ARTIFICIAL NEURAL NETWORK BASED COMPUTATIONAL FRAMEWORK TO SOLVE SI SYSTEM OF COVID-19 NON LINEAR EQUATION

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ISLAMABAD

September, 2023

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BSCS, University of Gujrat, Gujrat, 2021

A THESIS SUBMITTED IN PARTIAL FULFILMENT OF
THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF SCIENCE
In **Computer Science**

To
FACULTY OF ENGINEERING & COMPUTER SCIENCE

NATIONAL UNIVERSITY OF MODERN LANGUAGES ISLAMABAD

NATIONAL UNIUVERSITY OF MODERN LANGUAGES FACULTY OF ENGINEERIING & COMPUTER SCIENCE

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Thesis Title: Artificial Neural Network Based Computational Framework To Solve SI System Of Covid-19 Non Linear Equation

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Candidate of **Master of Science in Computer Science** at the National University of Modern Languages do hereby declare that the thesis **Artificial Neural Network Based Computational Framework To Solve SI System Of Covid-19 Non Linear Equation** submitted by me in partial fulfillment of MSCS degree, is my original work, and has not been submitted or published earlier. I also solemnly declare that it shall not, in future, be submitted by me for obtaining any other degree from this or any other university or institution. I also understand that if evidence of plagiarism is found in my thesis/dissertation at any stage, even after the award of a degree, the work may be cancelled and the degree revoked.

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ABSTRACT

Title: Artificial Neural Network Based Computational Framework To Solve SI System of Covid-19 Non Linear Equation

The aim of the present study is to design the artificial neural network framework to solve the Susceptible and infected (SI) model of the COVID-19 non-linear equation**.** Mathematical model for epidemic diseases are non-linear. The approximation of non-linear system of ODE is a challenging task. To model epidemic diseases, many analytical and numerical models are proposed in literature. In recent past, unsupervised artificial neural network gain much attention to solving ODE in different application. In the proposed method, the ReLU artificial neural network has been used for its effectiveness in training and modeling complex relationships of our proposed scheme. The fitness function of the unsupervised error function is used to determine how well the predictions provided by the ANN align with the actual data or the desired outcomes. With the help of an analytical model and numerical solver (ODE-45) result, the MAE is utilized to evaluate the accuracy and reliability of our suggested scheme.

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LIST OF ABBREVIATIONS

SI - Susceptible And Infected
ReLU - Rectified linear Unit
MAE - Mean Absolute Error
E1 - Error 1
E2 - Error 2

LIST OF SYMBOLS

$\boldsymbol{\beta}$	-	Rate of transmission
N	-	Total number of population
S	-	Number of susceptible individual
I	-	Number of infected individual
b		Bias
w		Weight
α		Unknown parameter

ACKNOWLEDGEMENT

First of all, I wish to express my gratitude and deep appreciation to Almighty Allah, who made this study possible and successful. This study would not be accomplished unless the honest espousal that was extended from several sources for which I would like to express my sincere thankfulness and gratitude. Yet, there were significant contributors for my attained success and I cannot forget their input, especially my research supervisors, Head of department. Dr. Sajjad Haider and co-supervisor Mr.Frahad M Riaz, who did not leave any stone unturned to guide me during my research journey.

I shall also acknowledge the extended assistance from the administrations of Department of Computer Sciences who supported me all through my research experience and simplified the challenges I faced. For all whom I did not mention but I shall not neglect their significant contribution, thanks for everything.

DEDICATION

This thesis work is dedicated to my parents and my teachers throughout my education career who have not only loved me unconditionally but whose good examples have taught me to work hard for the things that I aspire to achieve

CHAPTER 1

INTRODUCTION

1.1 Overview

Mathematical modeling and approximation have become challenging undertakings in recent years to combat epidemic diseases due to their non-linear and coupled nature. To solve the mathematical model, an analytical model must first be established. Second, these mathematical models are solved using the numerical solver. Compared to the analytical model, the numerical solver provides a superior approximation. The rise of Covid-19 and making predictions regarding its distribution are highly difficult tasks. The mathematical model of COVID-19 was solved using the analytical model and a numerical solver. To predict Covid-19 infections, many epidemic models have been created, including the Susceptible Infected (SI), Susceptible, Infected and Recovered (SIR), and Susceptible, Infected, Recovered and Deceased (SIRD) models. Differential equations serve as the foundation for all of these epidemic models. However, the nature of these differential equations is coupled and non-linear. These epidemic models are well approximated by the analytical model and numerical solver. Unsupervised artificial neural networks have received a lot of interest recently for solving ODE in many applications. We can reduce the error rate and raise accuracy if we employ artificial neural networks to handle these coupled, non-linear epidemic models. The goal of this research is to create an artificial neural network-based framework to solve the Covid-19 nonlinear equation's Susceptible and Infected (SI) model.

1.2 Motivation

A new kind of coronavirus called Covid-19 was first introduced and first appeared in Wuhanbefore spreading throughout China[1]. It spread quickly worldwide, and the World Health Organization (WHO) proclaimed it a pandemic[2]. The coronavirus, which causes the severe respiratory illness COVID-19, is very contagious. Because intimate human contact is the key factor contributing to Covid-19, the global government encouraged people to avoid it. The severity of COVID-19's symptoms might vary; if it is not treated quickly, it can be deadly. Expanding the network of COVID-19 diagnostic centers around the country would require increased participation from the commercial sector, especially in testing. As a key tactic to lower the infection rate, it is crucial to inform and inspire residents to take responsibility for the situation through the strict enforcement of preventative non-pharmaceutical methods. Effectively combating the COVID-19 epidemic in Nigeria requires a quick, thorough, coordinated, government-led strategy that involves active involvement from local communities and civil society[3]. Since it is difficult to predict how COVID-19 may develop in the future, it is essential to take precautions to stop it from spreading further. The Covid-19 potential spread of variants is clearly shown in Figure 1.

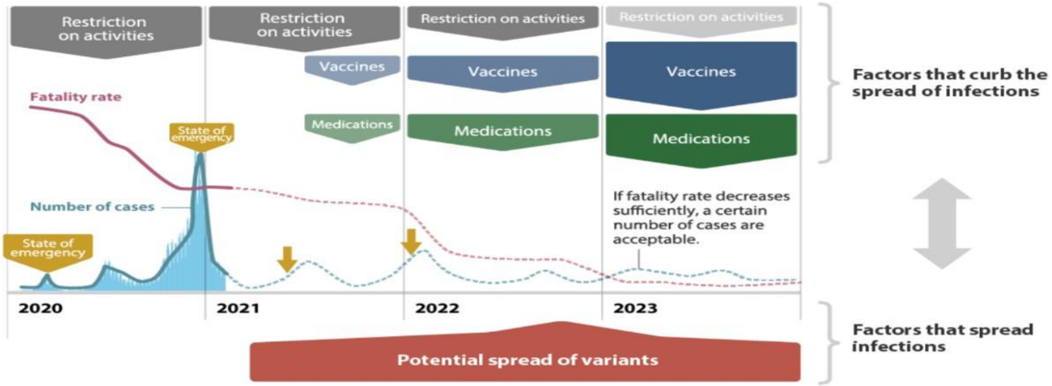


Figure 1.1 : Scenario of COVID-19 Pandemic [4]

Numerous research has attempted to forecast how this pandemic would develop in the future. Tocomprehend how this pandemic spreads, various tools have been used, one of which is the mathematical model[5]. Using a mathematical model to anticipate and analyze the Covid-19 outbreak is helpful for formulating measures that would reduce fatality rates while still protecting the community[3]. The compartmental approach is employed as the modeling's main method of controlling this pandemic’s separation. Compartmental models are built on mathematical Equations, and by comprehending these mathematical equations, we may learn more about how people interact inside different "compartments” of a population. The most popular compartmental model uses ordinary and partial differential equations to split an estimated population into several groups and then uses a set of mathematical rules to describe how people move between groups. [6] The Covid-19 of the non-linear equation is solved using many forms of epidemic models by using analytical models and numerical solvers [7-10]. Since most mathematical models of the dynamics of infectious illnesses are composed of a collection of nonlinear differential equations, it is difficult to derive a precise analytical solution for these models. Approximation and perturbation are employed to assess these mathematical models' solutions. The homotopy perturbation technique (HPM) and the homotopy analysis method are compared in this research to show that the former is more effective than the other. Additionally, by adding modern perturbation techniques, this research improves the HPM[11]. The approximation of the non- linear system of ordinary differential equations (ODE) is a challenging task. In the recent past, unsupervised artificial neural networks gain much attention for solving ODE in different applications[12]. Unsupervised artificial neural networks (ANNs) are a sort of neural network that can learn from the input without the need for labeled samples or direct supervision. Unsupervised learning involves training the machine using unlabeled data, providing no indications or hints about what it is observing. Several studies are shown in[13, 14] to describe the worth of unsupervised artificial neural network algorithm. Different types of activation function are used to make the output non-linearity like log sigmoid, tanh, softmax etc [15].

In this case, the ReLU artificial neural network is used in multiple dense layer. The objective of the proposed scheme is to design a neural network that are solved the Susceptible and infected (SI) Covid-19 non-linear equation. With the use of absolute error, we determine the outcome in the result phase. The comparison between Error 1 (E1) and Error 2 (E2), two results. E1 is between the analytical model and the ODE-45, whereas E2 lies between the artificial neural network and the analytical model. We can accurately predict Covid-19's future with the assistance of these errors.

1.2.1 Epidemic Models of Covid-19 nonlinear equation

A set of ordinary differential equations is the foundation of the commonly used model for explaining the spread of Covid-19 because it can accurately represent the rate of Change in each compartment over time. There are several types of epidemic models, including:

Susceptible infected and recovered model: A popular non-linear model, the SIR model dividesthe population into three groups:

Susceptible (S): People who are susceptible to the virus and might become infected if exposed areincluded in this compartment.

Infected (I): People who are infected and able to spread the virus to those who are vulnerable areconsidered to be infected in this compartment.

Recovered (R): Individuals in this compartment are no longer contagious since they have recovered from their illness[16].

Susceptible-Exposed-Infected-Recovered model: Susceptible-Exposed-Infectious-Recovered, or SEIR, is a crucial compartmental epidemiological model used to explain how infectious illnessespropagate within a community. By adding a new epidemic for those who have been exposed to the illness but are not yet contagious, it expands the SIR model. As a result, illness dynamics may be represented more thoroughly. A set of ordinary differential equations are used in the SEIR model to explain how quickly each compartment changes over time. Numerous factors, including the rate of transmission, the pace at which people move from the exposed to infected state, and the rate of recovery, affect these ODEs[17].

Susceptible infected recovered deceased model: The Susceptible-Infected-Recovered-Deceased(SIRD) model, frequently used to represent the dynamics of infectious illnesses within a population, is a crucial epidemiological compartmental model. It expands the SIR model by addinga new compartment to take into consideration those who sadly died from the condition. A set of ordinary differential equations (ODEs) are used in the SIRD model to describe the rate of change in each compartment over time. The initial number of infected people, the transmission rate, the survival rate, the death rate, and other characteristics are all taken into account by these ODEs[18].

1.2.2 Application of Epidemic Model

Epidemic models may be used to simulate and predict how infectious diseases will spread through a population. These models offer priceless insights into the mechanisms of disease transmission, aiding in the development of effective strategies for managing and containing epidemics. Here are a few common situations where epidemic models are helpful:

Gaining Knowledge of Disease Dynamics: Epidemic models help academics and public health professionals understand how illnesses develop over time across communities. Researchers can identify variables influencing the acceleration or slowing of disease spread by examining transmission patterns.

Disease Outbreak Prediction: Epidemic models are able to predict the track of current outbreaks or the possibility of impending ones. These forecasts are crucial for allocating resources effectively, creating reaction plans, and putting interventions into place on schedule.

Examining Intervention Strategies: Epidemic models may be used to evaluate public health initiatives including immunization campaigns, quarantine procedures, and travel restrictions. These models make it possible to assess the effectiveness of various measures for reducing diseasetransmission and controlling outbreaks.

Guiding Policy Formulation: Epidemic models may be used to help policymakers make informed decisions about the deployment of measures like lockdowns, school closings, and travel restrictions. These models provide information on the potential benefits and drawbacks of various policy options.

Analysis of Different Situations: Epidemic models enable researchers to compare the outcomes of various situations. This covers situations with different levels of adherence to preventative measures, diverse vaccine coverage rates, and the effects of various public health policies.

1.3 Problem Background

The rapid spread of Covid-19 around the world and its rapid development have highlighted the critical need for accurate models that can provide guidance for effective public health actions and provide clarity regarding the complexities of infectious disease dynamics. The Susceptible- Infectious (SI) model and other well-established epidemiological frameworks serve as essential tools for understanding disease transmission in communities. However, these models usually include non-linear differential equations that are difficult to solve analytically, particularly when confronted with the complex interaction of variables and factors found in real-world situations.

Although successful in certain cases, traditional approaches for solving non-linear differential equations fall short when it comes to fully capturing the complex dynamics of disease spread and the effects of changing parameters. The complexity of real-world scenarios, which include elements like the impact of public health measures, social interactions, and geographical inconsistencies, calls for a more flexible and adaptive approach to resolving modeling problems.

Artificial Neural Networks (ANNs) have emerged as effective tools for simulating complex non- linear relationships present in data, drawing inspiration from the structure and operation of the human brain. ANNs are adept at comprehending complex patterns, adapting to changing conditions, and making decisions from data that has been collected[19].This inspired us to develop the ANN framework to solve the SI system of the Covid-19 non-linear equation.

