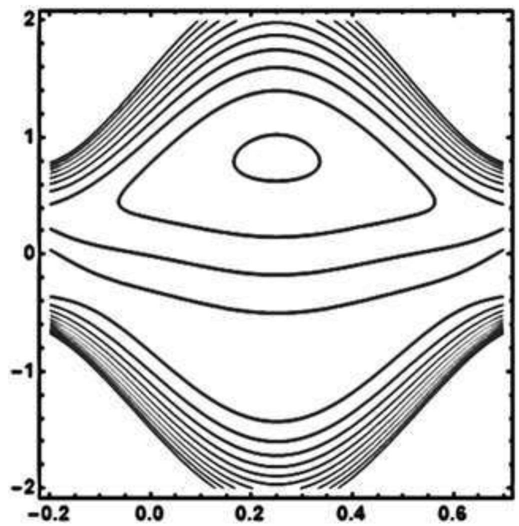


$B = 0.65$

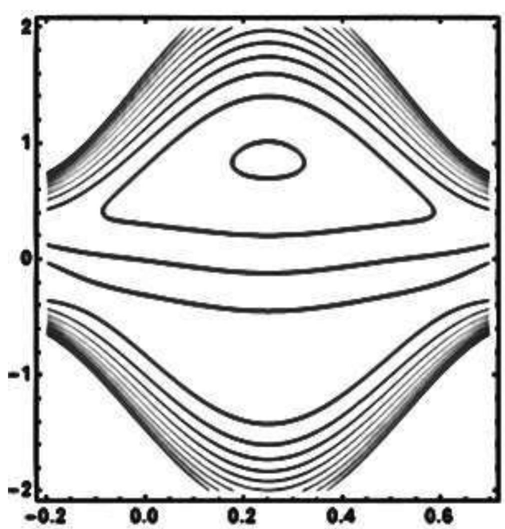
Figure 5.8 Stream lines for Ellis fluid parameter (B).



