A NOVEL STUDY OF DISTANCE BASED SIMILARITY MEASURES ON GLIVIFSESs

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A Novel Study of Distance Based Similarity Measures on GLIVIFSESs

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Candidate of **Master of Science in Mathematics (MS Mathematics)** at the National University of Modern Languages do hereby declare that the thesis **"A Novel Study of Distance Based Similarity Measures on GLIVIFSESs"** submitted by me in partial fulfillment of MS Math's degree, is result of my own research except as cited in references. This thesis has not been submitted or published earlier. I also solemnly declare that it shall not, in the future, be submitted by me for getting any other degree from this or any other university or institution. I also understand that if evidence of plagiarism is found in my thesis at any stage, even after the award of a degree, the work may be canceled and the degree revoked.

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ABSTRACT

A Novel Study of Distance Based Similarity Measures on GLIVIFSESs

Similarity plays an essential rule in pattern recognitions, in image processing and interdisciplinary fields such as statistics, information retrieval and data science. “Generalized linguistic interval valued intuitionistic fuzzy soft expert sets” (GLIVIFSESs) is comprehensive model in fuzzy algebra which allows flexi and more hesitant information in the form of intervals with expert expertise. We developed different types of similarity measures on GLIVIFSESs. Also separately for each similarity measure we constructed practical problems from real world data examples and checked-out the accuracy level of these measures. Behind similarity measures we attempted to apply dissimilarity measure, which plays an essential role in decision making problems. In which we firstly introduced the mathematical expression to measure dissimilarity for GLIVIFSESs and then tested the validity of that dissimilarity measure by considering the practical example related to judgments regarding the authorities of “X” state education department, and we obtained mostly accurate result. After that we used the idea of Entropy and employed it in similarity measurements which provided us comparatively most accurate results. We also introduced the concept of linguistic fuzzy implication for distance measure between GLIVIFSESs and then employed the exports opinions under linguistic fuzzy implication environment and obtained considerable accurate results.

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DEDICATION

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CHAPTER 1

INTRODUCTION

FUZZY, Uncertainty, Irresolute, Vagueness, Unclearity, Hesitancy, doubt in surety, these are common words in real life problems and frequently arises such that classical theories to dealt with problems which are either true or false, wrong or right, good or bad, 1 or 0, so that there are no other possibilities. The basic idea of fuzzy theory was introduced by L.A Zadeh in 1965 [1]. Then after that there are many extensions have been taken place. The best one among that is intuitionistic approach in which Attanasov considered the degree of non-membership at the same time with the degree of membership [2].

Now the linguistic approach to these fuzzy values, or fuzzy information has gained a lot of consideration because usually we prefer a linguistic word i.e. good, bad, very good, excellent, poor etc., to represent or to make opinion on certain quantity rather than a numerical value such that 0, 0.4, 0.25, 1 etc. with 1 is equivalent to excellent and 0 is equivalent to very poor. In linguistic approach basically we have a predefined term set from whom we take linguistic number for making opinion about some quantity with extreme subscript 't' which is a positive integer such that $s_i + s_j \leq s_t$ where $s_i \leq s_t$ & $s_j \leq s_t$ & $s_i, s_j, s_t \in \{s_0, \dots, s_i, \dots, s_j, \dots, s_t\}$. Similarly intuitionistic fuzzy sets we have linguistic intuitionistic fuzzy sets which were derived by Zhang in 2014, in which degree of membership and degree of non-membership both are considered in linguistic terms rather than a numerical numbers. Further extensions of these sets are into interval valued linguistic intuitionistic fuzzy sets. In linguistic approach there is concept of 1-D and 2-D which is used to fulfill the requirements of multiple attribute group decision making process. In which 1-D basically represents the decision maker's assessment values while 2-D represent the decision maker's validity on his/her assessment value or in other words 2-D represents the decision maker's knowledge or expertise about assassinator. The linguistic approach is as compared to numerical values is more reliable and flexible because in linguistic term we can store more than one numerical value. For instance consider the linguistic term “good”, such that this word as different approach according to different experts. For example one consider 0.7 as good number, the other one consider 0.8 as a good number and the other one considers 0.6 as a good number, so that the linguistic term “good” covers the range $0.6 \leq good \leq 0.8$ such that this approach shows flexi behavior rather than a fixed numerical quantity.

There are several extensions have been taken place in terms of linguistic approach such as interval valued intuitionistic fuzzy variables, in which both the degree of membership and degree of non-membership are considered in intervals in linguistic terms such that there is a flexibility in the choice of linguistic term selection or in other words the interval shows the hesitancy of expert for assigning the linguistic term to an alternative. Now the problem was to aggregate all the opinions given by different experts to obtain a final, competent, reliable and reality based result. For that there are many choices such as arithmetic mean, geometric mean, harmonic mean but the most real one was given by Yager [3] “an ordered weighted averaging aggregation operator” in which the opinions of experts are arranged in ascending order and then multiplied with weighting vector, in which associated weights are multiplied with expert values based on the value provided by the expert.

Later on there are many extensions have been taken place in ordered weighted averaging operators and used in the case of interval valued fuzzy sets, interval valued intuitionistic fuzzy sets and so on. The approach of ordered weighted averaging aggregation operators concludes the different linguistic variables into a single one variable. Now for all experts with same attributes for specific alternative and these calculations the decision making problem is then solved with the help of score and accuracy functions, such that if the value of score function of specific linguistic variable which represents the specific alternative is greater than the score function values of other linguistic variables then the most appropriate choice will be that alternative, and also if value of score function is greater or equal to other linguistic variables values then the value of accuracy function for it must be greater or equal than the other variables, and if the value of score function for two different linguistic variables will be same then the distinction will be made on the behalf of value of accuracy function.

Now we used the model "Generalized linguistic interval valued intuitionistic fuzzy soft expert sets" (GLIVIFSESs) [23] to define similarity measures on them. Basically generalized comes from the concept of accuracy, accuracy in terms of measuring or finding the most appropriate alternative. For that purpose we increase the number of dimensions for most relevant and realistic judgment. For instance if we select a team of three persons to select the proposal from "n" proposals which has most diversities or flexibilities for innovative work in the field fuzzy algebra, so for that purpose in 1-dimension we take their opinion regarding the degree of membership and degree of non-membership and for reality purposes we take 2-D in which the decision maker's express their knowledge about fuzzy algebra in linguistic terms, and so on we can add different factors by adding higher order dimensions to enhance the accuracy in selection process. In the study of similarity measures there are lot of choices are available in which some are similarity measures and some are distance based similarity measures. The similarity and distance based similarity measures are basically used to find the similarity index between two different structures which can be used further in many dimensions such as in pattern recognition, in face recognition and so on.

Firstly we take into account Type-I similarity measure [19, 22] and modify for under-study structure which is based on supremum and infimum properties. In which we separately calculate the infimum's w.r.t different criteria's and experts for each alternative and then find the supremum of all these obtained infimum's, similarly for all other alternatives and then calculate their average. On the same points we imply Type-II similarity measure [19] with shifting of infimum operation in the place of supremum and supremum on the place of infimum (in the way of struggling) to enhance the accuracy in results. In Type-III similarity measure [19] we take into account the concept of intersection and union of under-study structure. Such that for each alternative we calculate intersection and divide it by union of same sets with specific criteria and specific expert and then done it for all with at the end sum-up for all criteria's and after that for all experts, then finally we take the average of these similarity results to get the similarity between the opinions of experts over all the alternatives.

Type-IV similarity measures in which we propose a new similarity measure based on the idea from Type-III, such that in Type-III while taking intersection and union of two different under-study structures we convert the linguistic terms with membership values as an interval into a constant numerical number by taking the average value of extreme ends of interval which violates the structure of under-study structure. So in Type-IV to remove that violence we take the intervals as it is and then use the properties of fuzzy intervals for union and intersection. From Type-III similarity measure we make Type-V similarity measure from a point that linguistic terms in case of linguistic intuitionistic fuzzy sets have no relation with fuzzy interval such that the second term may be lower in order than the first-one with the property that $s_a + s_b \leq s_t$. Further to reduce the inaccuracy in similarity results between the opinions of experts we take in modified Type-V similarity measure only the intersections of under-study structures and then calculate their sum to find the similarity between opinions for specific alternative and then for collectively all alternatives we calculates similarities results(for specific alternatives) average. From the idea of entropy, which is used to measure the fuzziness of fuzzy sets, we modify the entropy measure for interval-valued intuitionistic fuzzy sets [4] to GLIVIFSESs and then calculate entropy based similarity measure for under-study structure. From [6] we take the idea of dissimilarity which was implied in [6] on intuitionistic fuzzy sets, which is based on the idea of differences between the opinions of experts. Here in case of under-study structure we extend that measure for such structure for specific proposals w.r.t different criteria's and apply it on practical example to see the reliability of that dissimilarity measure. As correlation represents a relationship or in other words a similarity measure [7], so we employed correlation to measure the similarity between different structures of GLIVIFSESs. We use [8, 9] to extend the idea of similarity measure on under-study structure using the operation of max-min for that structure.

Then we modify that max-min similarity measure by bringing the change in statement of similarity measure to enhance the accuracy in similarity results. Further we use classical and fuzzy implications [10] and extend these implications for linguistic cases & use these fuzzy implications with matrix norms and [13, 14] to measure the distance between under-study structures. After that we discus about distance measures [16, 17] and extend these distance measures for linguistic case and then generalizes for under-study structure and call these as modified hamming and Euclidean distances. On the basis of these modified distances we apply similarity measures on under-study structure and compare results with practical examples.

CHAPTER 2

LITERATURE REVIEW

In this chapter we will discuss about the previous work in the field of fuzzy algebra done up to that time. In which we take into account mainly the similarity measures proposed by different researchers in case of fuzzy expressions in the of form numerical quantities and then come to main point "linguistic approach" towards fuzzy information and mainly discuss about under-study structure also briefly consider into account linguistic approach extensions from simple fuzzy set towards under-study structure.

In literature the first start of fuzzy algebra/fuzzy mathematics was initiated by L.A Zadeh [1] in which he introduced the idea of "uncertainty for selection". Then after that it gains popularity by many researchers also by many well reputed agencies/companies and lot of extensions of fuzzy set theory has been taken place but the most popularity gained by "intuitionistic fuzzy theory" [20, 21, 2] in which K.T Attanasov considered about "uncertainty for rejection" at the same time with uncertainty for selection. Then after that Molodtsov [28] introduced the concept of softness in fuzzy structures by introducing the concept of parameters/attributes/criteria's such that for the purpose of assign a belongingness sign or non-belongingness the judgment will be based on criteria's/parameters related to the ideal structure, i.e. in the selection of valid university the parameters will be research excellency, competency of teachers, fee structure and so on. Then V.Torra [29] discussed about hesitancy of fuzzy sets and calls them as hesitant fuzzy sets, such that in these sets we take hesitancy as also a part of fuzzy information or in other words what amount of lack of knowledge present in selection or rejection of specific individual.

Further for a case of relation/link between fuzzy sets, here study of relations between fuzzy sets has great importance due to its applications in many fields, the major one's are medical diagnosis, data mining's, assigning a preference [30], and so on. Bustince and Burillo [31] introduced the entropy for intuitionistic fuzzy sets and interval-valued intuitionistic fuzzy sets to measure the fuzziness of these sets, which plays an essential role in measuring the relationship because when we are sure about fuzziness of two different individuals we can easily draw their similar portion. Ismat beg and Samina Ashraf [32] discussed about similarity measures for fuzzy sets and proposed a new axiomatic similarity measure and discuses about the relevance of these new proposed axioms with classical/already-present axioms. Li Yingfang [33] studied about interval-valued fuzzy sets and presented similarity measure for measuring similarity between IVFSs, here IVFSs have an over advantage on fuzzy sets that these covers information if decision makers have unclearity in the form of intervals.

Deqing Li [34] discussed about hesitant fuzzy sets and presented some new distance and similarity measures for hesitant fuzzy sets with comparison to classical similarity measures and applies them to pattern recognition problem. Here in measures we have distance measures and similarity measures, while we can draw similarity measures on the behalf of distance measures such that if two sets are same then there will be no distance between them but their similarity will be the perfect value so, by subtracting that similarity value from perfect value we get the distance measure and vice versa. Chong Wu [35] discussed about interval-valued intuitionistic fuzzy sets along with hesitancy degree and presented new similarity measure on the basis of entropy measure and hesitancy degree and then employ that similarity measure to expert system for pattern recognition problems.

Expert system is demanding research field due to its vast use in practical problems because we are facing a lot of situations in real life in which we have to make decisions for appropriate choice from the multiple choices set and expert systems plays fundamental role for appropriate choice or in other words we can say that expert system plays a role just like a program in which we substitute different values and after operation defined in it we get the result which meets our needs.