1.4 Problem Statement

The epidemical models are nonlinear in nature. Mathematical modeling and approximation are challenging tasks. COVID-19 is an epidemic disease and the mathematical models for this disease are nonlinear and coupled in nature. An unsupervised neural network is used to solve the nonlinear models[19, 20]. The goal of this work is to design a neural network to solve the SI system of nonlinear equations. We shall analyze the accuracy and reliability of the proposed scheme withanalytical and numerical solutions.

1.5 Research Question

- How to Design the ANN-based framework to solve the SI system of non-linear equations?

- How to validate the consistency and accuracy of the proposed framework?

1.6 Aim of This Research:

The objective of this study is to create a neural network to solve the SI COVID-19 system of nonlinear equations. We will evaluate the success of the suggested plan in terms of dependability and accuracy.

1.7 Research Objectives:

- To develop the neural network-based computational framework to solve the ODE of covid-19.
- To compare the developed model with analytical and numerical models.

1.8 Scope of Research Work

Using Artificial Neural Networks (ANNs) to solve the non-linear equations in the Susceptible- Infectious (SI) model relating to Covid-19 resides in their ability to provide higher levels of accuracy than traditional analytical methods. Unlike numerical approaches like ODE 45, ANNs can assimilate complex data patterns, integrate real-world complexities, and improve forecasting skills linked to the dynamics of disease transmission.

1.9 Thesis Organization

Following is how the rest of the thesis is structured:

Chapter 2 gives a thorough introduction of the field of study and the body of previous research. Literature is categorised according to various methods and approaches, including models, algorithms, and particularly the methodology employed. Further, Chapter 2 provides existing constraints and challenges which provide a platform for further research and the research gap which leads to the design of an ANN-based framework to solve the SI systemof the Covid-19 non-linear equation.

The methodology for this research is provided in Chapter 3 in full, together with information on all the benchmark methodologies now in use and strategies for overcoming the limits that are currently in place. The approach strategy selected for carrying out this research is described in this Chapter. Also, it gives details of the implementation tools and how the proposed model will be evaluated.

The suggested design of the model is presented in depth in Chapter 4, which may be thought of as the thesis's main section. examining the tools, models, and algorithms used to put the suggested architecture into practice to solve the SI system of the Covid-19 nonlinear equation. For a better understanding of the model's design, this chapter also includes flowcharts, figures, and diagrams.

The reader will receive an assessment of the suggested model in Chapter 5 along with undertaken parameters to assess the model's put-feed-forward design. The efficacy of this architecture is further ensured in this chapter by a comparison of the suggested model with the analytical model and numerical solver.. Find Absolute error and fitness value that tells the effectiveness of the proposed scheme in terms of reliability and accuracy. Presents summary of contributions of this research work with directions towards future work furthermore, in this chapter the summary of the proposed research is given with its significant achievements.

CHAPTER 2

LITERATURE REVIEW

2.1 Overview

Due to their non-linear and coupled in nature, mathematical modeling and approximation have become difficult tasks in recent years to tackle the epidemic disease. Additionally, a COVID-19 diseases are non-linear and coupled in nature. Different mathematical model is used to solve the COVID-19 model like SI model, SIR model and SIRD etc. With the use of analytic models, numerical models, and artificial neural networks, several researchers use differential equations to solve mathematical models. Numerous researchers have used numerical solvers to solve Covid-19 illnesses in recent years. In this chapter, we will talk about the analytical model and the different numerical solver used to solve the COVID-19 in sections 2.1 and 2.2. Additionally, we will talked about using artificial neural networks to solve differential equations in section 2.3. We will eventually be able to identify the research gap after discussing these sections.

2.2 Analytical Model

The objective of this paper was to examine the relationship between public health actions and the epidemiological characteristics of the outbreak by splitting the COVID-19 outbreak in Wuhan into 5 phases based on major events and interventions. They gathered the 32583 confirmed cases between December 8, 2019, and March 8, 2020 from the municipal Notifiable Disease Report System, and the results revealed that 16 817 (51.6%) of the patients were females. Older persons have a higher risk of contracting COVID-19, while younger ones have a lower risk.

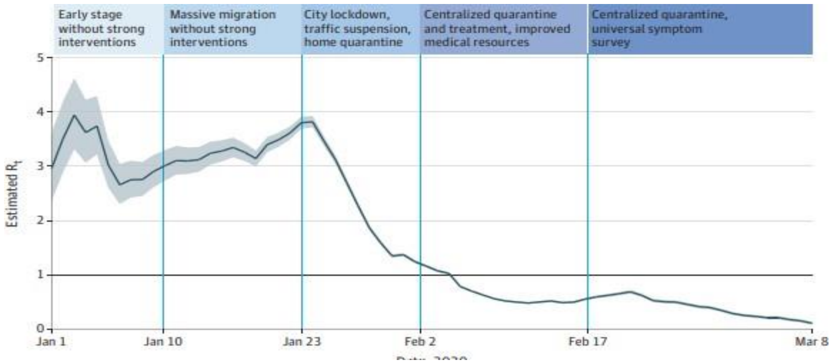


Figure 2.1 : The Effective Reproduction Number (Rt) Estimates Based on Laboratory-Confirmed Coronavirus Disease 2019 (COVID-19) Cases in Wuhan, China [21]

The Covid-19 in mainland China is examined using the susceptible and infected (SI) model in thispaper. The goal of this study is to compare the Susceptible Ad Infected (SI) to the epidemic data that has been publicly disclosed. The model can be used to forecast the future spread of Covid-19 and to assess various speculative scenarios of social mitigation strategies[22].

They collect the data of Covid-19 cases from Italy and Germany during the first half of 2020 to understand the time evaluation and developed the modified Susceptible-infected and recovered (SIR) mathematical model based on differential equation to tackle these two countries cases. The complexity to examine these pandemic diseases is not easily understandable, so this model is very suitable to handle the data during the application of social distancing and lockdown. They evaluate the advantages and disadvantages of various SIR model modifications. lastly, they evaluate the model's predictions for predicting the outbreak's progression over the short- and long-term[23]. This study found that 1715 participants in the first stage of the Covid-19 survey in Hong Kong had a significant risk of depression, anxiety, and personal hygiene issues as well as social withdrawal and travel-related indicators. The outcome demonstrates that if the government takes strong action to regulate gatherings of people during the early stages of the Covid-19 outbreak, it will decrease the spread of the outbreak in the future[24].

This study uses multi-agent technology and statistics on population migration to present the City-based Epidemic and Mobility Model (CEMM). The model showed its efficiency in replicating interstate transmission of the disease and offering greater geographic insights in comparison to conventional compartment models through a case study of COVID-19. The early stages of COVID-19's spatial and temporal evolution are successfully captured by the CEMM.

The transmission capacity of population flows during an epidemic is also impacted by the distance Decay effect, in addition to the population flow volume. The disease spread differently at various distances, although having the same seed numbers (number of infected cases) and mobility scales[25].

They collected the data of Italian women with the help of survey in April 2020. When working on housework, childcare, homeschooling, etc. during COVID-19, they examine the effects on both partners. The findings indicate that COVID-19 has increased the amount of housework that women must do[26]. In this study, they examine how a spatial SIR model's representation of road networksaffects the transmission of a disease. The road network facilitates the rapid spread of infectious disease. They introduced the new SIRT model in which addition of the compartment T for traveling infected on the road in the classical SIR Model.They solely talk about neighborhood Brownian diffusion and interactions[27].

The SEIR model of analytical solution was used to describe how COVID-19 infectious illnesses spread. The major goal of this technique is to pinpoint the peak and asymptotic values of the population affected by Covid-19 infectious diseases as well as to understand their features throughout time[28]. The purpose of this paper is to examine the unique fractional-order mathematical model that helps to explain the spread of the infectious disease Covid-19 in Ethiopia. The model is described as a system of non-linear fractional order differential equations using the Caputa fractional derivative. The outcome indicates that the government of Ethiopia needs to take important measures, such as lockdown and quarantine, to stop the spread of Covid-19. These actions aid in lowering the rate of reproduction[29].

The COVID-19 epidemic is now being described using a number of mathematical models that have recently surfaced. These models are useful tools for investigating the mechanisms behind coronavirus transmission and for predicting the course of the COVID-19 pandemic. They also fulfill the function of evaluating management and mitigation plans for this public health emergency. For understanding and predicting the patterns of infectious illnesses, especially COVID-19, compartmental models, such as the Susceptible-Infectious-Recovered model, have generally proven to be particularly effective. By using this calibration procedure, we can pinpoint the precise factors that are pertinent to each nation. Additionally, they evaluated the success of the policies and programs that the governments of each of these nations adopted in response to the pandemic's development. Their numerical models offer a useful tool for forecasting the disease's future spread, improving our comprehension of its dynamics, and guiding policy choices[30].

This paper uses the epidemiological SIR model for the infectious illness Covid-19. To determine the Infected rate, data are gathered from different country in the world.[31]. This study examines the global spread of Covid-19 infectious diseases using a variety of analytical models. To describe the mortality instances during COVID-19 globally, they employed the Death model, Extended SIR model, and Monte Carlo Planck model. The D-model and ESIR predict similar patterns of infected and fatal cases, according to the comparison approach[32]. In this work, the Sobol method are applied to demonstrate the existence, positivity, and uniqueness of the solution for existing proposed model of COVID-19. Using Global sensitive analysis Sobol method, they investigate the main effect, interaction effects, ranking/of key regulating parameter, and factor fixing[33]. Using the Laplace residual power series approach, they address the fractional SIR epidemic model solution in this study. Using Caputo's derivative method, which entails three interconnected fractional differentialequations with the third equation depending on the solutions of the first two, they introduce the fractional SIR model. They take on the suggested model using the Laplace residual power series method (LRPSM), and the solution is shown as a quickly convergent series expansion that approaches the precise answer. The outcomes and compare the resultant approximations to those attained using different approaches in order to appraise the findings[34]. A accurate closed-form solution for the SIR Epidemic Model is obtained by using asymptotic approximants. This solutionis created by analytically extending the diverging power series solution, aligning it with the epidemic model's long-term asymptotic behavior. Applying this analytical methodology to the COVID-19 epidemic provides an illustration of how useful it is. The findings of this research highlight the value of using an asymptotic approximant to get accurate analytical solutions for the SIR model. Future research should examine the asymptotic approximant technique's ability to provide closed-form answers for increasingly complex epidemic models[35]. This study investigates a fractional-order SEIR epidemic model created to study the dynamics of infectious disease transmission within a host population, including HIV and HBV. The Holling type-II saturation incidence rate is included in the model as the infection component and takes into consideration a limited total host population. We compute the crucial parameter, the basic reproduction number (R0), which determines whether the illness will be eradicated or persist across the population, using the SEIR epidemic model. Two equilibria—the disease-free equilibrium (DFE) and the endemic equilibrium (EE)—are shown to exist by the model analysis. We use the generalized Adams-Bash forth-Moulton approach to get numerical solutions. We next do numerical simulations to confirm and demonstrate the analytical results. The findings highlight the dynamic progression's strong interaction with the order of the fractional derivative. The parameter (infection rate) shows a positive association with R0, whereas the parameter (inhibition rate) shows a negative correlation with R0, according to sensitivity analysis. This suggests that, in comparison to other factors, these ones have a greater impact on the dynamics of the illness[36].

China had put in place a national emergency response to stop the spread of the severe acute respiratory syndrome-coronavirus 2 by restricting travel and enforcing social isolation rules among its citizens. The efficacy of particular actions is still debatable. In their study, a quantitative assessment of the impact of control strategies spanning in December 2019 to February 2020, a period that encompassed the Lunar New Year when numerous individuals embarked on journeys across China for familial gatherings. This finding suggests that collectively preventing community gatherings and closing public transit and entertainment centers at the municipal level prevented the emergence of hundreds of thousands of infection cases. Therefore, any easing of these control efforts might potentially lead to a comeback of infections. The following reduction in instances is unlikely to have occurred due to the depletion of vulnerable people[37]. Many European nations have put non-pharmaceutical solutions into place, including things like statewide lockdowns and school closures. This model takes into consideration the lag between infection and death by backtracking transmissions from recorded fatalities to calculate those that took place weeks earlier. They use a technique that incorporates both individual and shared impacts on the time-varying reproduction number (Rt) while only partially pooling data across nations. This pooling strategy makes it possible to use a broader variety of information, helps get around data quirks, and makes it easier to provide more accurate predictions more quickly. Their findings highlight the significant effects of significant non-pharmaceutical treatments, with a focus on the efficiency of lockdowns in reducing transmission. As a result, we support the idea of considering long-term measures to keep SARS-CoV-2 transmission under control[38].

Countries are looking for advice on how to relax restrictions while reducing the danger of future outbreaks in light of the drop in new cases of coronavirus disease 2019 (COVID-19) across Europe as a result of non-pharmaceutical measures including social distance regulations and lockdowns. To address this, we assess the potential benefit of coordinated departure tactics in postponing the reappearance of the virus and reducing community transmission using data on mobility and Covid-19 cases. Their findings show that if well-connected nations with stringent current interventions prematurely lift these efforts, a recurrence of the continental pandemic might arise up to 5 weeks early. Additionally, they determine that efficient collaboration can greatly increase the chances of eliminating community transmission throughout Europe[39].

2.3 Numerical Model

Different numerical solver are used to solve the mathematical model based on differential equation. Give an overview of the newest methods for solving differential equations utilizing various Runge-Kutta algorithm orders.. This survey includes an overview of recent articles based on the Runge-Kutta method's third, fourth, fifth, and sixth orders. The author provides commentary and observations on various Runge-Kutta methods. With the use of their own observations and comments, they compare several RK methods. When solving differential equations, the order of the RK method provides superior accuracy than the other Runge-Kutta methods[40].