$k = 3$

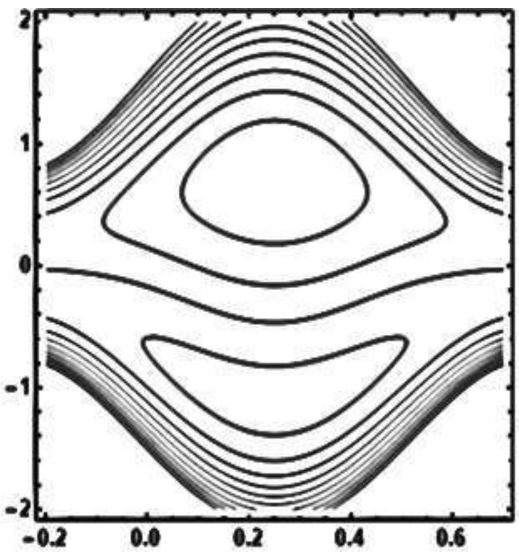


$k = 4$

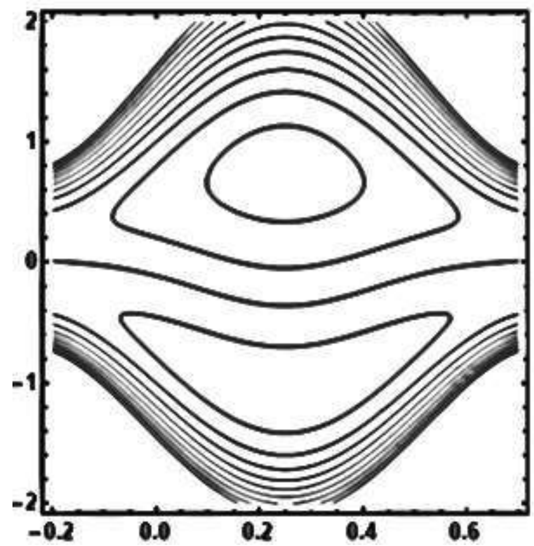


$k = 6$

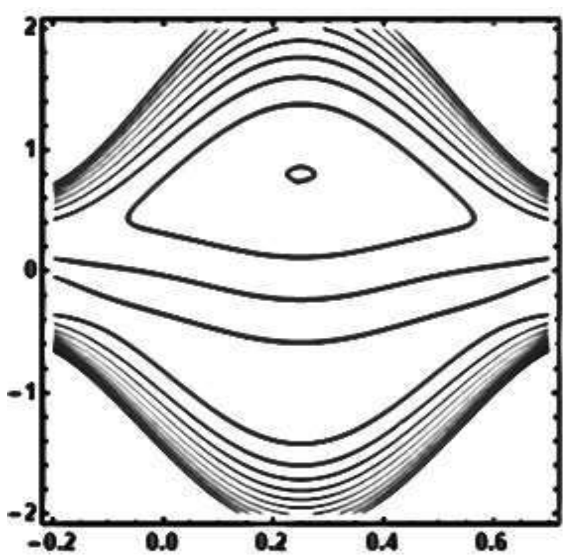
Figure 5.9 Streamlines for curvature of the channel (k).