Now in case of linguistic approach towards fuzzy theory, because some times in real-world situations it is not possible to depict the information in quantitative way i.e. to describe the patient condition after taking medicine will be in form of, well, bed, excruciating and so on. Zadeh [36] firstly introduced the concept of linguistic variables, which basically reflects the fact that humans reasoning or selection is not an exact value but an approximation and the values in these variables are words not a numbers. Zhang [37] proposed from linguistic fuzzy sets, linguistic intuitionistic fuzzy sets for the purpose of better dealing with unsurely information, along with the usage of t-norm and t-conorm some aggregation operators to aggregate linguistic intuitionistic fuzzy values and uses that structure in multiple attribute group decision making problems. Since the decision making process with the help of linguistic approach is good but at the same time the selection of linguistic term may be biased, may be inappropriate due to the weaknesses of experts in selecting the appropriate choices which leads to non-negligible inappropriate results. To overcome that drawback Zhu et al [38] give the idea of 2-D, in which decision makers have to give opinions in the form of 2-DLVs in which 1-D represents judgments and the second dimension (2-D) represents reliability of judgmental results or in other words it represents the expertise of decision maker, i.e. the innovator as an expert give the judgment as "excellent" and in familiarity "perfectly familiar".

Yu et al [39] further discussed on 2-DL and used concept of triangular fuzzy number to distinct from 1-DL and developed weighted averaging and ordered weighted averaging aggregation operators for 2-DL information and apply them in MCDM problems. As from linguistic intuitionistic fuzzy information we have a degree of freedom in selection of best alternative but its drawback is its lack of ability to take into account the reliability of experts, on the other hand 2-DL information, it only gives information on reliability of expert's opinions. So the both representations have limitations, to overcome these limitations Verma et al [3] presented the hybrid model containing both linguistic intuitionistic and 2-D linguistic fuzzy information structures and call it as 2-DLIFVs. They also introduced the operational laws for that hybrid structure, also with some aggregation operators which include 2-D linguistic intuitionistic fuzzy weighted averaging operator, 2-D linguistic intuitionistic fuzzy ordered weighted averaging operator, 2-D linguistic intuitionistic fuzzy weighted geometric operator, 2-D linguistic intuitionistic fuzzy ordered weighted geometric operator. With the help of averaging and ordered averaging operator they introduced the idea of 2-D linguistic intuitionistic fuzzy hybrid averaging operator, similarly from weighted geometric and ordered weighted geometric operator they introduced 2-D linguistic intuitionistic fuzzy hybrid geometric operator. Also they employ that new proposed 2-D structure in multi-criteria group decision making (MCGDM) problems and use the proposed aggregated results for aggregating the 2-DLIFVs.

Later on Tasaduq & Afshan Qayyum [23] studied the linguistic approach and by adopting the fact that dimensions order impacts the results, they used the term "generalized" for the order of dimensions. Also from linguistic intuitionistic fuzzy variables, due to the hesitancy of decision makers for exactly in the choice of specific linguistic term such that linguistic intuitionistic variables limitation in case of hesitancy in choice, they introduced the idea of intervals in linguistic prospective. So that it allows more flexibility in the choice of linguistic terms even judgments based on lack of surety. At the same time they take into account the concept of soft sets and its generalization into soft expert sets, such that introduced the concept of parameters with the selection of IVIFVs, which also improves the reliability of resulted information. In combing these proposed advancements, they call as "generalized linguistic interval-valued intuitionistic fuzzy soft expert sets".

CHAPTER 3

PRELIMINARIES

In this chapter we will consider some basic definitions about fuzzy sets, interval-valued fuzzy sets, intuitionistic fuzzy sets, interval-valued intuitionistic fuzzy sets, cubic soft expert sets which were presented for numerical data, defining definition of Type-I and Type-II similarity measures under cubic sets. Also discuss these under linguistic approach, finally discuss under-study structure.

Definition [1]

Suppose $X = \{x_i : i = 1,2,3, \dots, n\}$ represents a set of discourse and $A \subseteq X$ mathematically defined as

$$A = \{\langle x, \alpha_A(x)\rangle : x \in X \;\&\; \alpha_A(x) \in [0,1]\}$$

known as fuzzy set.

Definition [1]

Suppose $X = \{x_i : i = 1,2,3, \dots, n\}$ represents a set of discourse and $B \subseteq X$ mathematically defined as

$$B = \{\langle x, [\alpha_B(x), \alpha'_B(x)]\rangle : x \in X \;\&\; \alpha_B(x), \alpha'_B(x) \in [0,1] \; s.t \; \alpha_B(x) \leq \alpha'_B(x)\}$$

known as interval-valued fuzzy set.

Definition [20]

Suppose $X = \{x_i : i = 1,2,3, \dots, n\}$ represents a set of discourse and $C \subseteq X$ mathematically defined as

$$C = \{\langle x, \alpha_C(x), \beta_C(x)\rangle : x \in X \;\&\; \alpha_C(x), \beta_C(x) \in [0,1] \; s.t \; \alpha_C(x) + \beta_C(x) \leq 1\}$$

here $\alpha_C(x)$ represents membership function and $\beta_C(x)$ represents non-membership function. The above defined set is known as intuitionistic fuzzy set.

Definition [2]

Suppose $X = \{x_i : i = 1,2,3, \dots, n\}$ represents a set of discourse and $D \subseteq X$ mathematically defined as

$$D = \{\langle x, [\alpha_D(x), \alpha'_D(x)], [\beta_D(x), \beta'_D(x)] \rangle : x \in X \;\&\; \alpha_D(x), \alpha'_D(x), \beta_D(x), \beta'_D(x) \in [0,1] \;\&$$
$$\alpha_D(x) \leq \alpha'_D(x), \beta_D(x) \leq \beta'_D(x) \; s.t \; \alpha'_D(x) + \beta'_D(x) \leq 1\}$$

known as interval-valued intuitionistic fuzzy set.

Definition [19]

Suppose $X = \{x_i : i = 1,2,3, \dots, n\}$ represents a set of alternatives with $F \subseteq X$ and $E = \{e_j : j = 1,2,3, \dots, m\}$ represents a set of experts and $C = \{c_a : a = 1,2,3, \dots, w\}$ represents a set of criteria's then

$$F = \{\langle x, \alpha_F(x), [\alpha_{1_F}(x), {\alpha_1}'_F(x)] \rangle : x \in X \;\&\; \alpha_F(x), \alpha_{1_F}(x), {\alpha_1}'_F(x) \in [0,1]\}$$

known as cubic soft expert set, which is basically a combination of fuzzy set and interval-valued fuzzy set.

Definition [22]

Suppose $X = \{x_i : i = 1,2,3, \dots, n\}$ represents a set of alternatives and $E = \{e_j : j = 1,2,3, \dots, m\}$ represents a set of experts and $C = \{c_a : a = 1,2,3, \dots, w\}$ represents a set of criteria's and R & F are cubic soft expert sets of X then

$$S(R, F) =$$
$$\frac{1}{n}\sum_{i=1}^{n}\left[max\left[\sum_{j=1}^{m}\sum_{a=1}^{w} min\left\{\frac{\alpha_{1F}(x)+{\alpha_1}'_F(x)-\alpha_{1F}(x){\alpha_1}'_F(x)}{3}, \frac{\alpha_{1R}(x)+{\alpha_1}'_R(x)-\alpha_{1R}(x){\alpha_1}'_R(x)}{3}, \alpha_F(x), \alpha_R(x)\right\}\right]\right],$$
$$\frac{1}{n}\sum_{i=1}^{n}\left[max\left[\sum_{j=1}^{m}\sum_{a=1}^{w} min\left\{\frac{\alpha_{1F}(x)+{\alpha_1}'_F(x)-\alpha_{1F}(x){\alpha_1}'_F(x)}{3}, \frac{\alpha_{1R}(x)+{\alpha_1}'_R(x)-\alpha_{1R}(x){\alpha_1}'_R(x)}{3}, \alpha_F(x), \alpha_R(x)\right\}\right]\right]$$

known as Type-I & Type-II similarity measures for cubic soft expert sets respectively.

Definition [37]

Suppose $X = \{x_i : i = 1,2,3, \dots, n\}$ represents universe of discourse and $S = \{s_r : s_0 \leq s_r \leq s_t, \&t \in 2\mathbb{Z}\}$ represents a set of continuous linguistic terms, then

$$A = \{\langle x, s_i, s_j \rangle : x \in X \;\& s_i, s_j \in S\}$$

known as linguistic intuitionistic fuzzy set with s_i and s_j respectively represents degree of membership and non-membership.

Definition [38]

Suppose $X = \{x_i : i = 1,2,3,\dots,n\}$ represents universe of discourse and $S = \{\dot{s}_r : \dot{s}_0 \leq \dot{s}_r \leq \dot{s}_t, \& t \in 2\mathbb{Z}\}$ represents a set of continuous linguistic terms for the choice of linguistic terms in judgments. Also let $S^{\circ} = \{\ddot{s}_u : \ddot{s}_0 \leq \ddot{s}_u \leq \ddot{s}_{t'}, \& t' \in 2\mathbb{Z}\}$ which represents continuous linguistic terms for appropriate selection of reliability, then

$$B = \{\langle x, \dot{s}_i \rangle, \langle x, \ddot{s}_k \rangle : x \in X \;\&\; \dot{s}_i \in S, \ddot{s}_k \in S^{\circ}\}$$

known as 2-D linguistic fuzzy set with $(\dot{s}_i, \ddot{s}_k)$ respectively represents degree of membership.

Definition [3]

Suppose $X = \{x_i : i = 1,2,3,\dots,n\}$ represents universe of discourse and $S = \{\dot{s}_r : \dot{s}_0 \leq \dot{s}_r \leq \dot{s}_t, \& \, t \in 2\mathbb{Z}\}$ represents a set of continuous linguistic terms for the choice of linguistic terms in judgments. Also let $S^{\circ} = \{\ddot{s}_u : \ddot{s}_0 \leq \ddot{s}_u \leq \ddot{s}_{t'}, \& t' \in 2\mathbb{Z}\}$ which represents continuous linguistic terms for appropriate selection of reliability, then

$$C = \{\langle x, \dot{s}_i, \dot{s}_j \rangle, \langle x, \ddot{s}_k, \ddot{s}_l \rangle : x \in X \;\&\; \dot{s}_i, \dot{s}_j \in S, \ddot{s}_k, \ddot{s}_l \in S^{\circ}\}$$

known as 2-D linguistic intuitionistic fuzzy set with $(\dot{s}_i, \ddot{s}_k)$ and $(\dot{s}_j, \ddot{s}_l)$ respectively represents degree of membership and non-membership.

Definition [3]

Let

$$S_1 = \{\langle \dot{s}_\alpha, \dot{s}_\beta \rangle, \langle \ddot{s}_\gamma, \ddot{s}_\delta \rangle\}$$
$$S_2 = \{\langle \dot{s}_{\alpha'}, \dot{s}_{\beta'} \rangle, \langle \ddot{s}_{\gamma'}, \ddot{s}_{\delta'} \rangle\}$$

be two different generalized linguistic intuitionistic fuzzy soft sets, the score function for them is as under

$$S(S_1) = S_{\left(\frac{t+\alpha-\beta}{2t}\right) \times \left(\frac{t'+\gamma-\delta}{2t'}\right)}$$

and the accuracy function is defined as under

$$H(S_1) = S_{\left(\frac{\alpha+\beta}{t}\right) \times \left(\frac{\gamma+\delta}{t'}\right)}$$

With following properties

i. If $S(S_1) > S(S_2)$ then $S_1 > S_2$

ii. If $S(S_1) = S(S_2)$ and $H(S_1) > H(S_2)$ then $S_1 > S_2$

iii. If $S(S_1) = S(S_2)$ and $H(S_1) = H(S_2)$ then $S_1 = S_2$

Definition [23]

Suppose $X = \{x_i: i = 1,2,3, \dots, n\}$ represents universe of discourse and $E = \{e_j: j = 1,2,3, \dots, m\}$ represents a set of experts and $C = \{c_a: a = 1,2,3, \dots, w\}$ represents a set of criteria's and $S = \{\dot{s}_\alpha: \dot{s}_0 \leq \dot{s}_\alpha \leq \dot{s}_{\alpha'} \leq \dot{s}_t, \& \, t \in 2\mathbb{Z}\}$ represents a set of continuous linguistic terms for the choice of linguistic terms in judgments. Also let $S^{\circ} = \{\ddot{s}_\gamma: \ddot{s}_0 \leq \ddot{s}_\gamma \leq \ddot{s}_{\gamma'} \leq \ddot{s}_{t'}, \& \, t' \in 2\mathbb{Z}\}$ which represents continuous linguistic terms for appropriate selection of reliability, then

$$Y = \{\langle x, \dot{s}_{[\alpha,\alpha']}, \dot{s}_{[\beta,\beta']}\rangle, \langle x, \ddot{s}_{[\gamma,\gamma']}, \ddot{s}_{[\delta,\delta']}\rangle : x \in X \,\&\, \dot{s}_\alpha, \dot{s}_{\alpha'}, \dot{s}_\beta, \dot{s}_{\beta'} \in S, \ddot{s}_\gamma, \ddot{s}_{\gamma'}, \ddot{s}_\delta, \ddot{s}_{\delta'} \in S^{\circ}\}$$

known as generalized linguistic interval-valued intuitionistic fuzzy soft expert set, with the condition that $\dot{s}_{\alpha'} + \dot{s}_{\beta'} \leq \dot{s}_t$ and $\ddot{s}_{\gamma'} + \ddot{s}_{\delta'} \leq \ddot{s}_{t'}$ where $\{\dot{s}_{[\alpha,\alpha']}, \ddot{s}_{[\gamma,\gamma']}\}$ and $\{\dot{s}_{[\beta,\beta']}, \ddot{s}_{[\delta,\delta']}\}$ respectively represents degree of membership and degree of non-membership.