The author uses the RK 4 method and the Euler method to solve the initial value problem of an ordinary differential equation. The initial value problem of an ODE can be solved very effectively and practically using the Euler approach and RK 4 method. To ensure correctness, the numerical solution is compared to the exact solution answer. They contrast the numerical approaches of the fourth order Runge-Kutta method and the Euler method. After the method of computational work for these methods is complete, the error of the suggested approach is computed.

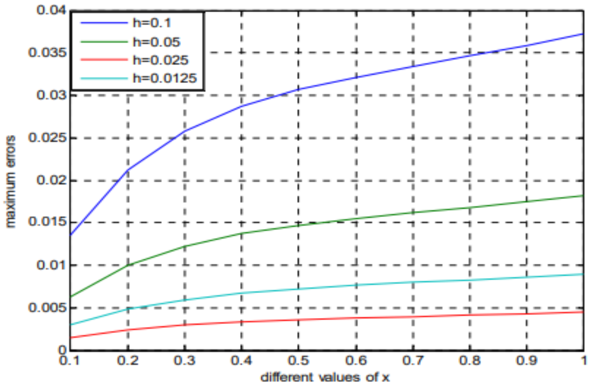


Figure 2.2: Error for different step size using Euler method

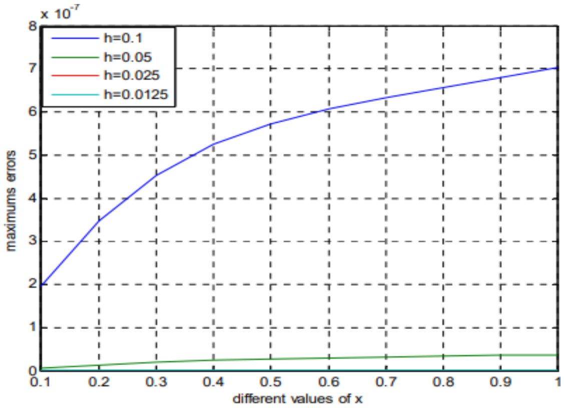


Figure 2.3: Error for different step size using Rk4 method

The accuracy of the methods for reducing the step size h is demonstrated in the aforementioned figures, which also compare the graphs of the approximate and exact solutions. After comparing the proposed approach, they note that the fourth-order Runge Kutta method's rate of convergence is O(h4), while Euler's method's rate of convergence is O(h). They are able to determine from these results that the Euler approach is less accurate than the rung-kutta method[41].

Three well-known models are utilized in this study to forecast COVID-19's future trends. For COVID-19, they applied the Euler technique, Runge-Kutta order 2 and Runge-Kutta order 4 numerical approach. They obtain a rough solution to this global problem by employing these strategies. The number of susceptible, infected, recovered, and quarantined individuals in the future can be predicted using the numerical results from the Euler method and Runge-Kutta orders 2 and 4. These numerical results significantly improve the international effort being made to address this global issue. They use a logistic model to analyze the forecasting epidemic size for Turkey and Iraq. After examine these result it can be the detail description about these epidemic disease[42].

In this research, they forecast the first and second waves of Covid-19 for Sri Lanka in the future. The first wave of Covid-19 cases were predicted using the stochastic model, while the second wave was predicted using the Susceptible-infected-Recovered(SIR) model and logistic model growth for Covid-19 cases[43].The fractional order mathematical modelling for Covid-19 transmission was reported in this paper. To solve numerical problems, they applied the Euler fractional method. The method of comparison based on the simulated result at various fractional orders against actual data for reported cases of infection and mortality[44]. This paper discusses the mathematical modelling of the isolation class in Covid-19. For the numerical solution, they employed the nonstandard finite difference (NFSD) scheme and Runge-Kutta method. They explore how quickly Covid-19 transferred from human to human by using their graphical results. The spread of Covid-19 can be slowed down by isolating the afflicted individuals[45].

In this paper, it is examined how a non-singular fractional order derivative is employed in a mathematical model to determine the transmissibility of the Covid-19 sickness. They used the fractional Adams-Bashforth approach to simulate the numerical results corresponding to different fractional orders. With the help of simulation, they observed that in lower fractional order of derivative the decrease in susceptible is faster and in smaller fractional order the infection is increase. The results that have been presented might assist us comprehend how to stop the Covid-19 sickness from spreading in the future[46]. This paper developed a mathematical model that incorporates a class in quarantine and government interventions meant to stop the disease from spreading. They examine the model's overall dynamic behavior in relation to the fundamental reproduction number. The short-term trend of Covid-19 for the three Indian states of Maharashtra, Delhi, and Tamil Nadu was predicted at the conclusion of this study. According to the findings, the first two states require additional surveillance to combat the disease[47].

This paper developed the mathematical model utilizing real data from China to determine the optimal method for vaccinating during the COVID-19 outbreak. One of the two ideal problems suggested by this model is to decrease the number of infected individuals, while the other is to decrease both the number of infected people and the concentration of the recommended vaccination during treatment[48].Bats-Hosts-Reservoir-People (BHRP) mathematical model under the Caputo fractional order derivative (CFOD) to developed the some result for transmission of Covid-19 diseases. To find the numerical solution of this approaches they used Euler method[49]. In this paper, a fractional-order dynamic model and a fractional-order stochastic model are createdin order to examine the randomness of COVID-19's temporal evolution. Beginning, they noticed that during early days of covid-19 people are also getting others vaccination alongside with Covid-19 first and second doses in the worldwide. This model tested the Saudi Arabia Covid-19 second wave. The Invasive Weed Optimization Algorithm is used in one case to optimize certain functions (IWO). To compute numerical solution they used fractional Euler method and the Euler-Murayama method[50].

Using a unique fractional-order COVID-19 model, this study examined the rise of COVID-19 in Jordan over a 40-day period from January 1 to February 10, 2022. Based on comparisons with actual data, it was shown that the dynamics were simulated using the Fractional Euler Method (FEM) to be more successful than classical models. It was discovered that the fractional-order COVID-19 model outperformed the integer-order model in terms of predicting the propagation of the virus. This could help decision-makers create efficient plans and strategies to combat a pandemic[51].

In the modern day, COVID-19 has placed a heavy burden on everyone, from detection to mitigation attempts. Lockdowns have an impact on the entire world's population because of the epidemic. To comprehend the nature of this pandemic and create effective control measures, researchers are tirelessly working. Using a powerful mathematical model that can accurately capture and control the pandemic's progress is a critical strategy for achieving this. Four control techniques are provided together with the requisite fractional order criteria, and it is shown that the fractional order COVID-19 SIDARTHE model has a stable solution. The researchers also examine how the fractional order COVID-19 SIDARTHE model responds to changes in the parameters governing the fractional order and infection rate. All experiments are numerically simulated using the MATLAB programmed and a fractional order differential equation solver in order to validate their results. In this study, the authors suggest a unique fractional order SIDARTHE model, which has not been previously investigated in the literature, to simulate the COVID-19 pandemic[52]. The recently constructed integer-order mathematical framework for COVID-19 is expanded upon by the newly proposed non-linear fractional-order model. They proved the existence and uniqueness of solutions inside our suggested model by using basic concepts like continuity and the Banach fixed-point theorem. They also looked at the disease-free equilibrium point's local asymptotic stability. They used the fractional Adam-Bashforth-Moulton technique to carry out numerical simulations of the created epidemic model in order to confirm the theoretical conclusions[53].

In order to analyze the dynamics of COVID-19 transmission and evaluate control measures, a deterministic model has been created and investigated in this study. The severity of the disease and the risk factors related to age and co-morbidities are strongly correlated, according to a body of data. They discriminate between two categories of vulnerable people in this model—a high-risk group and a low-risk group—in order to account for this link. They have calculated the fundamental reproduction number, or R0, which is crucial in defining how the illness behaves. R0 larger than 1 denotes that the disease will continue in the population, but R0 less than 1 suggests that the disease is likely to disappear with time. Their research shows that both high-risk and low-risk people must be treated and immunized in order to achieve successful disease management. Furthermore, their data suggest that the precise ratios of high- and low-risk populations do not significantly affect the efficiency of these management strategies[54].

A forecasting tool was created to estimate the total number of infections reported and the number of people who had the very lethal COVID-19 infection's Omicron form. They numericallysimulated the model using the data that was gathered to show the effects of various actions on the Omicron variant's spread throughout the metropolis. These precautions include both pharmaceutical and non-pharmaceutical ones, such as non-pharmaceutical ones like contact tracing leading to clinical testing of exposed people, consistent use of face masks, maintaining personal hygiene through hand sanitization, and regular hand washing. The results of the computersimulations showed that it is possible to significantly lower the prevalence of the highly contagious disease inside the city by increasing the rate of detecting affected people and guaranteeing broad adherence to safety standards[55]. The inquiry uses the q-homotopy analysis transform technique (q-HATM) to evaluate the susceptible-infected-recovered (SIR) epidemic model for pediatric illnesses. In order to capture the dynamics of pediatric illness throughout a population and its influence on susceptible, infected, and recovered compartments in the community, this model entails a set of three differential equations with fractional derivatives. The technique used combines components of the Laplace transform with the q-homotopy analysis method. We illustrate simulations through graphs while adjusting the fractional order values by taking into consideration two different illustrative instances. The study sheds insight on the behavior of several mathematical models using differential equations in the context of human disease, emphasizing the crucial roles played by both the approach and the applied derivative[56].

Lockdowns have been viewed as a double-edged tool in the fight against pandemics, providing some relief while wreaking havoc on the economy. These regulations successfully reduce human movement, but the negative effects on economic activity outweigh the benefits. Governments frequently lack thorough and precise standards as they struggle to balance the benefits and costs of lockdowns. We carefully tracked the development of spatiotemporal commercial networks and daily human movement by drawing conclusions from comprehensive financial transaction records and massive mobility data. As a result, we developed a carefully comprehensive two-sided framework that assesses the efficiency of various lockdown strategies from an epidemiological and economic perspective. According to their research, the tightest lockdown measures reduced the pandemic's duration by approximately two months when compared to the loosest lockdown measures, demonstrating their combined epidemiological and financial effectiveness[57].

Using an enhanced SEIR epidemic model that takes into account variables like vaccination, temporal delays, and random noise, they examine the dynamics of COVID-19 in the

UAE environment in this study. They look at the stationary ergodic distribution of positive solutions, where the sickness persists stochastically by orbiting around the equilibrium of the deterministic scenario. There are several circumstances that lead to infection-free states (extinction), where the likelihood of a disease occurring decreases exponentially to one. They verify the applicability of theoretical conclusions through numerical simulations and alignment with actual observations. The model's dynamics are enhanced by the combination of stochastic perturbations and temporal delays, with the level of white noise being crucial in tackling infectious illnesses[58].

2.4 Artificial Neural Network In Epidemic Model

An ANN is being used by many researchers to solve the differential equation. The ANN is more effective than traditional numerical solvers because it provides a differentiable solution, makes it possible to tackle problems in parallel architectures, can handle complex differential equations, and minimizes the amount of iterations[59].

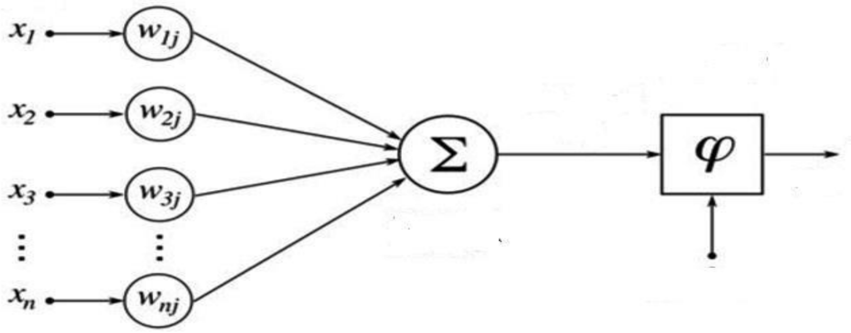


Figure 2.4: Simple architecture of Feed-Forward Neural Network[60]

Scaled conjugate gradient neural networks (SCGNNs) are a stochastic solver that are used in this study's investigation of the nonlinear mathematical model of COVID-19. Three essential components—verification processes, sample statistics, and testing and training methodologies are included in the stochastic SCGNNs methodology. A sizable dataset is used to do this, with 70% of the data being used for training, 16% for verification, and 14% for testing[61]. This study'smain goal is to provide numerical solutions for the susceptible, infected, and quarantine (SIQ) system's fractional mathematical model while taking the impacts of the lockdown during the coronavirus sickness into account. The generated numerical results are submitted to error histograms, state transition measurements, correlation/regression values, and mean square error assessments in order to validate and evaluate the dependability of the AI-SCG solver[62]. The goalof this research is to provide a computational framework that addresses the nonlinear dynamics of COVID-19 dissemination, as described by the SEIR-NDC model, using artificial neural networks(ANNs) and global and local search optimisation techniques. Statistics are used to establish the validity of the suggested technique, and performance index graphs and convergence metrics are used to demonstrate the precision and consistency of the stochastic system[63].

They presented a vectorized technique in this study that uses deep neural networks to solve systems of ordinary differential equations. They used Python code to run a number of tests, and graphical simulations were used to show the results. These tests give us important new information about the model's architectural features. Their research showed that, for some issue cases, a single hidden layer neuron can still achieve the requisite degree of accuracy. It's crucial to remember that adding more neurons can improve accuracy, but at the expense of necessitating more parameter learning rounds. It is important to be cautious while arbitrary expanding the number of neurons, as the ideal size should be established depending on the underlying issue. They conducted a comparison of their strategy with the well-known fourth-order Runge-Kutta technique in order to gauge its efficacy. Their findings showed that especially when working with a limited number of grid points, the artificial neural network technique outperformed the Runge-Kutta method. Furthermore, compared to the RK4 technique, their ANN approach continually produced greater accuracy as the domain size rose[64].