$\beta = 0$

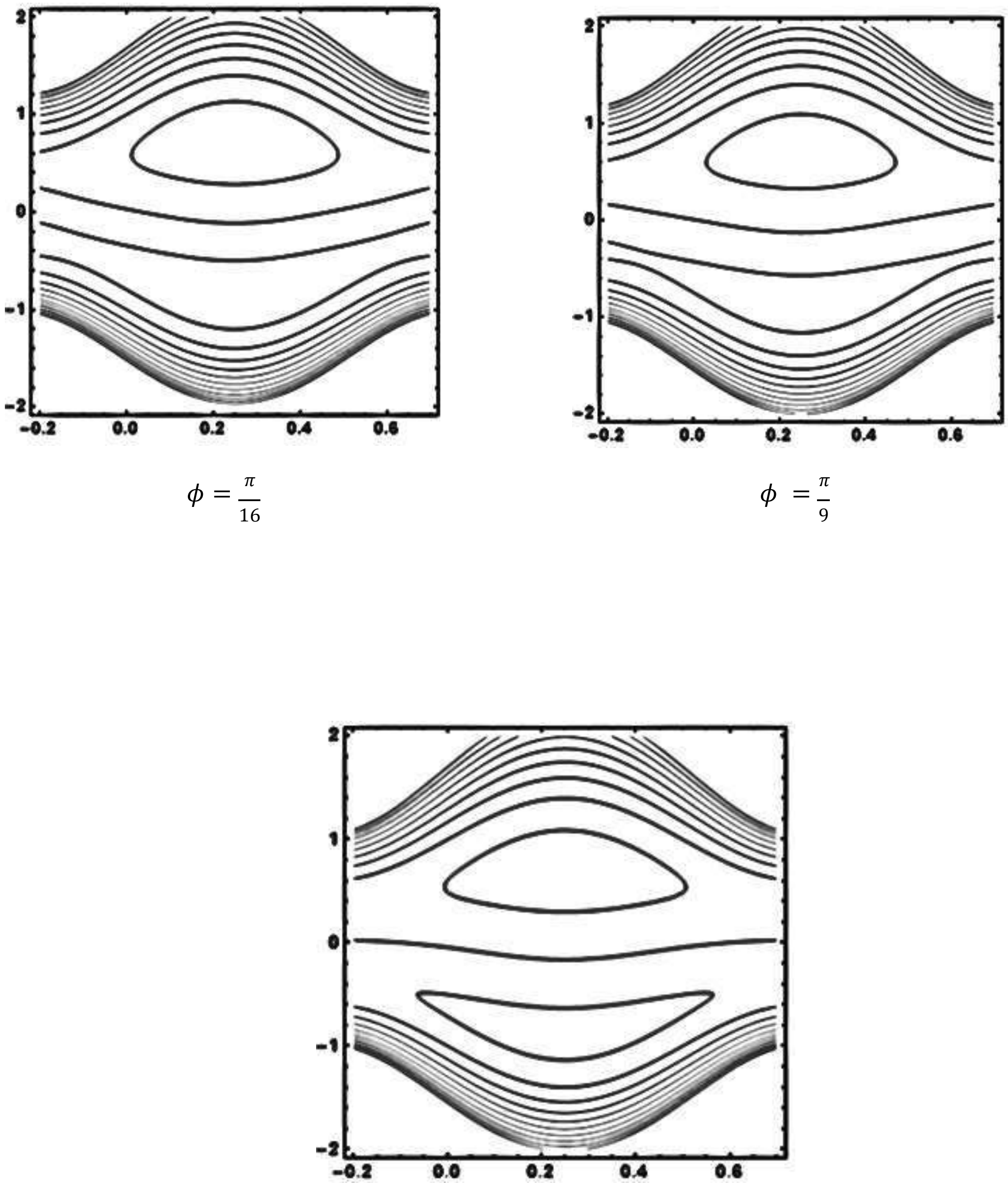


$\beta = 0.05$



$\beta = 0.1$

Figure 5.10 Streamlines for slip parameter (β).



$\phi = \frac{\pi}{16}$

$\phi = \frac{\pi}{9}$

$\phi = \frac{\pi}{4}$

Figure 5.11 Streamlines for inclination of magnetic field (ϕ).

CHAPTER 6

CONCLUSION

This thesis aims to explore the influence of inclined magnetic field and partial slip on peristaltic flow of an Ellis fluid. The complexity of the problem is addressed using perturbation techniques, leveraging widely accepted assumptions of low Reynolds numbers and long wavelengths to simplify the analysis. Graphical representations of longitudinal velocity, stream function, and pressure gradient are generated using Mathematica software, providing visual insights into the behavior of the peristaltic flow under varying conditions. The overall conclusion drawn from the current work is summarized as following:

Increasing the curvature (k) enhances pressure within the peristaltic pumping region, while a higher slip parameter value leads to a decrease in pressure within the co-pumping region. Furthermore, elevating the Ellis fluid parameter results in heightened pressure in the peristaltic pumping region. Conversely, as the inclination of the magnetic field increases, it eliminates any free pumping region.

Increasing the slip parameter β causes a reduction in fluid velocity near the centerline of the curved channel due to enhanced slip effects, altering the velocity profile and disrupting the boundary layer flow dynamics.

As the Ellis fluid thins out more readily with higher B values, it encounters less resistance to flow within the tube. This reduced resistance allows the bolus to move more freely through the peristaltic system. The curvature parameter K influences the trapping effect, with higher curvature leading to a larger bolus due to increased fluid concentration along the inner wall. On the other hand, the slip parameter β affects the slip behavior at the channel walls, with higher slip resulting in reduced trapping and a smaller bolus in the upper half of the curved channel due to decreased resistance and easier flow past the walls.

Future Work

The model could be expanded by incorporating additional factors such as energy activation, viscous dissipation, and various other body forces. This could involve exploring different fluid models like Jeffery and Johnson segalman, as well as other non-Newtonian fluid models, to analyze the influence of inclined magnetic field and boundary slip. While our current study focused on a curved channel, future research could explore alternative geometries such as symmetric, asymmetric, or planar channels to further investigate the effects of different fluid models. Additionally, incorporating realistic boundary conditions, such as rough or porous walls, could provide further insights into the behavior of peristaltic flow in practical scenarios. Moreover, investigating the interaction between peristaltic flow and external factors such as external electric or magnetic fields could offer valuable insights into potential applications in biomedical devices or microfluidic systems.