CHAPTER 4

SIMILARITY MEASURES ON GENERALIZED LINGUISTIC INTERVAL VALUED INTUITIONISTIC FUZZY SOFT EXPERT SETS (GLIVIFSESs)

In this chapter we will discuss/extend/propose similarity measures on under-study structure and check their validity by considering different practical problems examples. Also we consider a common practical problem example for comparison reasons between different similarity measures results. In doing so we firstly propose Type-I similarity measure over under-study structure and for validity of results obtained after that measure we generally consider a problem of finding the similarities between the opinions of experts/students against their teachers. Further we propose Type-II similarity measure over under-study structure and firstly compare similarity result by taking Example 4.1.1 and then consider a practical problem regarding the judgment of employees. Then we propose Type-III similarity measure and constructed a practical problem example regarding the recruitment of competitive teachers to bring into the mind of students about creativity rather than a usual process of just memorize the course contents and promote to the next level with high grades. Further we propose Type-IV similarity measure and compare these with Type-I, II under the data of Example 4.1.1, also constructed a practical problem example for resolving the issue of allocation budget to a specific department on the behalf of its performance results.

Similarly later on we propose Type-V and its extension as Modified Type-V similarity measure from Type-III similarity measure also compare these results by using Example 4.1.1 And constructed practical problem example regarding the issuance of mining certificate to do mining in specific areas of Balochistan and compare these two similarity measure results. Finally, in that section we propose Max-Min and Modified Max-Min similarity measure for under-study structure, also compare these similarities results with previous ones under Example 4.1.1 data, also we constructed practical problem example regarding selection of appropriate candidates for Hungarian-Stipend to Pakistani students for study in Hungry specified/sponsored institutions.

4.1. TYPE-I SIMILARITY MEASURE FOR GLIVIFSESs.

Definition. Let $S_t = \{S_0, S_1, S_2, \; . \; . \; . \; , S_t\}$ be a linguistic term set which is predefined, where $'t'$ is any positive integer with even cardinality such that

$$\mathrm{S}_1=\left\{\langle \dot{\mathrm{s}}_{[\alpha,\alpha']}, \dot{\mathrm{s}}_{[\beta,\beta']}\rangle \;, \langle \ddot{\mathrm{s}}_{[\gamma,\gamma']} \,, \ddot{\mathrm{s}}_{[\delta,\delta']}\rangle\right\},$$

which represents the generalized (specifically we taken 2-D) linguistic interval valued intuitionistic fuzzy soft expert set, similarly we have

$$\mathrm{S}_2=\left\{\langle \dot{\mathrm{s}}_{[\alpha,\alpha']}, \dot{\mathrm{s}}_{[\beta,\beta']}\rangle \;, \langle \ddot{\mathrm{s}}_{[\gamma,\gamma']} \,, \ddot{\mathrm{s}}_{[\delta,\delta']}\rangle\right\}.$$

Let $\mathrm{SM}(\mathrm{S}_1, \mathrm{S}_2)$ represents the Type-I similarity measure between these two sets, to calculate the similarity measure between them we have to calculate the similarity measure between each correspondence. Such that we have to find the similarity measure between the linguistic sets for a specific proposal with certain criteria from which S_1 and S_2 obtained by applying aggregation operators, for which we consider the proposal's be a finite set $\mathrm{P} = \{\mathrm{p_i}: \mathrm{i} = 1, . \;. \;. \;. \;\;, \mathrm{m}\}$ and the criteria for membership is again a finite set $\mathrm{C} = \left\{\mathrm{c_j}: \mathrm{j} = 1, . \;. \;. \;. \;\;, \mathrm{n}\right\}$ and the number of judges for appropriate judgment we again consider a set $\mathrm{J} = \{\mathrm{j_k}: \mathrm{k} = 1 \,, . \;. \;. \;. \;\;, \mathrm{o}\}$ where $\mathrm{m}, \mathrm{n}, \mathrm{o} \in \mathbb{Z}$ with the possibility of equality. Then we take $S_i(\mathrm{c_j}, \mathrm{j_k})$ as ith similarity measure based on the proposal $\mathrm{p_i}$ which is defined as follows

$$
\begin{gathered}
S_i(\mathrm{S}_1, \mathrm{S}_2, \ldots, \mathrm{S}_o) = \\
\sup\left[\begin{array}{c}
\inf\left[\begin{array}{c}
\left(\left\langle \dot{s}_1(c_j, j_k)_{\left[\frac{\alpha+\alpha'-\frac{\alpha\alpha'}{t}}{3}\right]}, \dot{s}_1(c_j, j_k)_{\left[\frac{\frac{\beta\beta'}{t}}{3}\right]}\right\rangle, \left\langle \ddot{s}_1(c_j, j_k)_{\left[\frac{\gamma+\gamma'-\frac{\gamma\gamma'}{t'}}{3}\right]}, \ddot{s}_1(c_j, j_k)_{\left[\frac{\frac{\delta\delta'}{t'}}{3}\right]}\right\rangle\right), \\
\left(\left\langle \dot{s}_2(c_j, j_{k+1})_{\left[\frac{\alpha+\alpha'-\frac{\alpha\alpha'}{t}}{3}\right]}, \dot{s}_2(c_j, j_{k+1})_{\left[\frac{\frac{\beta\beta'}{t}}{3}\right]}\right\rangle, \left\langle \ddot{s}_2(c_j, j_{k+1})_{\left[\frac{\gamma+\gamma'-\frac{\gamma\gamma'}{t'}}{3}\right]}, \ddot{s}_2(c_j, j_{k+1})_{\left[\frac{\frac{\delta\delta'}{t'}}{3}\right]}\right\rangle\right), \ldots, \\
\left(\left\langle \dot{s}_o(c_j, j_o)_{\left[\frac{\alpha+\alpha'-\frac{\alpha\alpha'}{t}}{3}\right]}, \dot{s}_o(c_j, j_o)_{\left[\frac{\frac{\beta\beta'}{t}}{3}\right]}\right\rangle, \left\langle \ddot{s}_o(c_j, j_o)_{\left[\frac{\gamma+\gamma'-\frac{\gamma\gamma'}{t'}}{3}\right]}, \ddot{s}_o(c_j, j_o)_{\left[\frac{\frac{\delta\delta'}{t'}}{3}\right]}\right\rangle\right)
\end{array}\right], \\
\inf\left[\begin{array}{c}
\left(\left\langle \dot{s}_1(c_{j+1}, j_k)_{\left[\frac{\alpha+\alpha'-\frac{\alpha\alpha'}{t}}{3}\right]}, \dot{s}_1(c_{j+1}, j_k)_{\left[\frac{\frac{\beta\beta'}{t}}{3}\right]}\right\rangle, \left\langle \ddot{s}_1(c_{j+1}, j_k)_{\left[\frac{\gamma+\gamma'-\frac{\gamma\gamma'}{t'}}{3}\right]}, \ddot{s}_1(c_{j+1}, j_k)_{\left[\frac{\frac{\delta\delta'}{t'}}{3}\right]}\right\rangle\right), \ldots, \\
\left(\left\langle \dot{s}_o(c_{j+1}, j_o)_{\left[\frac{\alpha+\alpha'-\frac{\alpha\alpha'}{t}}{3}\right]}, \dot{s}_o(c_{j+1}, j_o)_{\left[\frac{\frac{\beta\beta'}{t}}{3}\right]}\right\rangle, \left\langle \ddot{s}_o(c_{j+1}, j_o)_{\left[\frac{\gamma+\gamma'-\frac{\gamma\gamma'}{t'}}{3}\right]}, \ddot{s}_o(c_{j+1}, j_o)_{\left[\frac{\frac{\delta\delta'}{t'}}{3}\right]}\right\rangle\right)
\end{array}\right], \ldots, \\
\inf\left[\begin{array}{c}
\left(\left\langle \dot{s}_1(c_n, j_k)_{\left[\frac{\alpha+\alpha'-\frac{\alpha\alpha'}{t}}{3}\right]}, \dot{s}_1(c_n, j_k)_{\left[\frac{\frac{\beta\beta'}{t}}{3}\right]}\right\rangle, \left\langle \ddot{s}_1(c_n, j_k)_{\left[\frac{\gamma+\gamma'-\frac{\gamma\gamma'}{t'}}{3}\right]}, \ddot{s}_1(c_n, j_k)_{\left[\frac{\frac{\delta\delta'}{t'}}{3}\right]}\right\rangle\right), \ldots, \\
\left(\left\langle \dot{s}_o(c_n, j_o)_{\left[\frac{\alpha+\alpha'-\frac{\alpha\alpha'}{t}}{3}\right]}, \dot{s}_o(c_n, j_o)_{\left[\frac{\frac{\beta\beta'}{t}}{3}\right]}\right\rangle, \left\langle \ddot{s}_o(c_n, j_o)_{\left[\frac{\gamma+\gamma'-\frac{\gamma\gamma'}{t'}}{3}\right]}, \ddot{s}_o(c_n, j_o)_{\left[\frac{\frac{\delta\delta'}{t'}}{3}\right]}\right\rangle\right)
\end{array}\right]
\end{array}\right] \quad (4.1)
\end{gathered}
$$

here "t'" represents the length of term set for 2-D linguistic approach and "t" represents length of term set for 1-D. Also ‘i’ represents a specific proposal such that it varies from 0 to ‘m’ and by using the above equation we have to find $S_i(\mathrm{S}_1, \mathrm{S}_2)$ for each ‘i’.

After calculating all these values we obtain a Type-I similarity measure by using the following equation

$$\mathrm{SMS_1, S_2, \ldots, S_o}) = \frac{\sum_{\mathrm{i=0}}^{\mathrm{m}} S_i(\mathrm{S_1, S_2, \ldots, S_o})}{\mathrm{m}} \rightarrow (\mathrm{A}).$$

To better understanding the above methodology we will consider data from practical example to demonstrate the above presented methodology.

EXAMPLE 4.1.1.

Consider two students $\{a, b\}$ from a class are selected (here we take these two students as judges) they are allowed to characterized your teachers $\{t_1, t_2\}$ on the behalf of following two habit's

- Loyalty
- Humbleness

And we form a set of criteria's $\{h_1, h_2\}$ by assigning

$h_1 =$ Loyalty, $h_2 =$ Humbleness

Now, on the behalf of students experience with teachers, length of time spend with them we increase the order of dimension from 1-D to 2-D with term sets are as under

$$S_t = \{S_0 = poor, S_1 = very\ bad, S_2 = bad, S_3 = average, S_4 = above\ average(good), S_5 = very\ good, S_6 = excellent\}$$

$$S_{t'} = \{S_0 = new\ student(No\ time\ spend), S_1 = little\ time, S_2 = from\ last\ year, S_3 = long\ time, S_4 = living\ with\ them\}.$$

The general form a result obtained after their evaluation will be of the form

$$\mathrm{S_1}=\left\{\langle \dot{\mathrm{s}}_{[\alpha,\alpha']}, \dot{\mathrm{s}}_{[\beta,\beta']}\rangle\ , \langle \ddot{\mathrm{s}}_{[\gamma,\gamma']}\ , \ddot{\mathrm{s}}_{[\delta,\delta']}\rangle\right\}$$

and

$$\mathrm{S_2}=\left\{\langle \dot{\mathrm{s}}_{[\alpha,\alpha']}, \dot{\mathrm{s}}_{[\beta,\beta']}\rangle\ , \langle \ddot{\mathrm{s}}_{[\gamma,\gamma']}\ , \ddot{\mathrm{s}}_{[\delta,\delta']}\rangle\right\}.$$