Systems of nonlinear ordinary differential equations make up the suggested model, which represents a variety of classes including susceptible, exposed, and infectious, super spreaders, infection but asymptomatic, hospitalized, recuperating, death. The explicit Runge-Kutta numerical approach is used to create the reference dataset for the Covid-19 model for a number of Chinese and Pakistani cities. Assessments are made based on obtained accuracy, using MAE, histograms, and regression analyses to gauge the efficiency and dependability of the constructed LMANNs (Local and Metropolitan Artificial Neural Networks)[65]. Using a mix of Artificial Intelligence

(AI) and Operations Research (OR) methodologies, a solution has been created to help health managers with the Prioritization and Scheduling of COVID-19 Patients. For AI-based analysis, the tool uses Artificial Neural Networks (ANN), and for OR-based decision-making, it uses a fuzzy interval mathematical model. This strategy's main goal is to reduce death rates while respecting the resource limitations placed on each facility. The study shows how these integrated techniques may be used by health institutionsas effective management tools for vital resources[66].

In order to predict and anticipate the number of confirmed and recovered COVID-19 cases till September 17, 2020, a unique artificial neural network (ANN) model has been created. The Covid-19 condition in Saudi Arabia—Demographics is where the model gets its training data. The Prey-Predator technique is used in the study's training, and a Multilayer Perceptron Neural Network is used. The Prey-Predator algorithm is used to optimize the parameters, resulting in the MLPNN-PPA model, which improves the performance of MLPNN. Analysis of the correlation coefficient (R) and root mean squared error (RMSE) functions is required for the suggested

MLPNN-PPA model's performance evaluation. The model is also evaluated using additional available data from Saudi Arabia (testing data). The findings demonstrate that in estimating the number of sick and recovering persons in Saudi Arabia, the MLPNN-PPA model works better than alternative approaches. According to the data, the number of affected people is expected to rise in the following days, at least reaching 9789. At the same time, between 2000 and 4000 recoveries per day are anticipated[67].

A mathematical method called the SAIVR model was created to forecast the COVID-19 epidemic's growth during immunization programs. It improves on the popular Susceptible-Infectious- Removed (SIR) model by adding extra compartments for asymptomatic (A) and immunized (V) people. Semi-supervised machine learning is used to set the model's parameters and starting circumstances. The SAIVR differential equations are initially handled by an unsupervised neural network. Then, by comparing current infection data from 27 countries, a supervised framework is used to identify the circumstances and parameters that suit the data the best. Researchers thoroughly examined the temporal evolution of the pandemic under numerous scenarios using these findings as a foundation. They examined a range of daily vaccination rates, vaccine effectiveness, and societal degrees of vaccine hesitation and denial. Furthermore, in view of probable future scenarios including diverse vaccination campaigns and the advent of more contagious COVID-19 variations, the idea of herd immunity was examined[68].

This study aims to apply artificial neural networks (ANNs) to a COVID-19 SIQ-based mathematical model while accounting for the effects of lockdown measures. Three dimensions are included in the SIQ model: "susceptible," "infective," and "quarantined." Levenberg-Marquardt back propagation (LMB) and ANNs are used to perform numerical studies of the SIQ-based COVID-19 model. The SIQ system is solved using three pieces of data, including authentications, testing samples, and training samples. Statistics are used to split the data, with 80% going towards training and 10% each going towards testing and authentication. The SIQ model's numerical outcomes are then contrasted with a reference dataset produced using Adams solutions. With values ranging from 10-9 to 10-12, the evaluation of the nonlinear SIQ dynamical model shows a considerable reduction in the mean square error, demonstrating the efficacy of the suggested strategy[69].

In order to build the SITR epidemic differential equation (DE) model, which strives to comprehend the numerous facets of COVID-19 dissemination, the NAR-RBFs neural network paradigm offersa novel methodology. In order to successfully handle nonlinear input, a new set of transformations is developed to improve accuracy, stability, and convergence analysis. The modeling of resilient dynamics in multimodal systems is a challenging and expanding field of study. This study employsa hybrid model based on the NAR-RBFs neural network for the SITR model to examine the bimodal spread of COVID-19 while taking both chaotic and stochastic pandemic data into account. Before applying the bimodal paradigm, a unique class of transformations is performed to the system of ordinary differential equations (ODE) in order to quickly converge and increase accuracy. These transformations change local optimal values into global values, which improves the model's overall performance. With the addition of fragility in modeling stochastic variations for various situations and scenarios with limited variations, the suggested NAR-RBFs model is explicitly examined for the SITR model's bimodality. The suggested bimodal paradigm exhibits good agreement with a superb numerical solver after rigorous statistical analysis, with mean squareerror precision as low as 1E-25. This attests to the suggested model's consistency and stability. The computational method used has exceptional accuracy and convergence, which makes it useful for predicting the development of COVID-19 and comprehending the effect of different deciding parameters on its spread. The study's conclusions can help with preemptive planning, monitoring,and action to stop the spread of COVID-19[70].

This study examines the dynamics of COVID-19 transfer from human to human as well as from the environment to human. The researchers include a compartment to analyse the pathogen concentration in the environmental reservoir, such as droplet concentration in confined areas, to improve conventional epidemiological models. Deriving the fundamental reproduction number and evaluating the efficacy of facemask use mathematical studies, such as endemic equilibrium and next-generation methods. The development of physics-informed deep learning techniques (PINNs) to examine the dynamics of the illness is an important contribution of this research. When solving sets of differential equations that explain the dynamics of infectious diseases, PINNs provide an alternative to more conventional numerical techniques. The outcomes show that the suggested PINNs strategy is trustworthy for resolving such systems and locating crucial variables that control illness dynamics [71].

In the field of machine learning, a neural network is a powerful tool that can accurately continuous functions within predetermined bounds. When combined with automated differentiation techniques, it creates a solid foundation for addressing numerical solutions of ordinary differential equations and partial differential equations. They explore the use of neural network in function approximation and introduce a general solution designed for ordinary differential equation and PDEs in this paper. They rigorously evaluate this solution by putting it through initial value and boundary value ODE-related situations. The results are astounding and demonstrate astounding precision in approximating the unknown functions as well as in calculating their derivatives[72].

The limitations imposed had a considerable impact on how trends typically evolved, leading to diverse upsets. Rt (time-varying reproduction number) has been used to predict reproduction rates during epidemics all over the world. In the recent positive instances of SARS-CoV-2 (COVID- 19), this study offers a Deep Learning (DL) based approach for analyzing and forecasting epidemicpatterns. By modifying the data from the NN's output layer depending on the associated Rt estimation, the method combines a neural network (NN) and Rt estimation. Datasets from Italy, the USA, France, the UK, and Sweden were used to evaluate the solution. These datasets coveredpositive case registrations between 24 February 2020 and 11 January 2022. Their method outperformed other models with the same configuration but based on LSTM, GRU, RNN, ARIMA(1, 0,3) and ARIMA (7,2,4), or a NN without include Rt as a corrective index on the Italian dataset. Mean Absolute Percentage Error (MAPE) was reduced by 28.44%, 39.36%, 22.96%, 17.93%, 28.10%, and 24.50%. Additionally, compared to the same model without the Rt adjustment, it decreased MAPE by 17.93%, Mean Absolute Error (MAE) by 34.37%, and Root Mean Square Error (RMSE) by 43.76%. The method enabled average reductions in MAPE of 5.37%, 63.10%, 17.84%, and 14.91%, respectively, on datasets relating to the USA, France, the UK, and Sweden. These results underline how well the suggested DL-based method works to increase the precisionof epidemic trend analysis and predictions for COVID-19 cases[73].

The COVID-19 epidemic has quickly spread around the world and is now one of the major global health issues and causes of mortality. Like many other nations, Turkey has seen major negative effects as a result of COVID-19. The goal of this study is to create a prediction model using an artificial neural network (ANN) to predict the daily cases and fatalities of COVID-19 in the futurein a broad sense that can be used to outbreaks in various nations. The information used in this study spans the time period from 11 March 2020 to 23 January 2021 and includes data from several different nations. This ANN model's main goal is to help governments devise proactive preventative measures for hospitals and other healthcare institutions. The results show an overall accuracy of 86% in predicting death rate and 87% in estimating the number of COVID-19 cases, which is very promising. Decision-makers and authorities may benefit greatly from these predictive skills' insightful data in order to better plan for and handle the current pandemic crisis[74].

Deep neural network (DNN)-based artificial intelligence (AI) models and differential equations- based epidemic compartmental models are useful tools for comprehending and halting Covid-19transmission. Compartmental models, on the other hand, struggle with parameter estimation, while AI models lack interpretability and are unable to understand COVID-19's evolutionary patterns. In order to capture the intricate dynamics of COVID-19, this work offers a unique method dubbed Epi-DNNs, which combines compartmental models and DNNs. The

Runge-Kutta technique is used to solve the ordinary differential equations (ODEs) and supply the values of the ODEs at a particular time in the proposed Epi-DNNs approach, which uses a neural network to represent thecompartmental model's unknown parameters. The loss function takes into account the difference between predictions and observations, enabling the defined loss to be minimized and utilized to find the compartmental model's best-fitting parameters. Utilizing real-world reported COVID-19 data from Shanghai's Omicron outbreak, which lasted from February 25 to May 27, 2022, Epi- DNN performance is confirmed. The efficiency of Epi-DNNs in COVID-19 transmission modeling is demonstrated by experimental findings on simulated data. Additionally, the suggested Epi-DNNs method's inferred parameters result in a compartmental model that predicts future dynamics in the spread of COVID-19 [75].

The goal of this study was to develop an intelligent model utilizing artificial neural networks (ANNs) in order to diagnose COVID-19. The patient history and exposure characteristics that are commonly present in hospital medical records were used to create and evaluate an ANN model for COVID-19 diagnosis. The results of their investigation show that ANNs can diagnose COVID-19 with a high level of specificity and acceptable sensitivity. Additionally, the outcomes demonstrate that ANNs can accurately and efficiently discriminate COVID-19 from other viral pneumonia and flu-like diseases[76].

In this paper, they use a genetic algorithm to improve the typical BP neural network, allowing the neural network to avoid local optima during predictions. The empirical results show that the modified algorithm's loss value decreases faster, eventually reaching less than 0.09315. This decrease represents a respectable degree of simulation accuracy.Their technique to projecting medical resource needs for new patients involves the use of a linear function to simulate the distribution of individuals with various symptoms. In addition, for patients with varied symptoms, we use a normal distribution function to characterize the distribution of hospitalization lengths. They then compute the overall resource needs for various medical resources. Overall, the estimated values nearly match the real resource allocation, proving the model's usefulness[77].

The assessment of variables impacting both the intention to adopt and the actual use of health, as well as the analysis of how the actual use of health effects the mental well-being of its users, are the main objectives of this study. The study also explores the role that self-quarantine plays in influencing both intended and actual health usage, particularly in the context of the coronavirus disease (COVID-19) pandemic. Primary data were obtained from people utilizing health services in Bangladesh in order to get crucial information. In addition, this research used the Artificial Neural Network (ANN) approach to identify and rank relevant predictors acquired from the Structural Equation Modelling (SEM), taking into account nonlinearities seen in the dataset. In summary, this study contributes to the increasing corpus of health literature by illuminating the ways in which using health services might improve patients' mental health, particularly given the difficult conditions brought on by the current epidemic[78].

This study focuses on applying deep learning to solve optimum control problems and on using neural networks to construct optimal control functions. These neural networks embedded within differential equations may be trained by using adjoin sensitivity analysis. Notably, this method goes beyond usual epidemic control difficulties; it also finds use in epidemic prediction, reveals previously unidentified mechanisms, and provides fresh viewpoints for conventional mathematical epidemiological problems. It's important to note that this study has certain limitations, though. First of all, it lacks in-depth information on how to solve generic optimum control problems with path restrictions. The second shortcoming is that it doesn't offer sufficient theoretical support or a comprehensive examination of the problem's complexity, which may potentially show that deep learning techniques can effectively deal with high-dimensional optimum control issues[79].

Table 2.1 Summary of literature Review

Paper id	Technique	Advantage	Disadvantage
[55]	Genetic algorithm, ODE-45	• It provide insights on the dynamics of the disease in various situations.	•It doesn't address some of the complications and ambiguities connected to the dynamics of diseases in the actual world

[54]	ANN	- Their findings demonstrate that a larger number of high risk individuals causes a higher illness burden and significantly higher mortality.	- In this study, they do not explicitly examine reducing the number of fatalities.
[77]	ANN	- The model's predictions match the data well, showing an effective simulation effect.	- This paper are not solving the nonlinear equation
[62]	Deep Neural Network(DNN)	- For a smaller number of grid points, it generated more accurate results. - Compared to the RK4 approach, the accuracy of the DNN method is higher.	- This model is not solved the coupled equation
[30]	SIRD	- The model can accurately forecast the COVID-19 outbreak's future trends.	- It does not take into account the spatial distribution of each individuals and their displacement.
[22]	SI model	- It gives researchers and decision-makers access to a fundamental framework for modelling the spread of diseases.	- It could produce inaccurate results when used in more complicated epidemiological scenarios.
[71]	Neural Network(NN)	- It produces results with the same level of accuracy as traditional numerical methods.	- This model is not solving the equation that are coupled in nature.