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REFERENCES

1. Granger, R. A. (1995). Fluid mechanics. Courier Corporation.
2. Cohen, I. M., & Kundu, P. K. (2004). Fluid mechanics. Elsevier.
3. Granger, D. A. (2022). John Dewey, Eastern Philosophy, and the American Avant-Garde:Art and Education in Becoming. Beijing International Review of Education, 4(3), 366-395.
4. Tripathi, D., Pandey, S. K., & Das, S. (2010). Peristaltic flow of viscoelastic fluid with fractional Maxwell model through a channel. Applied Mathematics and Computation, 215(10), 3645-3654.
5. Sinha, A., Shit, G. C., & Ranjit, N. K. (2015). Peristaltic transport of MHD flow and heat transfer in an asymmetric channel: Effects of variable viscosity, velocity-slip and temperature jump. Alexandria Engineering Journal, 54(3), 691-704.
6. Sajid, M., Mahmood, K., & Abbas, Z. (2012). Axisymmetric stagnation-point flow with a general slip boundary condition over a lubricated surface. Chinese Physics Letters, 29(2), 024702
7. Javed, M., Qadeer, F., Imran, N., Kumam, P., & Sohail, M. (2022). Peristaltic mechanismof Ellis fluid in curved configuration with homogeneous and heterogeneous effects.Alexandria Engineering Journal, 61(12), 10677-10688.
8. Abbasi, A., Khan, S. U., Farooq, W., Mughal, F. M., Khan, M. I., Prasannakumara, B. C., & Galal, A. M. (2023). Peristaltic flow of chemically reactive Ellis fluid through an asymmetric channel: Heat and mass transfer analysis. Ain Shams Engineering Journal, 14(1), 101832.
9. Guedri, K., Mahmood, Z., Fadhl, B. M., Makhdoum, B. M., Eldin, S. M., & Khan, U. (2023). Mathematical analysis of nonlinear thermal radiation and nanoparticle aggregation on unsteady MHD flow of micropolar nanofluid over shrinking sheet. Heliyon, 9(3).
10. Hussain, Q., Latif, T., Alvi, N., & Asghar, S. (2018). Nonlinear radiative peristaltic flow of hydromagnetic fluid through porous medium. Results in Physics, 9, 121-134.

11. Ramesh, K., & Devakar, M. (2015). Magnetohydrodynamic peristaltic transport of couplestress fluid through porous medium in an inclined asymmetric channel with heat transfer. Journal of Magnetism and Magnetic Materials, 394, 335-348.
12. Latham, T. W. (1966). Fluid motions in a peristaltic pump (Doctoral dissertation, Massachusetts Institute of Technology).
13. Shapiro, A. H., Jaffrin, M. Y., & Weinberg, S. L. (1969). Peristaltic pumping with long wavelengths at low Reynolds number. Journal of fluid mechanics, 37(4), 799-825.
14. Ebaid, A. (2008). Effects of magnetic field and wall slip conditions on the peristaltic transport of a Newtonian fluid in an asymmetric channel. Physics letters A, 372(24), 4493-4499.
15. Kumari, S. K., Murthy, P. M. V. R., Kumar, Y. V. K. R., & Sreenadh, S. (2011). Peristaltic pumping of a Jeffrey fluid under the effect of magnetic field in an inclined channel. Applied Mathematical Sciences, 5(9), 447-458.
16. Tripathi, D., Bég, O. A., Gupta, P. K., Radhakrishnamacharya, G., & Mazumdar, J. (2015). DTM simulation of peristaltic viscoelastic biofluid flow in asymmetric porous media: a digestive transport model. Journal of Bionic Engineering, 12(4), 643-655.
17. Javid, K., Ali, N., & Asghar, Z. (2019). Numerical simulation of the peristaltic motion of a viscous fluid through a complex wavy non-uniform channel with magnetohydrodynamiceffects. Physica Scripta, 94(11), 115226.
18. Sinha, A., Shit, G. C., & Ranjit, N. K. (2015). Peristaltic transport of MHD flow and heat transfer in an asymmetric channel: Effects of variable viscosity, velocity-slip and temperature jump. Alexandria Engineering Journal, 54(3), 691-704.
19. Ali, N., Sajid, M., Abbas, Z., & Javed, T. (2010). Non-Newtonian fluid flow induced by peristaltic waves in a curved channel. European Journal of Mechanics-B/Fluids, 29(5), 387-394.
20. Nadeem, S., & Akram, S. (2010). Influence of inclined magnetic field on peristaltic flow of a Williamson fluid model in an inclined symmetric or asymmetric channel. Mathematical and Computer Modelling, 52(1-2), 107-119.
21. Srinivas, S., Gayathri, R., & Kothandapani, M. (2009). The influence of slip conditions, wall properties and heat transfer on MHD peristaltic transport. Computer physics communications, 180(11), 2115-2122.
22. Mekheimer, K. S., Salem, A. M., & Zaher, A. Z. (2014). Peristatcally induced MHD slip flow in a porous medium due to a surface acoustic wavy wall. Journal of the Egyptian Mathematical Society, 22(1), 143-151.