We consider the evaluation of student 'a' as $\mathrm{S_1}$ and the evaluation of student 'b' with $\mathrm{S_2}$. Now we wanted to find out the Type-I similarity measure between these above Generalized Linguistic Interval-valued Intuitionistic Fuzzy soft expert sets. For that first we will find out,

$$
\sup\left[\begin{array}{l}
\inf\left\{\begin{array}{l}
\left(\left\langle \dot{s}_1(h_1,a)_{\left[\frac{2+3-\frac{6}{6}}{3}\right]}, \dot{s}_1(h_1,a)_{\left[\frac{\frac{1\times 2}{6}}{3}\right]}\right\rangle, \left\langle \ddot{s}_1(h_1,a)_{\left[\frac{1+2-\frac{2}{4}}{3}\right]}, \ddot{s}_1(h_1,a)_{\left[\frac{\frac{1\times 2}{4}}{3}\right]}\right\rangle\right), \\
\left(\left\langle \dot{s}_2(h_1,b)_{\left[\frac{3+4-\frac{12}{6}}{3}\right]}, \dot{s}_2(h_1,b)_{\left[\frac{\frac{1\times 2}{6}}{3}\right]}\right\rangle, \left\langle \ddot{s}_2(h_1,b)_{\left[\frac{1+2-\frac{2}{4}}{3}\right]}, \ddot{s}_2(h_1,b)_{\left[\frac{\frac{1\times 2}{4}}{3}\right]}\right\rangle\right)
\end{array}\right\} \\
\inf\left\{\begin{array}{l}
\left(\left\langle \dot{s}_1(h_2,a)_{\left[\frac{1+3-\frac{3}{6}}{3}\right]}, \dot{s}_1(h_2,a)_{\left[\frac{\frac{2\times 3}{6}}{3}\right]}\right\rangle, \left\langle \ddot{s}_1(h_2,a)_{\left[\frac{1+2-\frac{2}{4}}{3}\right]}, \ddot{s}_1(h_2,a)_{\left[\frac{\frac{1\times 2}{4}}{3}\right]}\right\rangle\right), \\
\left(\left\langle \dot{s}_2(h_2,b)_{\left[\frac{1+2-\frac{2}{6}}{3}\right]}, \dot{s}_2(h_2,b)_{\left[\frac{\frac{3\times 4}{6}}{3}\right]}\right\rangle, \left\langle \dot{s}_2(h_2,b)_{\left[\frac{1+2-\frac{2}{4}}{3}\right]}, \ddot{s}_2(h_2,b)_{\left[\frac{\frac{1\times 2}{4}}{3}\right]}\right\rangle\right)
\end{array}\right\}
\end{array}\right],
$$

this implies after simplification and finding infimum between them we get

$$
sup\left\{\left(\left\langle \dot{s}_{\frac{1}{3}}, \dot{s}_{\frac{1}{9}}\right\rangle, \left\langle \ddot{s}_{\frac{2.5}{3}}, \ddot{s}_{\frac{1}{6}}\right\rangle\right), \left(\left\langle \dot{s}_{\frac{8}{9}}, \dot{s}_{\frac{2}{3}}\right\rangle, \left\langle \ddot{s}_{\frac{2.5}{3}}, \ddot{s}_{\frac{1}{6}}\right\rangle\right)\right\},
$$

which implies that the similarity between the opinions of students in case of teacher t_1

$$
=\left(\left\langle \dot{s}_{\frac{8}{9}}, \dot{s}_{\frac{1}{9}}\right\rangle, \left\langle \ddot{s}_{\frac{2.5}{3}}, \ddot{s}_{\frac{1}{6}}\right\rangle\right).
$$

Now we have to find out the similarity in the case of second teacher

$$
\sup\left[\begin{array}{l}
\inf\left\{\begin{array}{l}
\left(\left\langle \dot{s}_1(h_1,a)_{\left[\frac{1+2-\frac{2}{6}}{3}\right]}, \dot{s}_1(h_1,a)_{\left[\frac{\frac{2\times 4}{6}}{3}\right]}\right\rangle, \left\langle \ddot{s}_1(h_1,a)_{\left[\frac{1+2-\frac{2}{4}}{3}\right]}, \ddot{s}_1(h_1,a)_{\left[\frac{\frac{1\times 2}{4}}{3}\right]}\right\rangle\right), \\
\left(\left\langle \dot{s}_2(h_1,b)_{\left[\frac{2+4-\frac{8}{6}}{3}\right]}, \dot{s}_2(h_1,b)_{\left[\frac{\frac{1\times 2}{6}}{3}\right]}\right\rangle, \left\langle \ddot{s}_2(h_1,b)_{\left[\frac{1+2-\frac{2}{4}}{3}\right]}, \ddot{s}_2(h_1,b)_{\left[\frac{\frac{1\times 2}{4}}{3}\right]}\right\rangle\right)
\end{array}\right\} \\
\inf\left\{\begin{array}{l}
\left(\left\langle \dot{s}_1(h_2,a)_{\left[\frac{2+3-\frac{6}{6}}{3}\right]}, \dot{s}_1(h_2,a)_{\left[\frac{\frac{2\times 3}{6}}{3}\right]}\right\rangle, \left\langle \ddot{s}_1(h_2,a)_{\left[\frac{1+2-\frac{2}{4}}{3}\right]}, \ddot{s}_1(h_2,a)_{\left[\frac{\frac{1\times 2}{4}}{3}\right]}\right\rangle\right), \\
\left(\left\langle \dot{s}_2(h_2,b)_{\left[\frac{1+2-\frac{2}{6}}{3}\right]}, \dot{s}_2(h_2,b)_{\left[\frac{\frac{3\times 4}{6}}{3}\right]}\right\rangle, \left\langle \ddot{s}_2(h_2,b)_{\left[\frac{1+2-\frac{2}{4}}{3}\right]}, \ddot{s}_2(h_2,b)_{\left[\frac{\frac{1\times 2}{4}}{3}\right]}\right\rangle\right)
\end{array}\right\}
\end{array}\right],
$$

this implies that

$$
sup\left\{\left(\left\langle \dot{s}_{\frac{8}{9}}, \dot{s}_{\frac{4}{9}}\right\rangle, \left\langle \ddot{s}_{\frac{2.5}{3}}, \ddot{s}_{\frac{1}{6}}\right\rangle\right), \left(\left\langle \dot{s}_{\frac{8}{9}}, \dot{s}_{\frac{2}{3}}\right\rangle, \left\langle \ddot{s}_{\frac{2.5}{3}}, \ddot{s}_{\frac{1}{6}}\right\rangle\right)\right\},
$$

this implies that the similarity between opinions in case of second teacher is as under

$$
=\left(\left\langle \dot{s}_{\frac{8}{9}}, \dot{s}_{\frac{4}{9}}\right\rangle, \left\langle \ddot{s}_{\frac{5}{6}}, \ddot{s}_{\frac{1}{6}}\right\rangle\right).
$$

By adding these two results we get

$$=\left(\langle \dot{s}_{\frac{400}{243}}, \dot{s}_{\frac{14}{81}}\rangle, \langle \ddot{s}_{\frac{215}{144}}, \ddot{s}_{\frac{1}{144}}\rangle\right).$$

Now by multiplying above equation with $\frac{1}{2}$ we get

$$=\left(\langle \dot{s}_{\frac{8}{9}}, \dot{s}_{1.01835015}\rangle, \langle \ddot{s}_{\frac{5}{6}}, \ddot{s}_{\frac{1}{6}}\rangle\right).$$

Thus the similarity between the opinions of two different students for two different teachers is given as

$$\mathrm{SM}(\mathrm{S}_1, \mathrm{S}_2) = \left(\langle \dot{s}_{\frac{8}{9}}, \dot{s}_{1.01835015}\rangle, \langle \ddot{s}_{\frac{5}{6}}, \ddot{s}_{\frac{1}{6}}\rangle\right).$$

Theorem: - Let S_1 and S_2 be any GLIVIFSESs and $s_\sigma \in [s_0, s_t]$, then these are said to be s_σ similar if $\mathrm{SM}(\mathrm{S}_1, \mathrm{S}_2) \geq s_\sigma$. We call S_1 and S_2 as significantly similar if $\mathrm{SM}(\mathrm{S}_1, \mathrm{S}_2) \geq s_{\frac{t}{2}}$.

4.2. TYPE-II SIMILARITY MEASURE BETWEEN GLIVIFSESs.

Type-II similarity measures are same as Type-I similarity measures with variation in supremum and infimum usage, mathematically defined as

$$
S_i(\mathrm{S}_1,\mathrm{S}_2,\dots,\mathrm{S}_o)=
$$

$$
inf\left[\begin{array}{l}
sup\left[\begin{array}{c}
\left(\left\langle \dot{s}_1(c_j,j_k)_{\left[\frac{\alpha+\alpha'-\frac{\alpha\alpha'}{t}}{3}\right]}, \dot{s}_1(c_j,j_k)_{\left[\frac{\frac{\beta\beta'}{t}}{3}\right]}\right\rangle, \left\langle \ddot{s}_1(c_j,j_k)_{\left[\frac{\gamma+\gamma'-\frac{\gamma\gamma'}{t'}}{3}\right]}, \ddot{s}_1(c_j,j_k)_{\left[\frac{\frac{\delta\delta'}{t'}}{3}\right]}\right\rangle\right),\\
\left(\left\langle \dot{s}_2(c_j,j_{k+1})_{\left[\frac{\alpha+\alpha'-\frac{\alpha\alpha'}{t}}{3}\right]}, \dot{s}_2(c_j,j_{k+1})_{\left[\frac{\frac{\beta\beta'}{t}}{3}\right]}\right\rangle, \left\langle \ddot{s}_2(c_j,j_{k+1})_{\left[\frac{\gamma+\gamma'-\frac{\gamma\gamma'}{t'}}{3}\right]}, \ddot{s}_2(c_j,j_{k+1})_{\left[\frac{\frac{\delta\delta'}{t'}}{3}\right]}\right\rangle\right),\dots,\\
\left(\left\langle \dot{s}_o(c_j,j_o)_{\left[\frac{\alpha+\alpha'-\frac{\alpha\alpha'}{t}}{3}\right]}, \dot{s}_o(c_j,j_o)_{\left[\frac{\frac{\beta\beta'}{t}}{3}\right]}\right\rangle, \left\langle \ddot{s}_o(c_j,j_o)_{\left[\frac{\gamma+\gamma'-\frac{\gamma\gamma'}{t'}}{3}\right]}, \ddot{s}_o(c_j,j_o)_{\left[\frac{\frac{\delta\delta'}{t'}}{3}\right]}\right\rangle\right)
\end{array}\right],\\
sup\left[\begin{array}{c}
\left(\left\langle \dot{s}_1(c_{j+1},j_k)_{\left[\frac{\alpha+\alpha'-\frac{\alpha\alpha'}{t}}{3}\right]}, \dot{s}_1(c_{j+1},j_k)_{\left[\frac{\frac{\beta\beta'}{t}}{3}\right]}\right\rangle, \left\langle \ddot{s}_1(c_{j+1},j_k)_{\left[\frac{\gamma+\gamma'-\frac{\gamma\gamma'}{t'}}{3}\right]}, \ddot{s}_1(c_{j+1},j_k)_{\left[\frac{\frac{\delta\delta'}{t'}}{3}\right]}\right\rangle\right),\dots,\\
\left(\left\langle \dot{s}_o(c_{j+1},j_o)_{\left[\frac{\alpha+\alpha'-\frac{\alpha\alpha'}{t}}{3}\right]}, \dot{s}_o(c_{j+1},j_o)_{\left[\frac{\frac{\beta\beta'}{t}}{3}\right]}\right\rangle, \left\langle \ddot{s}_o(c_{j+1},j_o)_{\left[\frac{\gamma+\gamma'-\frac{\gamma\gamma'}{t'}}{3}\right]}, \ddot{s}_o(c_{j+1},j_o)_{\left[\frac{\frac{\delta\delta'}{t'}}{3}\right]}\right\rangle\right)
\end{array}\right],\dots,\\
sup\left[\begin{array}{c}
\left(\left\langle \dot{s}_1(c_n,j_k)_{\left[\frac{\alpha+\alpha'-\frac{\alpha\alpha'}{t}}{3}\right]}, \dot{s}_1(c_n,j_k)_{\left[\frac{\frac{\beta\beta'}{t}}{3}\right]}\right\rangle, \left\langle \ddot{s}_1(c_n,j_k)_{\left[\frac{\gamma+\gamma'-\frac{\gamma\gamma'}{t'}}{3}\right]}, \ddot{s}_1(c_n,j_k)_{\left[\frac{\frac{\delta\delta'}{t'}}{3}\right]}\right\rangle\right),\dots,\\
\left(\left\langle \dot{s}_o(c_n,j_o)_{\left[\frac{\alpha+\alpha'-\frac{\alpha\alpha'}{t}}{3}\right]}, \dot{s}_o(c_n,j_o)_{\left[\frac{\frac{\beta\beta'}{t}}{3}\right]}\right\rangle, \left\langle \ddot{s}_o(c_n,j_o)_{\left[\frac{\gamma+\gamma'-\frac{\gamma\gamma'}{t'}}{3}\right]}, \ddot{s}_o(c_n,j_o)_{\left[\frac{\frac{\delta\delta'}{t'}}{3}\right]}\right\rangle\right)
\end{array}\right]
\end{array}\right] \quad (4.2)
$$

with

$$
\mathrm{SM}(\mathrm{S}_1,\mathrm{S}_2,\dots,\mathrm{S}_o)=\frac{\sum_{i=0}^{m} S_i(\mathrm{S}_1,\mathrm{S}_2,\dots,\mathrm{S}_o)}{m}.
$$

To better understand firstly we see the result variation of Example 4.1.1 using that similarity measure and then construct an example to see its working.

EXAMPLE 4.2.1.