2.5 Research Gap And Direction

After an extensive survey of the existing literature on ANN to solve the SI model, multiple problems have been identified which led to the development of the proposed scheme discussed in the coming chapters. Existing issues are related to the model's complexity and minimization of the error that helps to achieve a high level of accuracy. Some identified problems and existing gaps are listed below:

- Non-linear equations, which are often more difficult to handle using standard methods, might be included in the SI model. ANNs might be used to effectively solve these equations given their capacity to capture non-linear relationships. However, diving into this complexity calls for more research into the precise architecture and training methods needed to handle this task.
- By using iterative numerical approximations, numerical solvers like ODE-45 provide accurate solutions for ordinary differential equations (ODEs). Although accurate, these solutions may have trouble solving complex nonlinear equations with a wide range of beginning circumstances and parameters.
- Analytical models offer theoretical insights but could potentially lack practical accuracy in real-world scenarios.
- Given sufficient and representative training data, ANNs have the capacity to recognize complex non-linear relationships within data, which may result in a reduction in MAE.
- Due to their reliance on mathematical equations, analytical models do not produce direct MAE values but rather accurate answers.
- It could be required to carefully tune solver parameters and deal with any numerical instability in order to achieve a low MAE.

2.6 Summary

This chapter provides a solid foundation of research by providing an extensive Literature Survey on different techniques that solved the epidemic model of Covid-19. This chapter offers a critical analysis of current models in relation to various methodologies. Strengths and weaknesses of each approach are also theoretically examined. Their shortcomings and limits are then explored, which should serve as a challenge for future study.

CHAPTER 3

DEVELOPED ANN TO SOLVE THE SI SYSTEM OF COVID-19 NON-LINEAR EQUATION

3.1 Overview

This chapter gives detailed documentation of our designed methodology ANN solve the ODE of SI system of COVID-19 non-linear equation. with the help of Artificial Neural Network with all phases which are applied explained as well as elaborated diagrammatically. Furthermore, how the proposed research objectives are achieved is also discussed. Section 3.2 gives the detailsof the research process followed in this thesis. The explanation of SI model, its parameter and application of SI model are shown in 3.2. Moreover, the feed forward Neural Network used for the implementation is described with all its number of neurons in each layer. The worth if ReLU artificial neural network are described in this chapter. The fitness function of the unsupervised error function is used to determine how well the predictions provided by the ANN align with the actual data or the desired outcomes. The comparison of our proposed scheme with benchmark are described in section 3.4 with the Mean absolute error.

3.2 Operational Framework

In this section, the operational workflow and step-by-step phases of the methodology are used in which ANN solves the ODE of the SI system of the Covid-19 non-linear equation are explained. Fig 3.1 shows the research phases and steady process to meet the research objectives. The proposed Research consists of three phases.

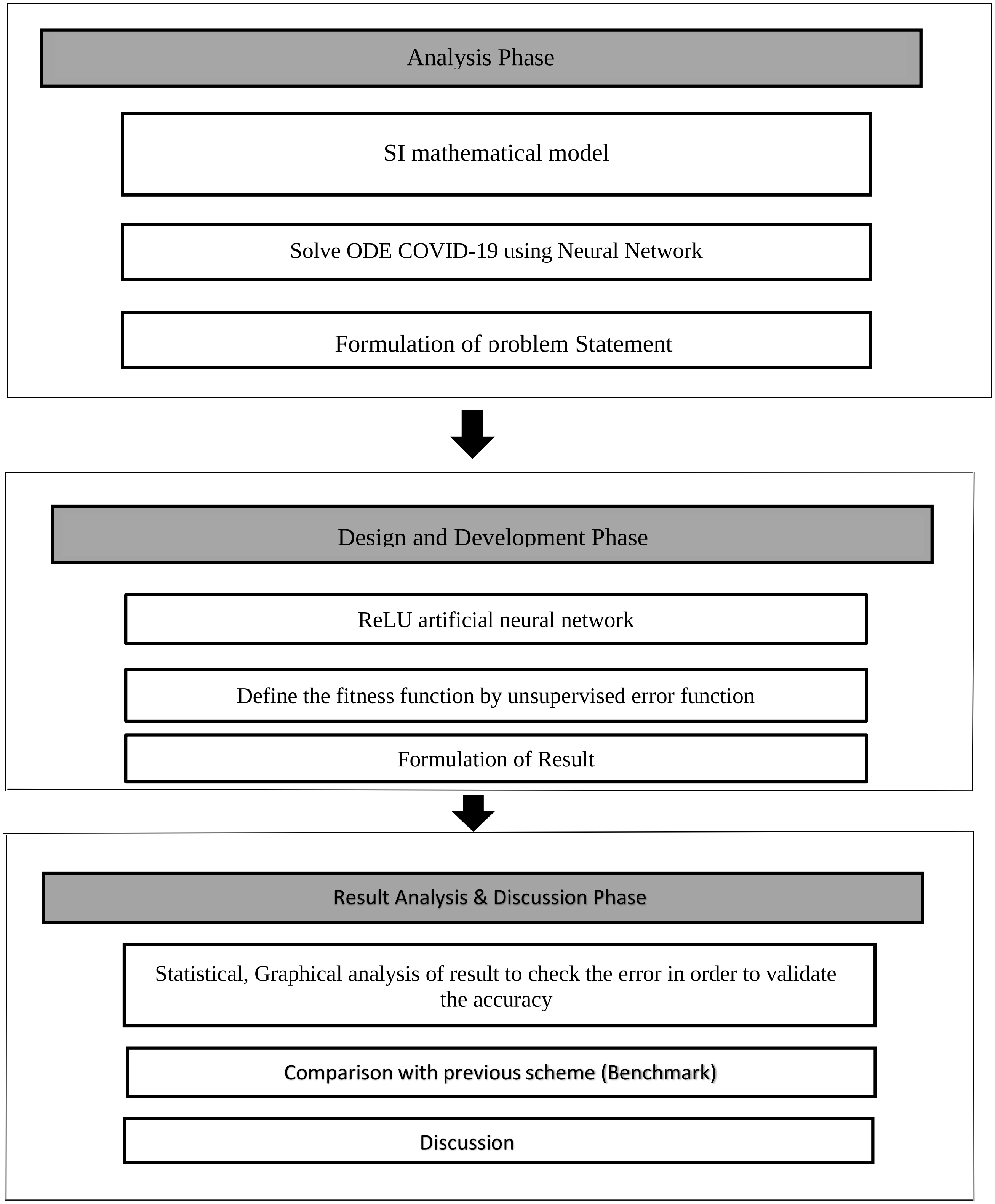


Figure 3.1: Operational Framework of the Research Work

The first phase is based on Analysis. During this Phase, all the previously available methods and solutions to solve ODE of the SI model are analyzed in brief. Since the equations in the SI model are nonlinear, they cannot always be resolved analytically. The integration of these equations is necessary in order to reach the analytical solution of this system. Nonetheless, directly solving these equations could pose challenges due to their nonlinear characteristics. Generally, achieving analytical solutions might be attainable primarily when working with simplified assumptions. The Susceptible- Infected (SI) model for Covid-19 is one dynamic system that can be solved using the numerical integration solver ode45, which is frequently used for solving ordinary differential equations (ODEs).Despite the SI model's simplicity, ode45 can produce reliable results if the dynamics of the system are well reflected in the model equations. Always keep in mind the limitations of the SI model and, ifrequired, take into account more complicated models. From the available Literature, several gaps are identified and limitations are found and described in section (2) that still require focus and research.

By investigating these constraints, the problem statement is developed and objectives are set.In this second phase, the design of the proposed model is discussed. The neural network gains non-linearity through the ReLU activation function, allowing it to understand complex correlations in the data. This is especially helpful when trying to replicate and understand the many factors that affect how a disease spreads. The fitness function of the unsupervised error function is used to determine how well the predictions provided by the ANN align with the actual data or the desired outcomes.

Finally, the third and last phase includes a discussion of results with the help of mean absolute error and benchmark parameters to evaluate the results and provides directions toward further research. Statistical and graphical methods are used to engage the audience in an interactive discussion of the findings. The next chapter goes into great length into the outcome’s discussion.

3.3 Research Design And Development

The artificial neural network is a computational method in artificial intelligence that is used to resolve a number of highly complex challenges that are stiff linear and non-linear problems in terms of scalability, reliability, and robustness. In this research, the feed-forward ANN is used to solve the SI model of the COVID-19 non-linear equation. The workflow of this research is described in below figure 2.

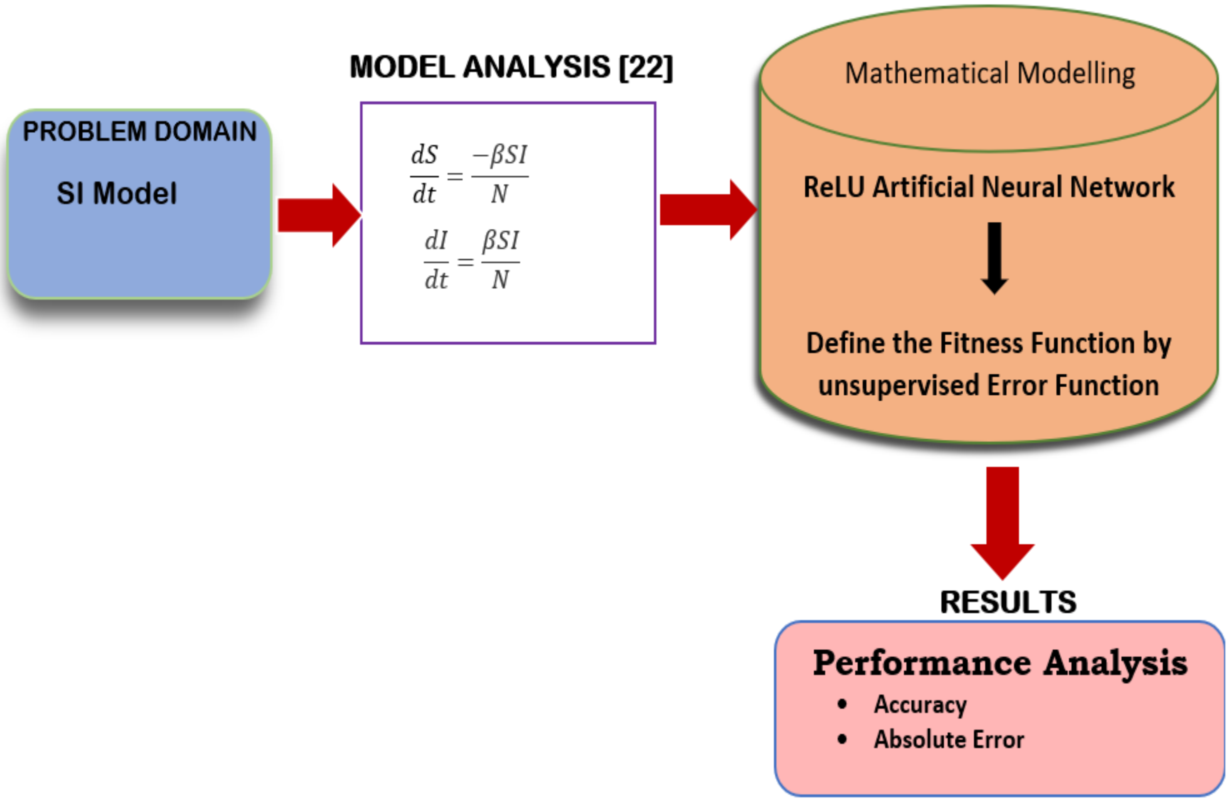


Figure 3.2: Developed model

The problem domain of our proposed research is to solve the SI mathematical model using ANN.The SI model is based on ODE that is coupled in nature. The population is split into two groups: susceptible persons (S), who are able to get the disease, and infected individuals (I), who can spread it to susceptible individuals. In the Susceptible-Infected (SI) model, the Rectified Linear Unit (ReLU) activation function has been utilized to improve the depiction of the complicated interactions and dynamics. Artificial neural networks (ANNs) frequently use this activation function. The fitness function of the unsupervised error function is used to determine how well the predictions provided by the ANN align with the actual data or the desired outcomes. The mean absolute error has been used to find the error rate between ANN and the analytical model, numerical solver (ODE-45). We have analyzed the accuracy and reliability of the proposed schemewith analytical and numerical solutions.

3.3.1 SI Mathematical Model

The SI model is the simplest form of all infectious diseases in which the people are categorized as either susceptible and infected (SI). The individual who are healthy is susceptible and the individual who are sick is infected. The susceptible individual can be infected if the individual contact with the infected individual. The SI model can be written in the form of ordinary differential equation (ODE) is [22]:

$$\frac{dS}{dt} = \frac{-\beta SI}{N} \qquad \text{Eq.1}$$

$$\frac{dI}{dt} = \frac{\beta SI}{N}$$

dS, dI is the rate of change in Susceptible individual and the rate of change of infected individual, **dt** is the rate of change of time. . The **S** is the number of susceptible and **I** is the number of infected. The **β** is the rate of transmission the individual of Susceptible become infected. The **N** is the number of population. The **–β** represented when the susceptible individual get infected its mean the decrease the susceptible individual and increase the infected individual.

3.3.2 Parameters of SI Model

The Susceptible-Infected (SI) model factors that describe the characteristics and course of disease transmission in a population are referred to as parameters. These factors have a big impact on the dynamics of the model, which has a big impact on how an epidemic develops. The SI model's important parameters are as follows:

Transmission rate (β): The infection rate in this particular simulation is set at 2.

Population: The total number of people in a population has an effect on how widely a diseasespreads. The total population has been set in this research is 100.

The initial number of Susceptible (S0) and infected (I0): S0 has been set at 100 as the initial number of susceptible individuals. The initial number of infected people (I0) has been set at 1. These represent the initial quantities of susceptible and infected individuals at the outset of the epidemic.