23. Bhatti, M. M., & Abbas, M. A. (2016). Simultaneous effects of slip and MHD on peristaltic blood flow of Jeffrey fluid model through a porous medium. Alexandria Engineering Journal, 55(2), 1017-1023.
24. Yasmeen, S., Okechi, N. F., Anjum, H. J., & Asghar, S. (2017). Peristaltic motion of magnetohydrodynamic viscous fluid in a curved circular tube. Results in Physics, 7, 3307-3314.
25. Jha, B., & Aina, B. (2018). Impact of induced magnetic field on magnetohydrodynamic (MHD) natural convection flow in a vertical annular micro-channel in the presence of radial magnetic field. Propulsion and Power Research, 7(2), 171-181.
26. Bhatti, M. M., Abbas, M. A., & Rashidi, M. M. (2016). Combine effects of magnetohydrodynamics (MHD) and partial slip on peristaltic blood flow of Ree–Eyring fluid with wall properties. Engineering science and technology, an international journal, 19(3), 1497-1502.
27. Hafez, N. M., Abd-Alla, A. M., & Metwaly, T. M. N. (2023). Influences of rotation and mass and heat transfer on MHD peristaltic transport of Casson fluid through inclined plane. Alexandria Engineering Journal, 68, 665-692.
28. Srinivas, S., & Muthuraj, R. (2011). Effects of chemical reaction and space porosity on MHD mixed convective flow in a vertical asymmetric channel with peristalsis. Mathematical and Computer Modelling, 54(5-6), 1213-1227.
29. Beleri, J., & Kotnurkar, A. S. (2022). Peristaltic Transport of Ellis Fluid under the Influence of Viscous Dissipation Through a Non-Uniform Channel by Multi-Step Differential Transformation Method. Journal of Advanced Research in Numerical Heat Transfer, 9(1), 1-18.
30. Longwell, P. A. (1966). Mechanics of fluid flow.
31. Ali, N., Abbasi, A., & Ahmad, I. (2015). Channel flow of Ellis fluid due to peristalsis. AIP Advances, 5(9).
32. Abbas, M. A., Bhatti, M. M., & Rashidi, M. M. (2017). Peristaltic blood flow of Ellis fluid through a nonuniform channel having compliant Walls. Journal of Nanofluids, 6(2), 318- 323.
33. Sajid, M., Rooman, M., Ali, N., & Sadiq, M. N. (2021). Flow of the Ellis fluid in the renal tubule. Journal of Applied Mechanics and Technical Physics, 62, 292-299.
34. Saravana, R., Sreenadh, S., Kumar, P. R., & Babu, V. R. (2020). Peristaltic pumping of Ellis fluid through a flexible tube with complete slip effects. Journal of Naval Architecture & Marine Engineering, 17(2).

35. Ali, N., Hussain, S., Ullah, K., & Bég, O. A. (2019). Mathematical modelling of two-fluid electro-osmotic peristaltic pumping of an Ellis fluid in an axisymmetric tube. The European Physical Journal Plus, 134(4), 141.
36. Kumar, K. T., Kavitha, A., & Saravana, R. (2018). Peristaltic flow of an Ellis fluid model in an inclined uniform tube with wall properties. International Journal of Mechanical Engineering and Technology, 9(2), 15-27.
37. Bhatti, M. M., Abbas, M. A., & Rashidi, M. M. (2018). Entropy generation in blood flow with heat and mass transfer for the Ellis fluid model. Heat Transfer Research, 49(8).
38. Javed, M. A., Ali, N., & Sajid, M. (2017). A theoretical analysis of the calendering of Ellis fluid. Journal of Plastic Film & Sheeting, 33(2), 207-226.
39. Ali, N., Kazmi, H. R., & Ullah, K. (2023). A study on the bifurcation of the peristaltic driven flow of Ellis fluid. Waves in Random and Complex Media, 1-18.
40. Celli, M., Barletta, A., & Brandão, P. V. (2021). Rayleigh–Bénard instability of an Ellis fluid saturating a porous medium. Transport in Porous Media, 138(3), 679-692.
41. Alam, M. K., Memon, K. N., Siddiqui, A. M., Shah, S. F., Farooq, M., Ayaz, M., & Ahmad, H. (2021). Modeling and analysis of high shear viscoelastic Ellis thin liquid film phenomena. Physica Scripta, 96(5), 055201.
42. Goud, J. S., & Reddy, R. H. (2018). Peristaltic motion of an Ellis fluid model in a vertical uniform tube with wall properties. International Journal of Civil Engineering and Technology, 9, 847-856.
43. Haroun, M. H. (2006). Effect of wall compliance on peristaltic transport of a Newtonian fluid in an asymmetric channel. Mathematical Problems in Engineering.
44. Mishra, M., & Ramachandra Rao, A. (2003). Peristaltic transport of a Newtonian fluid in an asymmetric channel. Zeitschrift für angewandte Mathematik und Physik ZAMP, 54, 532-550.
45. Srinivas, S., & Pushparaj, V. (2008). Non-linear peristaltic transport in an inclined asymmetric channel. Communications in Nonlinear Science and Numerical Simulation, 13(9), 1782-1795.
46. Khan, A. A., Ellahi, R., Gulzar, M. M., & Sheikholeslami, M. (2014). Effects of heat transfer on peristaltic motion of Oldroyd fluid in the presence of inclined magnetic field. Journal of magnetism and magnetic materials, 372, 97-106.
47. Kothandapani, M., & Srinivas, S. (2008). Peristaltic transport of a Jeffrey fluid under the effect of magnetic field in an asymmetric channel. International Journal of Non-Linear Mechanics, 43(9), 915-924.