Using data from Example 4.1.1, firstly to find the similarity between the opinions of student 'a' and student 'b' for a first teacher we use

$$\inf\left[\begin{array}{l} \sup\left\{\begin{array}{l} \left(\left\langle \dot{s}_1(h_1,a)_{\left[\frac{2+3-\frac{6}{6}}{3}\right]}, \dot{s}_1(h_1,a)_{\left[\frac{\frac{1\times 2}{6}}{3}\right]}\right\rangle, \left\langle \ddot{s}_1(h_1,a)_{\left[\frac{1+2-\frac{2}{4}}{3}\right]}, \ddot{s}_1(h_1,a)_{\left[\frac{\frac{1\times 2}{4}}{3}\right]}\right\rangle\right), \\ \left(\left\langle \dot{s}_2(h_1,b)_{\left[\frac{3+4-\frac{12}{6}}{3}\right]}, \dot{s}_2(h_1,b)_{\left[\frac{\frac{1\times 2}{6}}{3}\right]}\right\rangle, \left\langle \ddot{s}_2(h_1,b)_{\left[\frac{1+2-\frac{2}{4}}{3}\right]}, \ddot{s}_2(h_1,b)_{\left[\frac{\frac{1\times 2}{4}}{3}\right]}\right\rangle\right) \end{array}\right\} \\ \sup\left\{\begin{array}{l} \left(\left\langle \dot{s}_1(h_2,a)_{\left[\frac{1+3-\frac{3}{6}}{3}\right]}, \dot{s}_1(h_2,a)_{\left[\frac{\frac{2\times 3}{6}}{3}\right]}\right\rangle, \left\langle \ddot{s}_1(h_2,a)_{\left[\frac{1+2-\frac{2}{4}}{3}\right]}, \ddot{s}_1(h_2,a)_{\left[\frac{\frac{1\times 2}{4}}{3}\right]}\right\rangle\right), \\ \left(\left\langle \dot{s}_2(h_2,b)_{\left[\frac{1+2-\frac{2}{6}}{3}\right]}, \dot{s}_2(h_2,b)_{\left[\frac{\frac{3\times 4}{6}}{3}\right]}\right\rangle, \left\langle \ddot{s}_2(h_2,b)_{\left[\frac{1+2-\frac{2}{4}}{3}\right]}, \ddot{s}_2(h_2,b)_{\left[\frac{\frac{1\times 2}{4}}{3}\right]}\right\rangle\right) \end{array}\right\} \end{array}\right],$$

this gives after arithmetic evaluation and finding supremum between them we get

$$inf\left\{\left(\langle \dot{s}_{\frac{4}{3}}, \dot{s}_{\frac{1}{9}}\rangle, \langle \ddot{s}_{\frac{5}{6}}, \ddot{s}_{\frac{1}{6}}\rangle\right), \left(\langle \dot{s}_{\frac{7}{6}}, \dot{s}_{\frac{1}{3}}\rangle, \langle \ddot{s}_{\frac{5}{6}}, \ddot{s}_{\frac{1}{6}}\rangle\right)\right\} = \left(\langle \dot{s}_{\frac{7}{6}}, \dot{s}_{\frac{1}{3}}\rangle, \langle \ddot{s}_{\frac{5}{6}}, \ddot{s}_{\frac{1}{6}}\rangle\right),$$

this represents the similarity between opinions of two different students for first teacher.

Now to find the similarity between student's opinions for second teacher, we have

$$\inf\left[\begin{array}{l}\sup\left\{\begin{array}{l}\left(\langle \dot{s}_1(h_1,a)_{\left[\frac{1+2-\frac{2}{6}}{3}\right]}, \dot{s}_1(h_1,a)_{\left[\frac{\frac{2\times 4}{6}}{3}\right]}\rangle, \langle \ddot{s}_1(h_1,a)_{\left[\frac{1+2-\frac{2}{4}}{3}\right]}, \ddot{s}_1(h_1,a)_{\left[\frac{\frac{1\times 2}{4}}{3}\right]}\rangle\right),\\ \left(\langle \dot{s}_2(h_1,b)_{\left[\frac{2+4-\frac{8}{6}}{3}\right]}, \dot{s}_2(h_1,b)_{\left[\frac{\frac{1\times 2}{6}}{3}\right]}\rangle, \langle \ddot{s}_2(h_1,b)_{\left[\frac{1+2-\frac{2}{4}}{3}\right]}, \ddot{s}_2(h_1,b)_{\left[\frac{\frac{1\times 2}{4}}{3}\right]}\rangle\right)\end{array}\right\}\\ \sup\left\{\begin{array}{l}\left(\langle \dot{s}_1(h_2,a)_{\left[\frac{2+3-\frac{6}{6}}{3}\right]}, \dot{s}_1(h_2,a)_{\left[\frac{\frac{2\times 3}{6}}{3}\right]}\rangle, \langle \ddot{s}_1(h_2,a)_{\left[\frac{1+2-\frac{2}{4}}{3}\right]}, \ddot{s}_1(h_2,a)_{\left[\frac{\frac{1\times 2}{4}}{3}\right]}\rangle\right),\\ \left(\langle \dot{s}_2(h_2,b)_{\left[\frac{1+2-\frac{2}{6}}{3}\right]}, \dot{s}_2(h_2,b)_{\left[\frac{\frac{3\times 4}{6}}{3}\right]}\rangle, \langle \ddot{s}_2(h_2,b)_{\left[\frac{1+2-\frac{2}{4}}{3}\right]}, \ddot{s}_2(h_2,b)_{\left[\frac{\frac{1\times 2}{4}}{3}\right]}\rangle\right)\end{array}\right\}\end{array}\right],$$

that yields after simplification and finding supremum's we have

$$inf\left\{\left(\langle \dot{s}_{\frac{14}{9}}, \dot{s}_{\frac{1}{9}}\rangle, \langle \ddot{s}_{\frac{5}{6}}, \ddot{s}_{\frac{1}{6}}\rangle\right), \left(\langle \dot{s}_{\frac{4}{3}}, \dot{s}_{\frac{1}{3}}\rangle, \langle \ddot{s}_{\frac{5}{6}}, \ddot{s}_{\frac{1}{6}}\rangle\right)\right\} = \left(\langle \dot{s}_{\frac{4}{3}}, \dot{s}_{\frac{1}{3}}\rangle, \langle \ddot{s}_{\frac{5}{6}}, \ddot{s}_{\frac{1}{6}}\rangle\right),$$

which represents the similarity in opinions of both student's for second teacher.

Now by adding these two similarities, which are in the form of generalized linguistic intuitionistic fuzzy soft expert sets we get

$$\left(\langle \dot{s}_{\frac{7}{6}}, \dot{s}_{\frac{1}{3}}\rangle, \langle \ddot{s}_{\frac{5}{6}}, \ddot{s}_{\frac{1}{6}}\rangle\right) \oplus \left(\langle \dot{s}_{\frac{4}{3}}, \dot{s}_{\frac{1}{3}}\rangle, \langle \ddot{s}_{\frac{5}{6}}, \ddot{s}_{\frac{1}{6}}\rangle\right) = \left(\langle \dot{s}_{\frac{121}{54}}, \dot{s}_{\frac{1}{54}}\rangle, \langle \ddot{s}_{\frac{215}{144}}, \ddot{s}_{\frac{1}{144}}\rangle\right),$$

now by using Equation (A) we get

$$\mathrm{SM}(\mathrm{S}_1, \mathrm{S}_2) = \left(\langle \dot{s}_{\frac{18-\sqrt{203}}{3}}, \dot{s}_{\frac{1}{3}}\rangle, \langle \ddot{s}_{\frac{5}{6}}, \ddot{s}_{\frac{1}{6}}\rangle\right),$$

Which represents the similarity between opinions of two student's for two different teachers in the form generalized linguistic intuitionistic fuzzy soft expert set. By observation or by comparing it is clear that the linguistic terms appear in Example 4.1.1 similarity set and in Example 4.2.1 similarity set are not both are same.

EXAMPLE 4.2.2.

Consider a company wants to judge the performance of new enrolled employee's on the behalf of criteria fixed by the owner. For that purpose the executive committee made a committee of experts working with that company to judge the performance and present the reports to executive committee.

To get the reality based opinion’s the executive committee order the experts that also mention the own experience of working with them. The executive committee takes decision on the behalf of majority opinion from decision makers and for that purpose they wanted to calculate similarity between judgments of experts.

For simplicity reasons and to demonstrate the Type-II similarity measure we consider three new enrolled employees $\{e_1, e_2, e_3\}$ with two decision makers $\{j_1, j_2\}$ and two criteria’s $\{c_1, c_2\}$

- c_1= working speed
- c_2= focus on work

Here we consider the linguistic term set S_t with variation of t as $1 \leq t \leq 13$ and linguistic term set $S_{t'}$ which is predefined for 2-D linguistic approach, with variation of t' as$1 \leq t' \leq 12$.

Now firstly we suppose roughly the judgments of both experts in the form of generalized linguistic interval-valued intuitionistic fuzzy soft expert sets for employee e_1 and then find the similarity between them by using Type-II similarity measure for GLIVIFSESs.

$$\inf\left[\begin{array}{l} \sup\left\{\begin{array}{l} \left(\left\langle \dot{s}_1(c_1,j_1)_{\left[\frac{1+3-\frac{3}{13}}{3}\right]}, \dot{s}_1(c_1,j_1)_{\left[\frac{\frac{2\times 3}{13}}{3}\right]}\right\rangle, \left\langle \ddot{s}_1(c_1,j_1)_{\left[\frac{4+5-\frac{4\times 5}{12}}{3}\right]}, \ddot{s}_1(c_1,j_1)_{\left[\frac{\frac{1\times 2}{12}}{3}\right]}\right\rangle\right), \\ \left(\left\langle \dot{s}_2(c_1,j_2)_{\left[\frac{3+4-\frac{12}{13}}{3}\right]}, \dot{s}_2(c_1,j_2)_{\left[\frac{\frac{1\times 2}{13}}{3}\right]}\right\rangle, \left\langle \ddot{s}_2(c_1,j_2)_{\left[\frac{5+6-\frac{30}{12}}{3}\right]}, \ddot{s}_2(c_1,j_2)_{\left[\frac{\frac{4\times 5}{12}}{3}\right]}\right\rangle\right) \end{array}\right\} \\ \sup\left\{\begin{array}{l} \left(\left\langle \dot{s}_1(c_2,j_1)_{\left[\frac{1+3-\frac{3}{13}}{3}\right]}, \dot{s}_1(c_2,j_1)_{\left[\frac{\frac{2\times 3}{13}}{3}\right]}\right\rangle, \left\langle \ddot{s}_1(c_2,j_1)_{\left[\frac{2+4-\frac{8}{12}}{3}\right]}, \ddot{s}_1(c_2,j_1)_{\left[\frac{\frac{1\times 2}{12}}{3}\right]}\right\rangle\right), \\ \left(\left\langle \dot{s}_2(c_2,j_2)_{\left[\frac{3+4-\frac{12}{13}}{3}\right]}, \dot{s}_2(c_2,j_2)_{\left[\frac{\frac{4\times 5}{13}}{3}\right]}\right\rangle, \left\langle \ddot{s}_2(c_2,j_2)_{\left[\frac{1+2-\frac{2}{12}}{3}\right]}, \ddot{s}_2(c_2,j_2)_{\left[\frac{\frac{5\times 6}{12}}{3}\right]}\right\rangle\right) \end{array}\right\} \end{array}\right],$$

that yield’s after simplification

$$infimum\left\{\left(\left\langle \dot{s}_{\frac{79}{39}}, \dot{s}_{\frac{2}{39}}\right\rangle, \left\langle \ddot{s}_{\frac{17}{6}}, \ddot{s}_{\frac{1}{18}}\right\rangle\right), \left(\left\langle \dot{s}_{\frac{79}{39}}, \dot{s}_{\frac{2}{13}}\right\rangle, \left\langle \ddot{s}_{\frac{16}{9}}, \ddot{s}_{\frac{1}{18}}\right\rangle\right)\right\} = \left(\left\langle \dot{s}_{\frac{79}{39}}, \dot{s}_{\frac{2}{13}}\right\rangle, \left\langle \ddot{s}_{\frac{16}{9}}, \ddot{s}_{\frac{1}{18}}\right\rangle\right),$$

this represents the similarity between the opinions of experts for an employee e_1.