Time(t): The number of equally spaced values between 0 to 10 that define the time point tosimulate the model.

3.3.3 Application of SI Model

The Susceptible-Infected (SI) model is used in a variety of scenarios to understand and predict the spread of illnesses. Here are a few important areas where the SI model is useful:

- **Epidemiology:** The SI model serves as a fundamental tool in epidemiology for analyzing the early stages of infectious disease epidemics. It helps in estimating the number of infections and assessing the possibility of disease spread.

- **Public Health Planning:** Public health officials use the SI model to strategically allocate resources during disease outbreaks. Understanding the progression of infections enables them to decide on therapies and the allocation of resources with confidence.

- **Immunization Approaches:** The SI model helps assess the effects of various immunizationtactics. It makes it easier to foresee how vaccination rates affect the rate and range of diseasespread.

- **Research on Infectious Diseases:** To compare with more complex models, scientists use the SI model as a benchmark. It provides insights into basic transmission dynamics and laysthe foundation for understanding complex disease models.

- **Emergence of Novel Pathogens:** When data are limited at the beginning of a new disease outbreak, the SI model can offer early indications of the possible scope of the epidemic. This direction can direct future data gathering initiatives.

- **Utilization in Education:** The SI model is commonly used in educational settings to introduce students to epidemiology and disease transmission ideas. Complex dynamics are made simpler to improve understanding.

- **Modeling Non-Infectious Phenomena:** The SI model's ideas may be applied to various scenarios involving the spread of phenomena, even if its core design is on infectious illnesses.This covers situations like the spread of new ideas or information through social networks.

- **Foundational Disease Propagation Understanding:** The SI model serves as a stepping stone for individuals who are unfamiliar with epidemiology, helping them to have a fundamental understanding of how infectious illnesses may spread throughout society.

3.3.4 ANN In The Epidemic Model

The use of Artificial Neural Networks (ANNs) in epidemic models, including those used to study the spread of infectious illnesses, is becoming more common. ANNs are a kind of machine learning model that can identify complex patterns in data, making them potentially useful for analyzing and predicting the dynamics of diseases. Artificial neural networks (ANNs) must be used in a Covid-19 epidemic model in order to evaluate and predict the complex non-linear patterns regulating the spread of the disease. The use of ANNs in Covid-19 non-linear epidemic models has the potential to provide important details about complex disease dynamics, especially when the interplay between variables is complicated and characterized by non-linear correlations. In this proposed scheme, the ANN framework is used to solve the SI system of the COVID-19 non-linear equation.

3.4 ReLU Artificial Neural Network

In artificial neural networks (ANNs), the Rectified Linear Unit (ReLU) serves as an activation function, introducing nonlinearity into the network's computations. ReLU is a popular option in neural network topologies because of its simplicity and effectiveness in fixing some issues with other activation functions, such as sigmoid and tanh. Many study are shown in [80-82] todescribe worth of ReLU artificial neural network.

The mathematical representation of ReLU artificial neural network [80] is:

$$f(x) = \max(0, x) \qquad \text{Eq.2}$$

In other words, the ReLU function produces x for any input if x is larger than or equal to zero; if x is negative, the ReLU function outputs zero.

3.4.1 Dense Layer

4 dense layers are used to solve the SI model in this proposed research. Each neuron in this dense layer is affiliated with each and every other neuron of previous layer. In this layer a multiplication operation is performed in between the weights and input coming from the previous layer values produces after this operation into activation function in order to produce non-linearity in the final output layer also in this layer complex patterns of sequential patterns are learned via performing adjustment of bias and weight values. The neurons used in this layer represent the number in the output layer.

3.4.2 Updation of Neuron:

To achieve high accuracy and reduce error rate, two primary neuron updates have been applied. The initial update includes neuron ranges from 10 to 80. Neurons from 90 to 200 are included in the second updation.

3.4.3 Use of ReLU Artificial Neural Network In SI Model

Neural Network Architecture: The ReLU activation function has been used in neurons located within the hidden layers of the neural network in the context of embedding ANN techniques within the SI model. Through the introduction of non-linearity provided by this integration, the network is able to recognize complex connections and patterns contained within the data.

Extraction of Key Features: ReLU activation helps the neural network retrieve key features from the input data relevant to the SI model. For example, it helps with the identification of crucial elements influencing the spread of diseases, such as population density, contact rates, susceptibility,and infected people.

Model Complexity: Although the SI model is relatively simple, the dynamics of actual diseases can involve complex non-linear interactions. The ANN can handle this degree of complexity because to ReLU activation, which transforms linear combinations of input features into non-linear activations.

Simulation and Projection: Using ReLU activation inside the ANN can improve the model's capacity to more effectively mimic the spread of illness over time. This enhancement results from the network's skill in capturing the complex non-linear relationships between susceptible and infected populations.

Model Generalization: By including ReLU in the ANN, the model's ability to generalize knowledge from training data to unobserved data is increased, enhancing its adaptability and resistance to a variety of epidemic scenarios.

Predictive Ability: Incorporating ReLU into an ANN-based SI model allows the network to more accurately anticipate the spread of illness. The network's capacity to understand the complex non- linear interactions among variables like the vulnerable population, transmission rates, and initial infections leads to this enhancement.

3.4.4 Fitness Function By Unsupervised Error Function

In this proposed scheme, the fitness function of the unsupervised error function is used to determine how well the predictions provided by the ANN align with the actual data or the desired outcomes. The Susceptible-Infected (SI) model's fitness function uses an unsupervised error measure to evaluate the model's performance without depending on explicitly labeled input. This method does not require labeled data because the error function assesses the model's ability to capture patterns andreplicate the observed dynamics.

3.4.5 Mathematical Representation of ANN to Solve The SI System of Covid-19 Using Fitness Function

Equation 3 represents the mathematical representation of ODE of the SI model [22] and equation 4 represents the neural network of the SI model of Covid-19:

$$\frac{dS}{dt} = \frac{-\beta SI}{N} \qquad \text{Eq.3}$$
$$\frac{dI}{dt} = \frac{\beta SI}{N}$$

$$\frac{d\hat{S}}{dt} = \frac{-\beta \hat{S}\,\hat{I}}{N}$$
$$\frac{d\hat{I}}{dt} = \frac{\beta \hat{S}\,\hat{I}}{N} \qquad \text{Eq.4}$$

Rearrange equation 4 the equation (5,6) becomes,

$$\frac{d\hat{S}}{dt} = \frac{-\beta\hat{S}\hat{I}}{N} \quad \rightarrow \quad \frac{d\hat{S}}{dt} + \frac{\beta\hat{S}\hat{I}}{N} = 0$$ Eq.5

$$\frac{d\hat{I}}{dt} = \frac{\beta\hat{S}\hat{I}}{N} \quad \rightarrow \quad \frac{d\hat{I}}{dt} - \frac{\beta\hat{S}\hat{I}}{N} = 0$$ Eq.6

In this equation (7), y(t) represents the output of the neural network at time t. The equation is a summation (∑) over n neurons in the neural network. Each neuron has weights wi and bias bi, and f is the activation function. The term αi represents a coefficient associated with each neuron. This equation shows the mathematical representation of the output of the neural network[60]:

$$\left[\begin{array}{l} y(t) = \sum_{i=1}^{n} \alpha_i \, f(w_i t + b_i) \\ \\ \frac{d^n\hat{y}}{dt^n} = \sum_{i=1}^{n} \alpha_i \, \frac{d^n}{dt^n} f\,(w_i t + \, b_i) \end{array}\right]$$ Eq.7

This equation (8) define th Rectified Linear unit (ReLU) where f(t) is output of the activation function, and it is the maximum of 0 and *t*. This activation function introduces non-linearity to the network[80]:

$$f(\hat{t}) = \max(0, t)$$ Eq.8

These equations (9,10) use the ReLU activation function to represent the susceptible (S) and infected (I) populations as outputs of the neural network. Similar to Equation 5, the equations summation of neurons with weights, biases, and coefficients are:

$$\hat{S}(t) = \sum_{i=1}^{n} \alpha_i \, f(w_i t + b_i) \qquad \text{Eq.9}$$

$$\hat{I}(t) = \sum_{i=1}^{n} \alpha_i \, f(w_i t + b_i) \qquad \text{Eq.10}$$

These equations (11,12) provide the formulas for the susceptible and infected populations using the ReLUactivation function by replacing the ReLU activation function from Equation 8 into Equations 9 and 10.

$$\hat{S}(t) = \sum_{i=1}^{n} \alpha_i \, [\max(0, t)](w_i t + b_i) \qquad \text{Eq.11}$$

$$\hat{I}(t) = \sum_{i=1}^{n} \alpha_i \, [\max(0, t)](w_i t + b_i) \qquad \text{Eq.12}$$

This equation defines the fitness function ($\in SI$) used to assess the performance of the model. It's a sum of squared errors for susceptible, infected, and a term that measures the difference between predicted and actual initial values:

$$\in_{SI} = (\in S)^2 + (\in I)^2 + (\in)^2 \qquad \text{Eq.13}$$

These equations (14,15, and 16) break down the squared error terms for susceptible ($\in s$), infected($\in I$), and the initial values ($\in$) in the fitness function. The terms involve derivatives and squared differences between model predictions and actual values:

$$(\in S)^2 = \frac{1}{1+N} \sum_{m=0}^{N} \left(\frac{d\hat{S}}{dt} + \frac{\beta \hat{S} \hat{I}}{dt} \right) \qquad \text{Eq.14}$$

$$(\in I)^2 = \frac{1}{1+N} \sum_{m=0}^{N} \left(\frac{d\hat{S}}{dt} + \frac{\beta \hat{S} \hat{I}}{dt} \right) \qquad \text{Eq.15}$$

$$(\in)^2 = \frac{1}{2}\left[(\hat{S}0 - S0)^2 + (\hat{I}0 - I0)^2\right] \qquad \text{Eq.16}$$

Putting values in Equation (14,15,16) the Eq (17,18,19) becomes:

$$(\in S)^2 = \frac{1}{10}\sum_{m=0}^{N}\left(\frac{d\hat{S}}{dt} + \frac{(2.0)\hat{S}\hat{I}}{dt}\right) \qquad \text{Eq.17}$$

$$(\in I)^2 = \frac{1}{10}\sum_{m=0}^{N}\left(\frac{d\hat{S}}{dt} - \frac{(2.0)\hat{S}\hat{I}}{dt}\right) \qquad \text{Eq.18}$$

$$(\in)^2 = \frac{1}{2}\left[(\hat{S}0 - 100)^2 + (\hat{I}0 - 1)^2\right] \qquad \text{Eq.19}$$

The total fitness function, which is the sum of squared errors for susceptible and infected populations as well as the beginning value differences, is represented by the final equation, $\in SI$. The fitness function's weighting of various error factors is determined by the coefficients in these equations.

$$(\in_{SI})^2 = \frac{1}{10}\sum_{m=0}^{N}\left(\frac{d\widehat{S}}{dt} + \frac{(2.0)\widehat{S}\widehat{I}}{dt}\right)^2 + \frac{1}{10}\sum_{m=0}^{N}\left(\frac{d\widehat{S}}{dt} + \frac{(2.0)\widehat{S}\widehat{I}}{dt}\right)^2 + \frac{1}{2}\left[(\widehat{S}0 - 100)^2 + (\widehat{I}0 - 1)^2\right] \qquad \text{Eq.20}$$

3.5 Performance Measures

The Mean Absolute Error (MAE) metrics for the susceptible and infected populations are represented by the two equations Eq. 21 and 22, respectively. The average absolute differences between predicted and actual values over several data points are measured by these formulae. They provide an evaluation of the model's accuracy in capturing the complex behaviors of susceptible andinfected people within the model's environment. The $S0$ represent the predicted value generate from the ANN and the S0 represent the actual value taken from the analytical model. N represent the number of populations.[73]

$$\text{Mean Absolute Error of Susceptible Individual} = \frac{\sum_{i=1}^{n}\left|\tilde{S}0 - S0\right|}{n} \qquad \text{Eq.21}$$

$$\text{Mean Absolute Error of Infceted Individual} = \frac{\sum_{i=1}^{n}\left|\tilde{I}0 - I0\right|}{n} \qquad \text{Eq.22}$$

3.5.1 Mean Absolute Error (MAE)

Mean Absolute Error (MAE) is used to evaluate the average size of discrepancies or variances between predicted values and actual values that were observed. It is widely used in many fields, including machine learning, statistics, and economics, to assess the accuracy of models or theconsequences of predictions. Mean Absolute Error (MAE) is widely used in several prediction models in addition to the Susceptible-Infected (SI) model. The average absolute difference between predicted values and actual observations is measured by MAE. The MAE offers a measure of how well the model's predictions for both susceptible and infected populations match the real-world datain the context of the SI model. In this proposed scheme, the MAE is used to evaluate the error rate. We have compared the ANN result with the benchmark consisting of the analytical model and Ode-45 are clearly shown in Chapter 4.

3.5.2 Comparison Result with An Analytical Model And ODE-45(Benchmark)

Mathematical models with a closed-form solution are called analytical models. This suggeststhat by using analytical methods, it is possible to express the solutions to the equations used to representsystem changes as mathematical functions.[34] An analytical model would include a set of differential equations that explain how susceptible and infected rates change over time for the SI model, which shows the interaction between susceptible and infected people in a population. These equations are derived based on presumptions and simplifications regarding the population dynamics and disease transmission processes. Ordinary differential equations (ODEs) are often solved numerically using the ODE-45 numerical method[83]. This approach, which may be found in several libraries for scientific computing, including MATLAB, serves as a solver algorithm. ODE-45 is useful for computationally resolving the set of differential equations regulating the dynamics of susceptible and infected populations as they change over time within the context of the SI model.