48. Arooj, A., Javed, M., Imran, N., Sohail, M., & Yao, S. W. (2021). Pharmacological and engineering biomedical applications of peristaltically induced flow in a curved channel. Alexandria Engineering Journal, 60(6), 4995-5008.
49. Nadeem, S., & Akram, S. (2009). Peristaltic transport of a hyperbolic tangent fluid model in an asymmetric channel. Zeitschrift für Naturforschung A, 64(9-10), 559-567.
50. Hayat, T., Iqbal, R., Tanveer, A., & Alsaedi, A. (2018). Variable viscosity effect on MHD peristaltic flow of pseudoplastic fluid in a tapered asymmetric channel. Journal of Mechanics, 34(3), 363-374.
51. Lachiheb, M. (2014). Effect of coupled radial and axial variability of viscosity on the peristaltic transport of Newtonian fluid. Applied Mathematics and Computation, 244, 761- 771.
52. Kothandapani, M., & Srinivas, S. (2008). On the influence of wall properties in the MHD peristaltic transport with heat transfer and porous medium. Physics letters A, 372(25), 4586-4591.
53. Khan, A. A., Ellahi, R., & Usman, M. (2013). The effects of variable viscosity on the peristaltic flow of non-Newtonian fluid through a porous medium in an inclined channel with slip boundary conditions. Journal of Porous media, 16(1).
54. Vijayalakshmi, E. A., Santra, S. S., Botmart, T., Alotaibi, H., Loganathan, G. B., Kannan, M., & Govindan, V. (2022). Analysis of the magnetohydrodynamic flow in a porous medium. *AIMS Mathematics*, *7*(8), 15182-15194.
55. Abd-Alla, A. M., Thabet, E. N., Bayones, F. S., & Alsharif, A. M. (2023). Heat transfer in a non-uniform channel on MHD peristaltic flow of a fractional Jeffrey model via porous medium. Indian Journal of Physics, 97(6), 1799-1809.
56. Abd-Alla, A. M., Abo-Dahab, S. M., Thabet, E. N., & Abdelhafez, M. A. (2022). Peristaltic pump with heat and mass transfer of a fractional second grade fluid through porous medium inside a tube. Scientific reports, 12(1), 10608.
57. Hayat, T., Saleem, A., Tanveer, A., & Alsaadi, F. (2017). Numerical study for MHD peristaltic flow of Williamson nanofluid in an endoscope with partial slip and wall properties. International Journal of Heat and Mass Transfer, 114, 1181-1187.
58. Shehzad, S. A., Abbasi, F. M., Hayat, T., Alsaadi, F., & Mousa, G. (2015). Peristalsis in a curved channel with slip condition and radial magnetic field. International Journal of Heat and Mass Transfer, 91, 562-569.
59. Ali, N., Hussain, Q., Hayat, T., & Asghar, S. (2008). Slip effects on the peristaltic transport of MHD fluid with variable viscosity. Physics Letters A, 372(9), 1477-1489.