Now for an employee e_2 we have

$$\inf\left[\begin{array}{l}\sup\left\{\begin{array}{l}\left(\left\langle \dot{s}_1(c_1,j_1)_{\left[\frac{4+5-\frac{20}{13}}{3}\right]}, \dot{s}_1(c_1,j_1)_{\left[\frac{\frac{5\times 6}{13}}{3}\right]}\right\rangle, \left\langle \ddot{s}_1(c_1,j_1)_{\left[\frac{1+2-\frac{2}{12}}{3}\right]}, \ddot{s}_1(c_1,j_1)_{\left[\frac{\frac{3\times 4}{12}}{3}\right]}\right\rangle\right),\\ \left(\left\langle \dot{s}_2(c_1,j_2)_{\left[\frac{2+3-\frac{6}{13}}{3}\right]}, \dot{s}_2(c_1,j_2)_{\left[\frac{\frac{1\times 5}{13}}{3}\right]}\right\rangle, \left\langle \ddot{s}_2(c_1,j_2)_{\left[\frac{2+3-\frac{6}{12}}{3}\right]}, \ddot{s}_2(c_1,j_2)_{\left[\frac{\frac{5\times 6}{12}}{3}\right]}\right\rangle\right)\end{array}\right\}\\ \sup\left\{\begin{array}{l}\left(\left\langle \dot{s}_1(c_2,j_1)_{\left[\frac{1+4-\frac{4}{13}}{3}\right]}, \dot{s}_1(c_2,j_1)_{\left[\frac{\frac{2\times 6}{13}}{3}\right]}\right\rangle, \left\langle \ddot{s}_1(c_2,j_1)_{\left[\frac{2+4-\frac{8}{12}}{3}\right]}, \ddot{s}_1(c_2,j_1)_{\left[\frac{\frac{2\times 3}{12}}{3}\right]}\right\rangle\right),\\ \left(\left\langle \dot{s}_2(c_2,j_2)_{\left[\frac{3+4-\frac{12}{13}}{3}\right]}, \dot{s}_2(c_2,j_2)_{\left[\frac{\frac{1\times 5}{13}}{3}\right]}\right\rangle, \left\langle \ddot{s}_2(c_2,j_2)_{\left[\frac{2+6-\frac{12}{12}}{3}\right]}, \ddot{s}_2(c_2,j_2)_{\left[\frac{\frac{1\times 4}{12}}{3}\right]}\right\rangle\right)\end{array}\right\}\end{array}\right],$$

after simplification and finding supremum's we get

$$inf\left\{\left(\left\langle \dot{s}_{\frac{97}{39}}, \dot{s}_{\frac{5}{39}}\right\rangle, \left\langle \ddot{s}_{\frac{3}{2}}, \ddot{s}_{\frac{1}{3}}\right\rangle\right), \left(\left\langle \dot{s}_{\frac{61}{39}}, \dot{s}_{\frac{5}{39}}\right\rangle, \left\langle \ddot{s}_{\frac{7}{3}}, \ddot{s}_{\frac{1}{9}}\right\rangle\right)\right\} = \left(\left\langle \dot{s}_{\frac{97}{39}}, \dot{s}_{\frac{5}{39}}\right\rangle, \left\langle \ddot{s}_{\frac{3}{2}}, \ddot{s}_{\frac{1}{3}}\right\rangle\right),$$

the above equation shows the similarity between expert's opinions for employee e_2.

Now to find the similarity in opinions of experts for employee e_3 we have

$$\inf\left[\begin{array}{l}\sup\left\{\begin{array}{l}\left(\left\langle \dot{s}_1(c_1,j_1)_{\left[\frac{2+6-\frac{12}{13}}{3}\right]}, \dot{s}_1(c_1,j_1)_{\left[\frac{\frac{1\times 4}{13}}{3}\right]}\right\rangle, \left\langle \ddot{s}_1(c_1,j_1)_{\left[\frac{1+3-\frac{3}{12}}{3}\right]}, \ddot{s}_1(c_1,j_1)_{\left[\frac{\frac{3\times 4}{12}}{3}\right]}\right\rangle\right),\\ \left(\left\langle \dot{s}_2(c_1,j_2)_{\left[\frac{2+5-\frac{10}{13}}{3}\right]}, \dot{s}_2(c_1,j_2)_{\left[\frac{\frac{1\times 6}{13}}{3}\right]}\right\rangle, \left\langle \ddot{s}_2(c_1,j_2)_{\left[\frac{2+3-\frac{6}{12}}{3}\right]}, \ddot{s}_2(c_1,j_2)_{\left[\frac{\frac{3\times 6}{12}}{3}\right]}\right\rangle\right)\end{array}\right\}\\ \sup\left\{\begin{array}{l}\left(\left\langle \dot{s}_1(c_2,j_1)_{\left[\frac{1+7-\frac{7}{13}}{3}\right]}, \dot{s}_1(c_2,j_1)_{\left[\frac{\frac{2\times 6}{13}}{3}\right]}\right\rangle, \left\langle \ddot{s}_1(c_2,j_1)_{\left[\frac{2+4-\frac{8}{12}}{3}\right]}, \ddot{s}_1(c_2,j_1)_{\left[\frac{\frac{2\times 6}{12}}{3}\right]}\right\rangle\right),\\ \left(\left\langle \dot{s}_2(c_2,j_2)_{\left[\frac{3+7-\frac{21}{13}}{3}\right]}, \dot{s}_2(c_2,j_2)_{\left[\frac{\frac{2\times 5}{13}}{3}\right]}\right\rangle, \left\langle \ddot{s}_2(c_2,j_2)_{\left[\frac{2+6-\frac{12}{12}}{3}\right]}, \ddot{s}_2(c_2,j_2)_{\left[\frac{\frac{1\times 5}{12}}{3}\right]}\right\rangle\right)\end{array}\right\}\end{array}\right],$$

that yields after simplification and finding supremum's between them we get

$$inf\left\{\left(\left\langle \dot{s}_{\frac{92}{39}}, \dot{s}_{\frac{4}{39}}\right\rangle, \left\langle \ddot{s}_{\frac{3}{2}}, \ddot{s}_{\frac{1}{3}}\right\rangle\right), \left(\left\langle \dot{s}_{\frac{109}{39}}, \dot{s}_{\frac{10}{39}}\right\rangle, \left\langle \ddot{s}_{\frac{7}{3}}, \ddot{s}_{\frac{5}{36}}\right\rangle\right)\right\} = \left(\left\langle \dot{s}_{\frac{92}{39}}, \dot{s}_{\frac{10}{39}}\right\rangle, \left\langle \ddot{s}_{\frac{3}{2}}, \ddot{s}_{\frac{1}{3}}\right\rangle\right),$$

this represents the similarity between expert's opinions for employee e_3.

Now by adding these similarities we get

$$\left(\langle \dot{s}_{\frac{79}{39}}, \dot{s}_{\frac{2}{13}}\rangle, \langle \ddot{s}_{\frac{16}{9}}, \ddot{s}_{\frac{1}{18}}\rangle\right) \oplus \left(\langle \dot{s}_{\frac{97}{39}}, \dot{s}_{\frac{5}{39}}\rangle, \langle \ddot{s}_{\frac{3}{2}}, \ddot{s}_{\frac{1}{3}}\rangle\right) \oplus \left(\langle \dot{s}_{\frac{92}{39}}, \dot{s}_{\frac{10}{39}}\rangle, \langle \ddot{s}_{\frac{3}{2}}, \ddot{s}_{\frac{1}{3}}\rangle\right)$$

$$=\left(\langle \dot{s}_{5.73567}, \dot{s}_{2.99\times10^{-5}}\rangle, \langle \ddot{s}_{\frac{601}{144}}, \ddot{s}_{4.28669\times10^{-5}}\rangle\right),$$

now by using the Equation (A)

$$\mathrm{SM}(\mathrm{S}_1, \mathrm{S}_2) = \left(\langle \dot{s}_{2.292352}, \dot{s}_{0.171601}\rangle, \langle \ddot{s}_{1.59342}, \ddot{s}_{0.18344}\rangle\right).$$

4.3. TYPE-III SIMILARITY MEASURE FOR GLIVIFSESs.

Definition. Let $U = \{u_i : i = 1, . . . , n\}$ be a set of alternatives and $E = \{e_j : j = 1, . . . , m\}$ represents the set of decision makers and $C = \{c_k : k = 1, . . . , r\}$ represents the set of criteria where n, r, m $\in \mathbb{Z}$ with the property that either 'n', 'r', 'm' are same or different. Suppose

$$\mathrm{S}_1=\left\{\langle \dot{\mathrm{s}}_{[\alpha,\alpha']}, \dot{\mathrm{s}}_{[\beta,\beta']}\rangle, \langle \ddot{\mathrm{s}}_{[\gamma,\gamma']}, \ddot{\mathrm{s}}_{[\delta,\delta']}\rangle\right\},$$

and

$$\mathrm{S}_2=\left\{\langle \dot{\mathrm{s}}_{[\alpha,\alpha']}, \dot{\mathrm{s}}_{[\beta,\beta']}\rangle, \langle \ddot{\mathrm{s}}_{[\gamma,\gamma']}, \ddot{\mathrm{s}}_{[\delta,\delta']}\rangle\right\},$$

the general form of generalized linguistic interval-valued intuitionistic fuzzy soft expert sets. According to Type-III similarity measure between different GLIVIFSESs we have firstly to calculate the similarity between GLIVIFSESs for a specific alternative, in mathematically

$$\sum_{j=1}^{m}\sum_{k=1}^{r} \frac{\left\{\langle \dot{\mathrm{S}}_{1(e_j,c_k)_{\frac{\alpha\oplus\alpha'}{2}}}, \dot{\mathrm{S}}_{1(e_j,c_k)_{\frac{\beta\oplus\beta'}{2}}}\rangle, \langle \ddot{\mathrm{S}}_{1(e_j,c_k)_{\frac{\gamma\oplus\gamma'}{2}}}, \ddot{\mathrm{S}}_{1(e_j,c_k)_{\frac{\delta\oplus\delta'}{2}}}\rangle\right\} \wedge \left\{\langle \dot{\mathrm{S}}_{2(e_{j+1},c_k)_{\frac{\alpha\oplus\alpha'}{2}}}, \dot{\mathrm{S}}_{2(e_{j+1},c_k)_{\frac{\beta\oplus\beta'}{2}}}\rangle, \langle \ddot{\mathrm{S}}_{2(e_{j+1},c_k)_{\frac{\gamma\oplus\gamma'}{2}}}, \ddot{\mathrm{S}}_{2(e_{j+1},c_k)_{\frac{\delta\oplus\delta'}{2}}}\rangle\right\}}{\left\{\langle \dot{\mathrm{S}}_{1(e_j,c_k)_{\frac{\alpha\oplus\alpha'}{2}}}, \dot{\mathrm{S}}_{1(e_j,c_k)_{\frac{\beta\oplus\beta'}{2}}}\rangle, \langle \ddot{\mathrm{S}}_{1(e_j,c_k)_{\frac{\gamma\oplus\gamma'}{2}}}, \ddot{\mathrm{S}}_{1(e_j,c_k)_{\frac{\delta\oplus\delta'}{2}}}\rangle\right\} \vee \left\{\langle \dot{\mathrm{S}}_{2(e_{j+1},c_k)_{\frac{\alpha\oplus\alpha'}{2}}}, \dot{\mathrm{S}}_{2(e_{j+1},c_k)_{\frac{\beta\oplus\beta'}{2}}}\rangle, \langle \ddot{\mathrm{S}}_{2(e_{j+1},c_k)_{\frac{\gamma\oplus\gamma'}{2}}}, \ddot{\mathrm{S}}_{2(e_{j+1},c_k)_{\frac{\delta\oplus\delta'}{2}}}\rangle\right\}} \tag{4.3}$$

which shows the similarity between the opinions of expert's for specific alternative u_i where 'i' range goes from one to 'n'. Here the point is to be noticed that division of intervals is undefined when zero belongs to the denominator interval in case interval-valued intuitionistic fuzzy sets and other interval-valued fuzzy sets but in our case such that in linguistic approach this problem doesn't interrupt such that S_0 instead of '0' doesn't produces the undetermined case. Similarly for each alternative we will use the above mentioned formula to calculate the similarity between the opinions of experts.

Now to find the combined similarity in the opinions of experts for all alternatives we will use the following relation

$$\mathrm{SM}(\mathrm{S}_1, \mathrm{S}_2, \dots, \mathrm{S}_\mathrm{o}) = \frac{\sum_{\mathrm{i}=0}^{\mathrm{m}} S_i(\mathrm{S}_1, \mathrm{S}_2, \dots, \mathrm{S}_\mathrm{o})}{\mathrm{m}},$$

here the point should be note that in general not only two GLIVIFSESs $(\mathrm{S}_1, \mathrm{S}_2)$ but these goes up to a finite number such that their quantity depends upon the number of experts involved in certain problem. To illustrate the above mentioned technique for measuring similarity between GLIVIFSESs we will firstly consider the Example 4.1.1 and then construct a separate example to briefly demonstrate that technique.

EXAMPLE 4.3.1.