In this proposed scheme, with the help of Mean Absolute Error (E1) and Mean Absolute Error (E2), the comparison in this suggested approach is based. The Error (E1) represents the average difference between the benchmark that shows the mean absolute error of susceptible and infected individuals between Ode-45 and the analytical model. The Error (E2) is the key contribution of our suggested method that represents the mean absolute error of susceptible and infected people between the analytical model and the ANN that are exhibited in performance measurements.

3.6 Summary

This chapter describes the details of techniques and steps taken to solve the SI model of the COVID-19 nonlinear equation using ANN. At first, the SI mathematical equation is explained. The Rectified Linear Unit (ReLU) activation function has been utilized to improve the depiction of complicated interactions and dynamics. The fitness function of the unsupervised error function is usedto determine how well the predictions provided by the ANN align with the actual data or the desired outcomes are explained in above. The Mean absolute error is used to find the error rate between the proposed scheme and the benchmark. Finally, the diagrammatical explanation is done with the help ofthis workflow.

CHAPTER 4

PERFORMANCE EVALUATION OF SI MODEL OF COVID-19

4.1 Overview

In this chapter, the implementation results of the proposed ANN framework to solve the SI system of COVID-19 nonlinear equation are presented in Chapter 3 will be discussed. Minimizing theerror to validate the accuracy and evaluate the reliability of our proposed scheme is the core of the proposed research. Furthermore, this chapter describes the implementation platform, experimental settings, and parameters used to evaluate the performance of the model. The organization of this chapter is like in Section 4.2 gives details about evaluation parameters and obtained results, and analysis and discussion are described in Section 4.3. The significant achievement and comparison with other benchmark is illustrated in Section 4.4. Finally, this chapter is summarized in Section 4.5.

4.2 Evaluation Parameter

An evaluation parameter is a measurement or statistic used to assess the effectiveness, quality, or performance of a system, process, model, algorithm, or solution. It provides a quantifiable way to assess how well the assessed object complies with predetermined standards or objectives. The importance of evaluation parameters resides in their function in determining the success or suitability of a solution, which is frequently used to compare different techniques or models. The proposed ANNto solve the SI system of the Covid-19 non-linear equation is evaluated on the basis of different parameters described in the sections below:

4.2.1 Mean Absolute Error

Mean Absolute Error (MAE), the most popular metric for assessing how well Deep Learning and Machine Learning models perform, is utilized as the second parameter to verify the performance of the proposed model. In cases where there are regression problems, MAE exhibits the greatest outcomes. It determines the exact gaps between real data or the ground truth and forecasted values. The benefit of MAE is that it does not depend on outliers and that it provides insights into the model's development by averaging out individually estimated absolute errors. MAE is calculated mathematically as follows [73]:

$$Mean\ Absolute\ Error\ = \frac{\sum_{i=1}^{n}|Predicted\ value - Actual\ value|}{n} \qquad \text{Eq.23}$$

- It identifies which parameters have the most significant impact on your model's behavior."
- The difference between each data point's actual value and the value predicted by the model was measured in absolute terms.
- The average was calculated by adding all of the outcome values.

The total number of occurrences in the whole dataset that is labeled and unlabeled is denoted by the number n in the equation above. Actual and predicted are the original and predicted values of any marked variable or instance, respectively. The procedures below were carried out when computing MAE for the proposed ANN to solve the SI system of the COVID-19 nonlinear equation.

4.3 Experimental Settings

During the experimentation and implementation phase following are some important settingsused for this experimenting research.

Table 4.1 Experimental Settings

Setting	**Value**	**Description**
Batch normalization	In 4 dense layer	To improve the training stability
Device	Jupiter notebook	Jupiter notebook is used for implementation and analyzing the proposed model.
Epochs	900	900 Epochs were used for fine training of the model which means the entire unlabeled dataset was passed 900 times through the model for training.
Learning rate	0.01	The model's parameters have been adjusted at a learning rate of 0.01 during each training phase.
Independent run	30	30 independent runs" means-tested the result 30 times to ensure accuracy.

4.4 Result and discussion

The detailed explanation of the results and important interpretation are given here to solve the SI model of covid-19 non-linear equation. The numerical results are based on the comparison method generated from analytical and ODE-5 solver and design artificial neural network proposed scheme. To evaluate performance measures, statistical observations are also listed. The result is obtained through 30 independent runs to achieve a higher level of accuracy to verify the stability and reliability. The statistical measure is based on mean absolute error. The graphical representation between analytical and ODE-45 and the proposed ANN are shown in given figure

4.1. To validate the proposed approach the graph of absolute error using proposed results is presented.

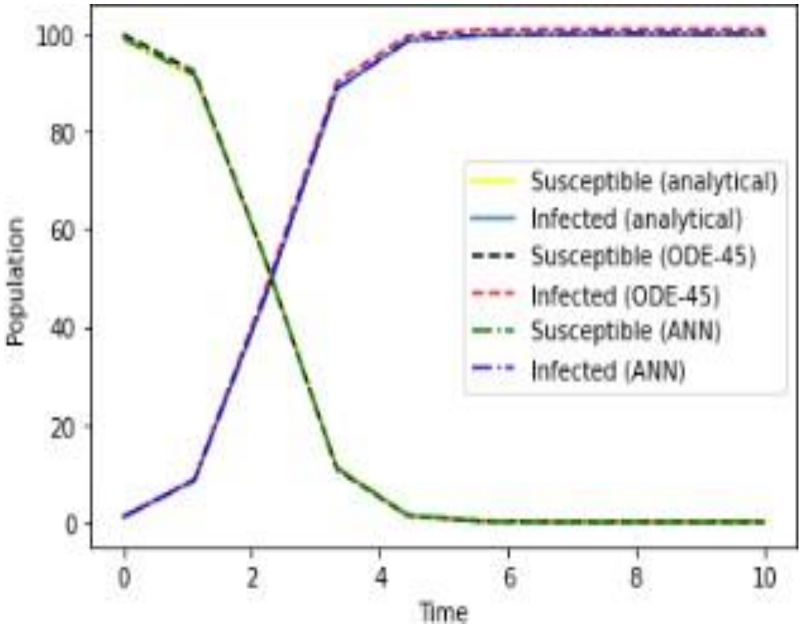


Figure 4.1: Graphically comparison between ANN and other benchmark when neuron range is10-80.

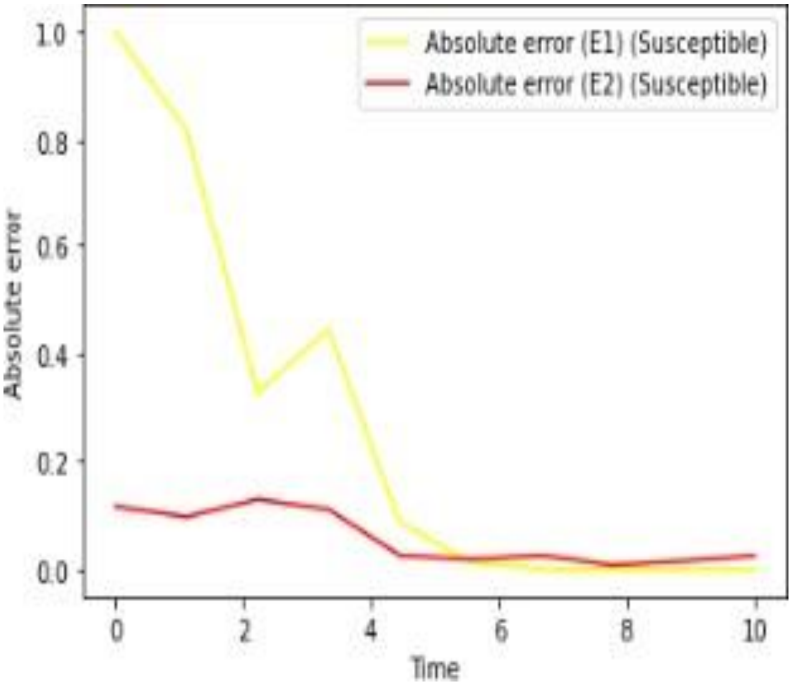


Figure 4.2: MAE (E1) and (E2) of susceptible individual

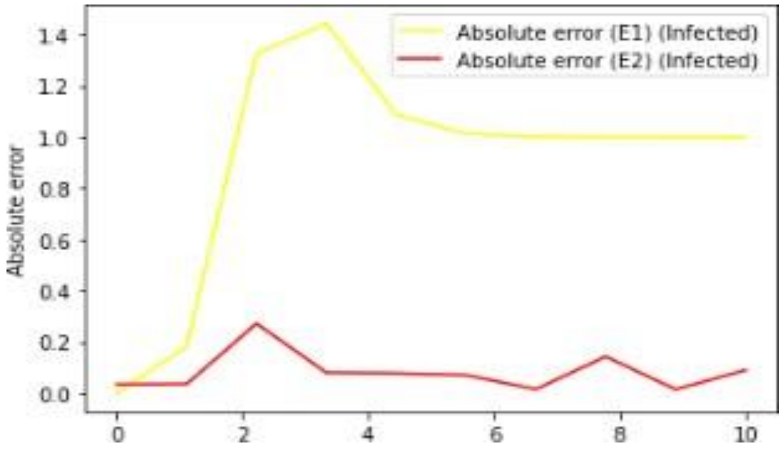


Figure 4.3: MAE (E1) and (E2) of Infected individual

Figure 4.1, in which graphically result are shown between the analytical model, ODE-45 solver and ANN to solve the SI model of Covid-19 non-linear equation. Number of population is 100 including susceptible and infected individual. . The number of equally spaced values is between 0to10 that define the time point to simulate the model. Clearly seen that the number of susceptible individuals is decreased in the analytical model, ODE-45 solver, and ANN, and the number of infected increases in the analytical model, ODE-45 solver and ANN. All the graph lines look likesame but with the help of absolute error, the differentiation between all results is exploited in Figures 4.2 and 4.3.

Figure 4.2, in which the yellow line indicates the absolute error of susceptible individuals between ODE-45 and the analytical model named Error 1(E1). And the red line indicates the absolute error of susceptible individuals between ANN and the analytical model named Error 2(E2). But after time 6 the number of susceptible E2 is increased then the number of susceptible E1 so to achieve a higher level of accuracy we increase the number of neurons from 90-200. The previous number of neurons range is 10-80. Figure 4.3, in which the yellow line indicates the absolute error of infected individuals between ODE-45 and the analytical model named Error 1(E1). And the red line indicates the absolute error of the infected individual between ANN and the analytical model named Error 2(E2). Clearly seen that the error rate of infected individuals of E2 is minimized thanE1.

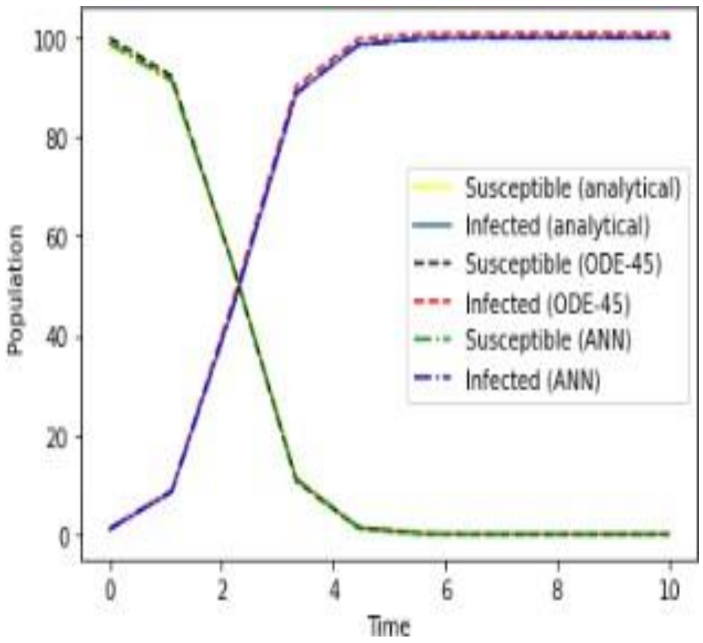


Figure 4.4: Graphically comparison between ANN and other benchmark when neuron range is90-200.

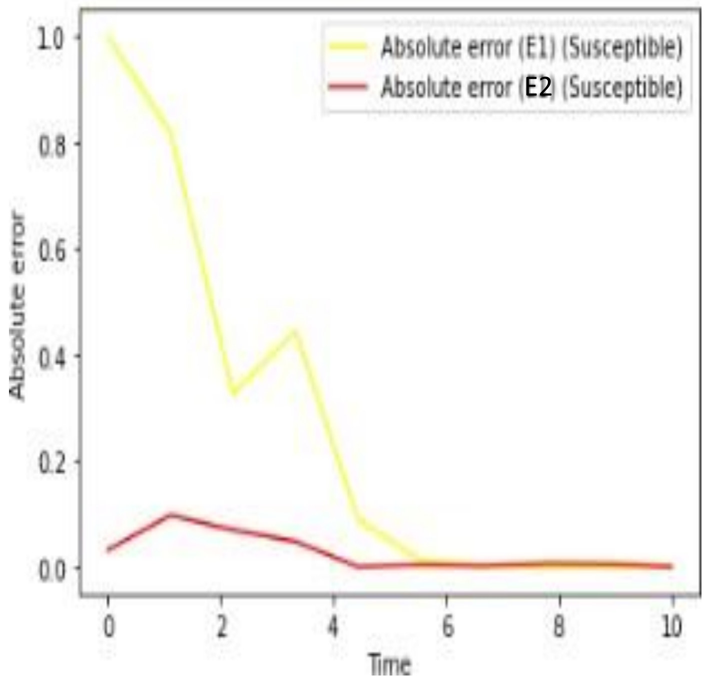


Figure 4.5: MAE (E1) and (E2) of susceptible individual

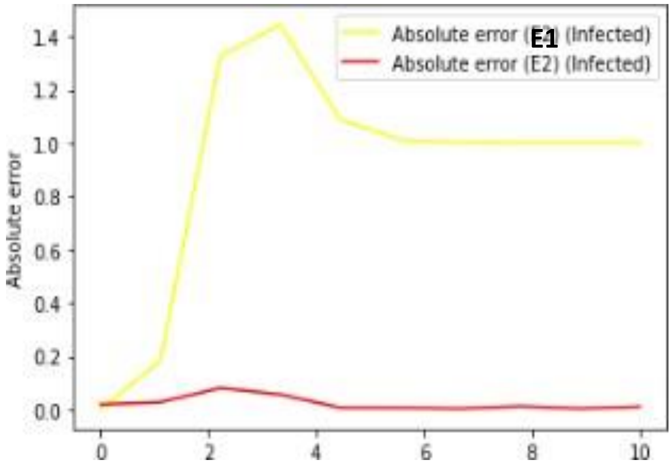


Figure 4.6: MAE (E1) and (E2) of infected individual

Figure 4.4, in which graphically result are shown the analytical model, ODE-45 solver and ANN to solve the SI model of Covid-19 non-linear equation when neuron range is 90-200. All the graph line look like same but with the help of absolute error, the differentiation between all results is exploit in figure 4.5 and 4.6.

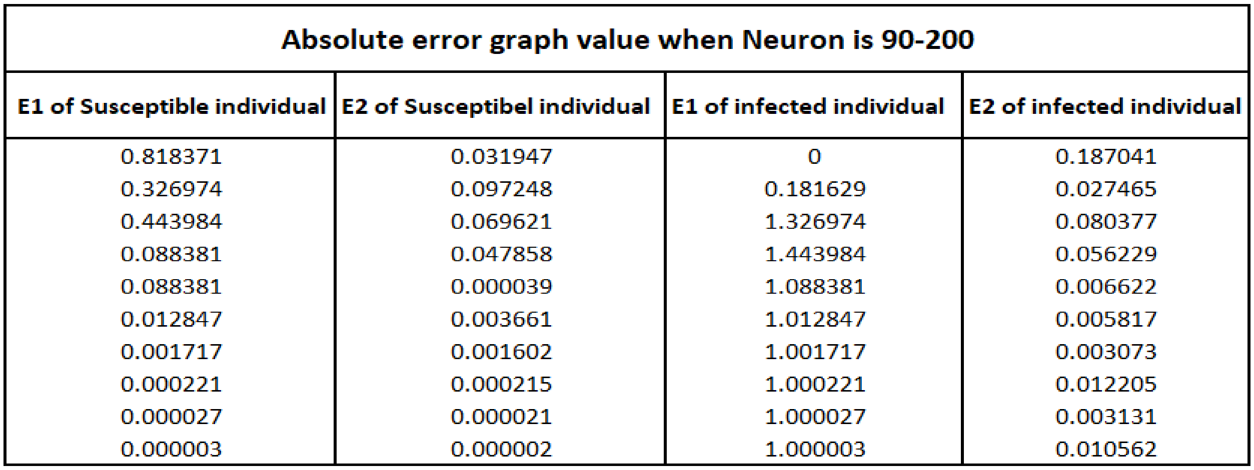
In figure 4.5, after increase the number of neuron from 90-200, the error rate of E2 is less than the number of susceptible individual of E1. Further explanation of values changes in table 4.3 in given below that give the clarity of error minimization after increase the number of neuron. In figure 4.6, the error rate of E2 is more minimize than E1.

Table 4.2. Absolute error graph value when Neuron is 10-80

Absolute error graph value when neuron is 10-80			
E1 of Susceptible individual	**E2 of Susceptibel individual**	**E1 of infected individual**	**E2 of infected individual**
0.818371	0.116684	0	0.033399
0.326974	0.097248	0.181629	0.034979
0.443984	0.129548	1.326974	0.27139
0.088381	0.111115	1.443984	0.079384
0.088381	0.026466	1.088381	0.077186
0.012847	0.018576	1.012847	0.069585
0.001717	0.026305	1.001717	0.014261
0.000221	0.007551	1.000221	0.142431
0.000027	0.015589	1.000027	0.013708
0.000003	0.025728	1.000003	**0.089254**

The graphical values of absolute error of figure figure (4.2) and figure (4.3) for susceptible and infected individual are shown in table 4.2 The table displayed the difference between E1 and E2 of susceptible and infected when we set the neuron range of 10-80. Our proposed model error rate (E2) are minimum than other benchmark error rate E1 for susceptible and infected. However, compared to E1 error rate of susceptible individual, error rate (E2) of susceptible individual is higher in the end. To minimize more this error rate we update the number of neuron range from 90-200 to achieve more good level of accuracy which are clearly shown in table 4.3

Table 4.3 Absolute error graph value when Neuron is 90-200



Absolute error graph value when Neuron is 90-200			
E1 of Susceptible individual	**E2 of Susceptibel individual**	**E1 of infected individual**	**E2 of infected individual**
0.818371	0.031947	0	0.187041
0.326974	0.097248	0.181629	0.027465
0.443984	0.069621	1.326974	0.080377
0.088381	0.047858	1.443984	0.056229
0.088381	0.000039	1.088381	0.006622
0.012847	0.003661	1.012847	0.005817
0.001717	0.001602	1.001717	0.003073
0.000221	0.000215	1.000221	0.012205
0.000027	0.000021	1.000027	0.003131
0.000003	0.000002	1.000003	0.010562

The graphical values of absolute error of figure (4.2) and figure (4.3) for susceptible and infected individual are shown in table 4.3. In table 4.3, we clearly see that the Error (E2) of susceptible and infected individual is more minimize than Error (E1) of susceptible and infected individual.

4.4.1 Benchmark Comparison Result with our Developed model

A benchmark is a previous study or a well-known model that serves as a standard for evaluating the effectiveness and performance of your proposed model. A benchmark model or study article provides a framework for evaluating the performance of your new strategy or model, much like benchmarks in various circumstances. This makes it easier to assess any improvements, new features, or limitations that your suggested model introduces. To determine the high accuracy and reliability of the proposed scheme, we evaluate our proposed model with a benchmark that are shown in table 4.4. The average value of MAE is identified the worth of our propose model. When the neuron range is 10-80 the MAE value of susceptible individual is 0.0574 and infected individual is 0.0825 and the other benchmark value of susceptible and infected individual is MAE value is 0.269 and 0.905. This clearly shows that the average MAE value of our proposed model is minimize as compared to benchmark but in table 4.2 we can see that at end of our model MAE value is increased as compared to benchmark value. So to minimize this error we update the number of neuron range from 90-200. That clearly show that the number of susceptible individual and infected individual MAE value of our proposed model is minimize as compared to the benchmark.

Table 4.4 Benchmark comparison result with our developed model

The average value ofMAE	**When the neuron range is 10-80**		**When the neuron range is 90-200.**	
	Susceptible individual	**Infected individual**	**Susceptibl eindividual**	**Infected individual**
Our proposed model(E2)	0.0574	0.0825	0.0264	0.0224
Benchmark (E1)	0.269	0.905	0.269	0.905

4.6 Summary

In this chapter, the experiments and implementation of the proposed research in this thesis have been discussed. The experimental phase was performed in the Jupiter notebook. The proposed model was tested on a benchmark MAE value and showed better performance as compared to other analytical models and ODE 45 solvers.

CHAPTER 5

CONCLUSION AND FUTURE WORK

5.1 Overview

The principal emphasis of this research was mainly to investigate the problem of minimizing the error to validate the accuracy and reliability of our proposed scheme and provide a solution in the form of an ANN techniques that can solve the SI system and validate the accuracy via MAE. The proposed ANN framework to solve SI system of COVID-19 is to magnify the previous benchmark and enhance them with new techniques and methodology. In this chapter complete summary of the research is provided more briefly in Section 5.2 the significant contributions of this research are summarized Section 5.3 describes certain issues and the directions that can be taken to further enhance the research. The center of this thesis is to design and ANN framework to solve the SI system of COVID-19 nonlinear equation that can provide validate the accuracy via MAE value. Furthermore, the influence of the proposed model is inspected by performing comparison with the available state of the art techniques.

5.2 Summary And Contribution

Using Artificial Neural Networks (ANNs) to solve the susceptible-infected (SI) model for COVID-19 offers a number of important advances, especially when compared to conventional analytical and numerical techniques like ODE-45:

- **ANN to solve the SI system of the non-linear equation**: The SI model of Covid-19 has been resolved using the research's proposed model. To ensure that their predictions closely match the actual data, the artificial neural network has been trained to minimize the Mean absolute error. This improves the chances of the model's limitation. In this study, ANN delivers more accurate modeling than analytical equations and numerical solvers (ODE-45), which could oversimplify certain issues. On the basis of a graphical depiction of absolute error for the susceptible and infected person of Error (E1) and Error (E2), we have gotten results in terms of the accuracy and reliability of our suggested scheme.

- **High accuracy**: The machine learning model's accuracy is a crucial output that reveals the effectiveness and performance of the model. The ReLU artificial neural network has been used for its effectiveness in training and modeling complex relationships of our proposed scheme. The fitness function of the unsupervised error function is used to determine how well the predictions provided by the ANN align with the actual data or the desired outcomes. With the use of an analytical and numerical solver (ODE-45), the MAE is utilized to evaluate the accuracy and reliability of our suggested scheme.

- **Mean absolute error:** We compared our suggested solution with an analytical and numerical solver (ODE-45) with the aid of Errors (E1) and (E2). The absolute error (E2) between the analytical model of the susceptible individual and the artificial neural network is 0.0574, while it is 0.08255 for the infected individual. Between an analytically susceptible individual and Ode-45, the absolute error (E1) is 0.269, while between an analytically infected individual and Ode-45, the absolute error (E1) is 0.9055. It is clearly shown that the mean absolute Error(E2) is greater than the Error(E1) of susceptible and infected individuals. Chapter 4 provides an illustration of how the increase in the number of neurons decreases error further.

5.3 Application

An interesting area for research is the use of Artificial Neural Networks (ANN) to address the Susceptible and Infected (SI) system for COVID-19. ANN is a powerful machine learning method that is excellent at identifying complex correlations and patterns in data. The use of ANN in epidemiological modelling, particularly the SI system for COVID-19, offers up a number of possible benefits and directions for additional research, including:

- **Improved Prediction Accuracy:** When compared to traditional compartmental models like SI, the use of ANN has the potential to improve the accuracy of COVID-19 spread predictions. Its ability to incorporate complex and non-linear connections between variables enables more accurate pandemic dynamics modelling.

- **Through data driven estimation:** The capacity to infer model parameters from empirical data is provided by ANN. This lessens the need for manual parameter adjusting and enables a more data-centric approach to modelling.

- **Model generalization:** The robust generalization and flexibility of ANN to fresh data allows for its implementation across many areas or nations with different COVID-19 dynamics.

- **Incorporate external factor:** The effectiveness of ANN's evaluation of external elements like social isolation policies, immunization rates, and other treatments depends on how well they are integrated.

- **Ensemble method:** Combining numerous ANN models through the use of ensemble techniques improves prediction accuracy and lowers model uncertainty.

- **Analysis of Uncertainties:** Research can look at methods for calculating uncertainties in ANN predictions, which is crucial for epidemiological models where uncertainties are frequent.

- **Short-term and long term prediction:** Both short- and long-term forecasts of the COVID-19 spread may be produced using ANN models, which offers policymakers useful information for resource allocation and decision-making.

- **Real-time forecasting:** Real-time COVID-19 case predictions using ANN models is possible, allowing for prompt interventions and reaction plans.

- **Spatial analysis:** When analyzing spatial data, ANN can take into account regional differences in COVID-19 dynamics.

- **Comparative analysis:** To determine their individual advantages and disadvantages, research might compare ANN models to other machine learning methods or to established epidemiological models.

5.4 Limitation

The presented ANN to solve the SI system of the covid-19 non-linear equation produced ahigh level of accuracy yet it is under certain limitations. This approach is limited to predicting the COVID-19 outcome. These limitations can lead to exploring more directions for further research.

5.5 Future Work

There is no limitation in the area of research. Artificial intelligence is the only field within computer science where academics and researchers have a wide range of pathways and issues to explore. The study that is currently being discussed in this thesis has several potential extensions. Following are these:

- **Making Policies and Allocating Resources:** In response to disease outbreaks, policy makers use the predictions of the SI model to make educated decisions regarding resource allocation, preventative measures, and healthcare planning. This data-driven strategy supports in efficiently prioritizing response efforts and optimizing resource use.

- **Modeling of various infectious disease:** Although infectious diseases like influenza are frequently associated with the SI model, it is flexible in how it may be adapted to examine a wide variety of infectious diseases. Researchers look into and examine diseases like

HIV, measles, TB, and other infectious diseases using this approach.

- **Planning of Global Health:** The SI model plays a crucial role in global health planning because it evaluates the possible worldwide effect of infectious illnesses. It encourages worldwide collaboration and coordination for efficient disease management actions by identifying susceptible zones.

- **Planning for the capacity of the healthcare system**: The SI model, which assists in anticipating the number of infected persons, improves healthcare capacity planning. This preemptive strategy guarantees that hospitals and healthcare institutions are well-equipped to properly handle possible patient surges.

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