By taking data from Example 4.1.1 we have the following generalized linguistic interval-valued intuitionistic fuzzy soft expert sets

$$\mathrm{S}_1(\mathrm{h}_1, \mathrm{a})=\{\langle \dot{\mathrm{s}}_{[2,3]}, \dot{\mathrm{s}}_{[1,2]}\rangle, \langle \ddot{\mathrm{s}}_{[1,2]}, \ddot{\mathrm{s}}_{[1,2]}\rangle\},$$
$$\mathrm{S}_1(\mathrm{h}_2, \mathrm{a})=\{\langle \dot{\mathrm{s}}_{[3,4]}, \dot{\mathrm{s}}_{[1,2]}\rangle, \langle \ddot{\mathrm{s}}_{[1,2]}, \ddot{\mathrm{s}}_{[1,2]}\rangle\},$$
$$\mathrm{S}_2(\mathrm{h}_1, \mathrm{b})=\{\langle \dot{\mathrm{s}}_{[1,3]}, \dot{\mathrm{s}}_{[2,3]}\rangle, \langle \ddot{\mathrm{s}}_{[1,2]}, \ddot{\mathrm{s}}_{[1,2]}\rangle\},$$
$$\mathrm{S}_2(\mathrm{h}_2, \mathrm{b})=\{\langle \dot{\mathrm{s}}_{[1,2]}, \dot{\mathrm{s}}_{[3,4]}\rangle, \langle \ddot{\mathrm{s}}_{[1,2]}, \ddot{\mathrm{s}}_{[1,2]}\rangle\},$$

these sets represent the evaluation values by students 'a' and 'b' for the teacher t_1. Now for the second teacher t_2 the evaluation values are as under

$$\mathrm{S}_1(\mathrm{h}_1, \mathrm{a})=\{\langle \dot{\mathrm{s}}_{[1,2]}, \dot{\mathrm{s}}_{[2,4]}\rangle, \langle \ddot{\mathrm{s}}_{[1,2]}, \ddot{\mathrm{s}}_{[1,2]}\rangle\},$$
$$\mathrm{S}_1(\mathrm{h}_2, \mathrm{a})=\{\langle \dot{\mathrm{s}}_{[2,4]}, \dot{\mathrm{s}}_{[1,2]}\rangle, \langle \ddot{\mathrm{s}}_{[1,2]}, \ddot{\mathrm{s}}_{[1,2]}\rangle\},$$
$$\mathrm{S}_2(\mathrm{h}_1, \mathrm{b})=\{\langle \dot{\mathrm{s}}_{[2,3]}, \dot{\mathrm{s}}_{[2,3]}\rangle, \langle \ddot{\mathrm{s}}_{[1,2]}, \ddot{\mathrm{s}}_{[1,2]}\rangle\},$$
$$\mathrm{S}_2(\mathrm{h}_2, \mathrm{b})=\{\langle \dot{\mathrm{s}}_{[1,2]}, \dot{\mathrm{s}}_{[3,4]}\rangle, \langle \ddot{\mathrm{s}}_{[1,2]}, \ddot{\mathrm{s}}_{[1,2]}\rangle\}.$$

Now by using the Type-III similarity measure we will firstly find the similarity between opinions of experts in case of t_1.

$$\frac{\left\{\langle \dot{S}_{1(h_1,a)_{\frac{2\oplus3}{2}}}, \dot{S}_{1(h_1,a)_{\frac{1\oplus2}{2}}}\rangle, \langle \ddot{S}_{1(h_1,a)_{\frac{1\oplus2}{2}}}, \ddot{S}_{1(h_1,a)_{\frac{1\oplus2}{2}}}\rangle\right\} \wedge \left\{\langle \dot{S}_{2(h_1,b)_{\frac{1\oplus3}{2}}}, \dot{S}_{2(h_1,b)_{\frac{2\oplus3}{2}}}\rangle, \langle \ddot{S}_{2(h_1,b)_{\frac{1\oplus2}{2}}}, \ddot{S}_{2(h_1,b)_{\frac{1\oplus2}{2}}}\rangle\right\}}{\left\{\langle \dot{S}_{1(h_1,a)_{\frac{2\oplus3}{2}}}, \dot{S}_{1(h_1,a)_{\frac{1\oplus2}{2}}}\rangle, \langle \ddot{S}_{1(h_1,a)_{\frac{1\oplus2}{2}}}, \ddot{S}_{1(h_1,a)_{\frac{1\oplus2}{2}}}\rangle\right\} \vee \left\{\langle \dot{S}_{2(h_1,b)_{\frac{1\oplus3}{2}}}, \dot{S}_{2(h_1,b)_{\frac{2\oplus3}{2}}}\rangle, \langle \ddot{S}_{2(h_1,b)_{\frac{1\oplus2}{2}}}, \ddot{S}_{2(h_1,b)_{\frac{1\oplus2}{2}}}\rangle\right\}} \oplus$$

$$\frac{\left\{\langle \dot{S}_{1(h_2,a)_{\frac{3\oplus4}{2}}}, \dot{S}_{1(h_2,a)_{\frac{1\oplus2}{2}}}\rangle, \langle \ddot{S}_{1(h_2,a)_{\frac{1\oplus2}{2}}}, \ddot{S}_{1(h_2,a)_{\frac{1\oplus2}{2}}}\rangle\right\} \wedge \left\{\langle \dot{S}_{2(h_2,b)_{\frac{1\oplus2}{2}}}, \dot{S}_{2(h_2,b)_{\frac{3\oplus4}{2}}}\rangle, \langle \ddot{S}_{2(h_2,b)_{\frac{1\oplus2}{2}}}, \ddot{S}_{2(h_2,b)_{\frac{1\oplus2}{2}}}\rangle\right\}}{\left\{\langle \dot{S}_{1(h_2,a)_{\frac{3\oplus4}{2}}}, \dot{S}_{1(h_2,a)_{\frac{1\oplus2}{2}}}\rangle, \langle \ddot{S}_{1(h_2,a)_{\frac{1\oplus2}{2}}}, \ddot{S}_{1(h_2,a)_{\frac{1\oplus2}{2}}}\rangle\right\} \vee \left\{\langle \dot{S}_{2(h_2,b)_{\frac{1\oplus2}{2}}}, \dot{S}_{2(h_2,b)_{\frac{3\oplus4}{2}}}\rangle, \langle \ddot{S}_{2(h_2,b)_{\frac{1\oplus2}{2}}}, \ddot{S}_{2(h_2,b)_{\frac{1\oplus2}{2}}}\rangle\right\}}$$

$$=\frac{\left(\langle \dot{s}_2, \dot{s}_{\frac{3}{2}}\rangle, \langle \ddot{s}_{\frac{3}{2}}, \ddot{s}_{\frac{3}{2}}\rangle\right)}{\left(\langle \dot{s}_{\frac{5}{2}}, \dot{s}_{\frac{3}{2}}\rangle, \langle \ddot{s}_{\frac{3}{2}}, \ddot{s}_{\frac{3}{2}}\rangle\right)} \oplus \frac{\left(\langle \dot{s}_{\frac{3}{2}}, \dot{s}_{\frac{7}{2}}\rangle, \langle \ddot{s}_{\frac{3}{2}}, \ddot{s}_{\frac{3}{2}}\rangle\right)}{\left(\langle \dot{s}_{\frac{7}{2}}, \dot{s}_{\frac{3}{2}}\rangle, \langle \ddot{s}_{\frac{3}{2}}, \ddot{s}_{\frac{3}{2}}\rangle\right)} = \left(\langle \dot{s}_{\frac{2}{9}}, \dot{s}_{\frac{9}{5}}\rangle, \langle \ddot{s}_{\frac{1}{4}}, \ddot{s}_{\frac{23}{12}}\rangle\right) \oplus \left(\langle \dot{s}_{\frac{1}{6}}, \dot{s}_{\frac{76}{21}}\rangle, \langle \ddot{s}_{\frac{9}{16}}, \ddot{s}_{\frac{39}{16}}\rangle\right)$$

$$=\left(\langle \dot{s}_{\frac{31}{81}}, \dot{s}_{\frac{38}{35}}\rangle, \langle \ddot{s}_{\frac{199}{256}}, \ddot{s}_{\frac{299}{256}}\rangle\right),$$

this represents the similarity between the opinions of experts for the first teacher. Now for the second teacher t_2, by substituting the opinions of expert in expression of Type-III similarity measure we get

$$\frac{\left\{\langle \dot{S}_{1(h_1,a)_{\frac{1\oplus2}{2}}}, \dot{S}_{1(h_1,a)_{\frac{2\oplus4}{2}}}\rangle, \langle \ddot{S}_{1(h_1,a)_{\frac{1\oplus2}{2}}}, \ddot{S}_{1(h_1,a)_{\frac{1\oplus2}{2}}}\rangle\right\} \wedge \left\{\langle \dot{S}_{2(h_1,b)_{\frac{2\oplus3}{2}}}, \dot{S}_{2(h_1,b)_{\frac{2\oplus3}{2}}}\rangle, \langle \ddot{S}_{2(h_1,b)_{\frac{1\oplus2}{2}}}, \ddot{S}_{2(h_1,b)_{\frac{1\oplus2}{2}}}\rangle\right\}}{\left\{\langle \dot{S}_{1(h_1,a)_{\frac{1\oplus2}{2}}}, \dot{S}_{1(h_1,a)_{\frac{2\oplus4}{2}}}\rangle, \langle \ddot{S}_{1(h_1,a)_{\frac{1\oplus2}{2}}}, \ddot{S}_{1(h_1,a)_{\frac{1\oplus2}{2}}}\rangle\right\} \vee \left\{\langle \dot{S}_{2(h_1,b)_{\frac{2\oplus3}{2}}}, \dot{S}_{2(h_1,b)_{\frac{2\oplus3}{2}}}\rangle, \langle \ddot{S}_{2(h_1,b)_{\frac{1\oplus2}{2}}}, \ddot{S}_{2(h_1,b)_{\frac{1\oplus2}{2}}}\rangle\right\}} \oplus$$

$$\frac{\left\{\langle \dot{S}_{1(h_2,a)_{\frac{2\oplus4}{2}}}, \dot{S}_{1(h_2,a)_{\frac{1\oplus2}{2}}}\rangle, \langle \ddot{S}_{1(h_2,a)_{\frac{1\oplus2}{2}}}, \ddot{S}_{1(h_2,a)_{\frac{1\oplus2}{2}}}\rangle\right\} \wedge \left\{\langle \dot{S}_{2(h_2,b)_{\frac{1\oplus2}{2}}}, \dot{S}_{2(h_2,b)_{\frac{3\oplus4}{2}}}\rangle, \langle \ddot{S}_{2(h_2,b)_{\frac{1\oplus2}{2}}}, \ddot{S}_{2(h_2,b)_{\frac{1\oplus2}{2}}}\rangle\right\}}{\left\{\langle \dot{S}_{1(h_2,a)_{\frac{2\oplus4}{2}}}, \dot{S}_{1(h_2,a)_{\frac{1\oplus2}{2}}}\rangle, \langle \ddot{S}_{1(h_2,a)_{\frac{1\oplus2}{2}}}, \ddot{S}_{1(h_2,a)_{\frac{1\oplus2}{2}}}\rangle\right\} \vee \left\{\langle \dot{S}_{2(h_2,b)_{\frac{1\oplus2}{2}}}, \dot{S}_{2(h_2,b)_{\frac{3\oplus4}{2}}}\rangle, \langle \ddot{S}_{2(h_2,b)_{\frac{1\oplus2}{2}}}, \ddot{S}_{2(h_2,b)_{\frac{1\oplus2}{2}}}\rangle\right\}}$$

after simplifications we get

$$=\frac{\left(\langle \dot{s}_{\frac{3}{2}}, \dot{s}_{3}\rangle, \langle \ddot{s}_{\frac{3}{2}}, \ddot{s}_{\frac{3}{2}}\rangle\right)}{\left(\langle \dot{s}_{\frac{5}{2}}, \dot{s}_{\frac{5}{2}}\rangle, \langle \ddot{s}_{\frac{3}{2}}, \ddot{s}_{\frac{3}{2}}\rangle\right)} \oplus \frac{\left(\langle \dot{s}_{\frac{3}{2}}, \dot{s}_{\frac{7}{2}}\rangle, \langle \ddot{s}_{\frac{3}{2}}, \ddot{s}_{\frac{3}{2}}\rangle\right)}{\left(\langle \dot{s}_{3}, \dot{s}_{\frac{3}{2}}\rangle, \langle \ddot{s}_{\frac{3}{2}}, \ddot{s}_{\frac{3}{2}}\rangle\right)} = \left(\langle \dot{s}_{\frac{1}{10}}, \dot{s}_{\frac{16}{5}}\rangle, \langle \ddot{s}_{\frac{1}{4}}, \ddot{s}_{\frac{23}{12}}\rangle\right) \oplus \left(\langle \dot{s}_{\frac{1}{6}}, \dot{s}_{\frac{131}{36}}\rangle, \langle \ddot{s}_{\frac{9}{16}}, \ddot{s}_{\frac{39}{16}}\rangle\right)$$

$$=\left(\langle \dot{s}_{\frac{19}{72}}, \dot{s}_{\frac{262}{135}}\rangle, \langle \ddot{s}_{\frac{199}{256}}, \ddot{s}_{\frac{299}{256}}\rangle\right).$$

Now by using Equation (A) we obtain

$$=\frac{\left(\langle \dot{s}_{\frac{31}{81}}, \dot{s}_{\frac{38}{35}}\rangle, \langle \ddot{s}_{\frac{199}{256}}, \ddot{s}_{\frac{299}{256}}\rangle\right) \oplus \left(\langle \dot{s}_{\frac{19}{72}}, \dot{s}_{\frac{262}{135}}\rangle, \langle \ddot{s}_{\frac{199}{256}}, \ddot{s}_{\frac{299}{256}}\rangle\right)}{2} = \frac{\left(\langle \dot{s}_{\frac{22037}{34992}}, \dot{s}_{\frac{4978}{14175}}\rangle, \langle \ddot{s}_{1.40362167}, \ddot{s}_{0.3410378}\rangle\right)}{2},$$

this implies that

$$\mathrm{SM}(\mathrm{S}_1,\mathrm{S}_2) = \left(\langle \dot{s}_{0.3236134}, \dot{s}_{1.4515819}\rangle, \langle \ddot{s}_{\frac{199}{256}}, \ddot{s}_{\frac{299}{256}}\rangle\right),$$

form above equation it is clear that the generalized (2-D) linguistic intuitionistic fuzzy soft expert set obtained from Type-III similarity measure is different from Type-II and Type-I similarity measures.

Now to find the relation between these similarity measures types such that from which type we get the more similarity between the opinions, we will use a score function and accuracy function [3] to rank these types of similarity measures for generalized linguistic intuitionistic fuzzy soft expert sets. Now by applying score function on a set obtained by Type-I similarity measure we have

$$S(\mathrm{SM}(\mathrm{S}_1,\mathrm{S}_2)) = \mathrm{s}_{\left(\frac{6+\frac{8}{9}-1.01835015}{12}\right)\times\left(\frac{4+\frac{5}{6}-\frac{1}{6}}{8}\right)} = \mathrm{s}_{0.2853734}.$$

Now by applying score function on Type-II similarity measure we get

$$S(\mathrm{SM}(\mathrm{S}_1,\mathrm{S}_2)) = \mathrm{s}_{\left(\frac{6+\frac{18-\sqrt{203}}{3}-\frac{1}{3}}{12}\right)\times\left(\frac{4+\frac{5}{6}-\frac{1}{6}}{8}\right)} = \mathrm{s}_{0.33626239}.$$

Now by applying score function on Type-III similarity measure we get

$$S(\mathrm{SM}(\mathrm{S}_1,\mathrm{S}_2)) = \mathrm{s}_{\left(\frac{6+0.3236134-1.4515819}{12}\right)\times\left(\frac{4+\frac{199}{256}-\frac{299}{256}}{8}\right)} = \mathrm{s}_{0.1831769}.$$

From above calculations it is clear that the Type-II similarity measure gives the value for similarity greater than the values of similarities by other two types. Now to briefly demonstrate the above similarity measure we consider another example.

EXAMPLE 4.3.2.

Government "A" wanted to improve the education system by adopting creative learning styles and for that purpose they wanted to hire the teachers how will fulfill their dreams. To achieve those objective authorities made a committee of experts for selection of candidates with the criteria fixed by the authorities and for reality based recruitment government restricts the experts that also mention their own expertise. For simplicity reasons we consider a set of two experts $\{e_1, e_2\}$ with only two candidates applied for recruitment process $\{t_1, t_2\}$ with two criteria's have been set by authorities

A. Creativeness in their own work

B. Experience in respective field

we call them as c_1 and c_2 respectively, such that $\{c_1, c_2\}$ represents a set of criteria. While the final decision for their selection of specific candidate is based on the majority opinion in favor of that candidate. Since the experts opinions will be in the form of linguistic terms and due to restriction on experts to also give their own expertise so, the opinions will be in the form of 2-D linguistic intuitionistic interval-valued fuzzy soft expert set because of hesitation and opinion against certain candidate.

For linguistic terms we consider a predefined linguistic term set S_t with variation of 't' as $0 \leq t \leq 16$, similarly linguistic term set for information about expertise we consider a linguistic term set $S_{t'}$ with variation of 't'' as $0 \leq t' \leq 14$.

The opinions of experts regarding t_1 are

$$\mathrm{S}_1(\mathrm{c}_1, \mathrm{e}_1)=\{\langle \dot{s}_{[4,5]}, \dot{s}_{[5,7]} \rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,5]} \rangle\},$$
$$\mathrm{S}_1(\mathrm{c}_2, \mathrm{e}_1)=\{\langle \dot{s}_{[3,4]}, \dot{s}_{[1,6]} \rangle, \langle \ddot{s}_{[1,4]}, \ddot{s}_{[1,3]} \rangle\},$$
$$\mathrm{S}_2(\mathrm{c}_1, \mathrm{e}_2)=\{\langle \dot{s}_{[2,3]}, \dot{s}_{[2,5]} \rangle, \langle \ddot{s}_{[3,5]}, \ddot{s}_{[4,5]} \rangle\},$$
$$\mathrm{S}_2(\mathrm{c}_2 \mathrm{e}_2,)=\{\langle \dot{s}_{[4,6]}, \dot{s}_{[3,4]} \rangle, \langle \ddot{s}_{[5,6]}, \ddot{s}_{[6,7]} \rangle\},$$

and the opinions of experts regarding t_2 are

$$\mathrm{S}_1(\mathrm{c}_1, \mathrm{e}_1)=\{\langle \dot{s}_{[1,5]}, \dot{s}_{[3,7]} \rangle, \langle \ddot{s}_{[3,6]}, \ddot{s}_{[3,5]} \rangle\},$$
$$\mathrm{S}_1(\mathrm{c}_2, \mathrm{e}_1)=\{\langle \dot{s}_{[1,4]}, \dot{s}_{[1,6]} \rangle, \langle \ddot{s}_{[3,4]}, \ddot{s}_{[1,3]} \rangle\},$$
$$\mathrm{S}_2(\mathrm{c}_1, \mathrm{e}_2)=\{\langle \dot{s}_{[2,3]}, \dot{s}_{[3,5]} \rangle, \langle \ddot{s}_{[2,5]}, \ddot{s}_{[1,5]} \rangle\},$$
$$\mathrm{S}_2(\mathrm{c}_2 \mathrm{e}_2,)=\{\langle \dot{s}_{[4,5]}, \dot{s}_{[3,4]} \rangle, \langle \ddot{s}_{[4,6]}, \ddot{s}_{[3,7]} \rangle\}.$$

Now by using Type-III similarity measure the similarity between opinions of experts in case of $'t_1'$ is

$$\frac{\left\{\langle \dot{S}_{1(c_1,e_1)_{\frac{4\oplus 5}{2}}}, \dot{S}_{1(c_1,e_1)_{\frac{5\oplus 7}{2}}}\rangle, \langle \ddot{S}_{1(c_1,e_1)_{\frac{1\oplus 2}{2}}}, \ddot{S}_{1(c_1,e_1)_{\frac{1\oplus 5}{2}}}\rangle\right\} \wedge \left\{\langle \dot{S}_{2(c_1,e_2)_{\frac{2\oplus 3}{2}}}, \dot{S}_{2(c_1,e_2)_{\frac{2\oplus 5}{2}}}\rangle, \langle \ddot{S}_{2(c_1,e_2)_{\frac{3\oplus 5}{2}}}, \ddot{S}_{2(c_1,e_2)_{\frac{4\oplus 5}{2}}}\rangle\right\}}{\left\{\langle \dot{S}_{1(c_1,e_1)_{\frac{4\oplus 5}{2}}}, \dot{S}_{1(c_1,e_1)_{\frac{5\oplus 7}{2}}}\rangle, \langle \ddot{S}_{1(c_1,e_1)_{\frac{1\oplus 2}{2}}}, \ddot{S}_{1(c_1,e_1)_{\frac{1\oplus 5}{2}}}\rangle\right\} \vee \left\{\langle \dot{S}_{2(c_1,e_2)_{\frac{2\oplus 3}{2}}}, \dot{S}_{2(c_1,e_2)_{\frac{2\oplus 5}{2}}}\rangle, \langle \ddot{S}_{2(c_1,e_2)_{\frac{3\oplus 5}{2}}}, \ddot{S}_{2(c_1,e_2)_{\frac{4\oplus 5}{2}}}\rangle\right\}} \oplus$$

$$\frac{\left\{\langle \dot{S}_{1(c_2,e_1)_{\frac{3\oplus 4}{2}}}, \dot{S}_{1(c_2,e_1)_{\frac{1\oplus 6}{2}}}\rangle, \langle \ddot{S}_{1(c_2,e_1)_{\frac{1\oplus 4}{2}}}, \ddot{S}_{1(c_2,e_1)_{\frac{1\oplus 3}{2}}}\rangle\right\} \wedge \left\{\langle \dot{S}_{2(c_2,e_2)_{\frac{4\oplus 6}{2}}}, \dot{S}_{2(c_2,e_2)_{\frac{3\oplus 4}{2}}}\rangle, \langle \ddot{S}_{2(c_2,e_2)_{\frac{5\oplus 6}{2}}}, \ddot{S}_{2(c_2,e_2)_{\frac{6\oplus 7}{2}}}\rangle\right\}}{\left\{\langle \dot{S}_{1(c_2,e_1)_{\frac{3\oplus 4}{2}}}, \dot{S}_{1(c_2,e_1)_{\frac{1\oplus 6}{2}}}\rangle, \langle \ddot{S}_{1(c_2,e_1)_{\frac{1\oplus 4}{2}}}, \ddot{S}_{1(c_2,e_1)_{\frac{1\oplus 3}{2}}}\rangle\right\} \vee \left\{\langle \dot{S}_{2(c_2,e_2)_{\frac{4\oplus 6}{2}}}, \dot{S}_{2(c_2,e_2)_{\frac{3\oplus 4}{2}}}\rangle, \langle \ddot{S}_{2(c_2,e_2)_{\frac{5\oplus 6}{2}}}, \ddot{S}_{2(c_2,e_2)_{\frac{6\oplus 7}{2}}}\rangle\right\}}$$

$$=\left(\langle \dot{s}_{\frac{5}{144}}, \dot{s}_{\frac{221}{36}}\rangle, \langle \ddot{s}_{\frac{1}{28}}, \ddot{s}_{\frac{523}{112}}\rangle\right) \oplus \left(\langle \dot{s}_{\frac{1}{16}}, \dot{s}_{\frac{117}{32}}\rangle, \langle \ddot{s}_{\frac{5}{56}}, \ddot{s}_{\frac{508}{77}}\rangle\right) = \left(\langle \dot{s}_{\frac{1193}{12288}}, \dot{s}_{\frac{2873}{2048}}\rangle, \langle \ddot{s}_{\frac{2739}{21952}}, \ddot{s}_{\frac{66421}{30184}}\rangle\right)$$

the above equation represents the similarity between the opinions of experts in the process of selecting t_1. Similarly, similarity between opinions of experts in case of $'t_2'$ using Type-III similarity measure

$$\frac{\left\{\langle \dot{S}_{1(c_1,e_1)_{\frac{1\oplus 5}{2}}}, \dot{S}_{1(c_1,e_1)_{\frac{3\oplus 7}{2}}}\rangle, \langle \ddot{S}_{1(c_1,e_1)_{\frac{3\oplus 6}{2}}}, \ddot{S}_{1(c_1,e_1)_{\frac{3\oplus 5}{2}}}\rangle\right\} \wedge \left\{\langle \dot{S}_{2(c_1,e_2)_{\frac{2\oplus 3}{2}}}, \dot{S}_{2(c_1,e_2)_{\frac{3\oplus 5}{2}}}\rangle, \langle \ddot{S}_{2(c_1,e_2)_{\frac{2\oplus 5}{2}}}, \ddot{S}_{2(c_1,e_2)_{\frac{1\oplus 5}{2}}}\rangle\right\}}{\left\{\langle \dot{S}_{1(c_1,e_1)_{\frac{1\oplus 5}{2}}}, \dot{S}_{1(c_1,e_1)_{\frac{3\oplus 7}{2}}}\rangle, \langle \ddot{S}_{1(c_1,e_1)_{\frac{3\oplus 6}{2}}}, \ddot{S}_{1(c_1,e_1)_{\frac{3\oplus 5}{2}}}\rangle\right\} \vee \left\{\langle \dot{S}_{2(c_1,e_2)_{\frac{2\oplus 3}{2}}}, \dot{S}_{2(c_1,e_2)_{\frac{3\oplus 5}{2}}}\rangle, \langle \ddot{S}_{2(c_1,e_2)_{\frac{2\oplus 5}{2}}}, \ddot{S}_{2(c_1,e_2)_{\frac{1\oplus 5}{2}}}\rangle\right\}} \oplus$$

$$\frac{\left\{\langle \dot{S}_{1(c_2,e_1)_{\frac{1\oplus 4}{2}}}, \dot{S}_{1(c_2,e_1)_{\frac{1\oplus 6}{2}}}\rangle, \langle \ddot{S}_{1(c_2,e_1)_{\frac{3\oplus 4}{2}}}, \ddot{S}_{1(c_2,e_1)_{\frac{1\oplus 3}{2}}}\rangle\right\} \wedge \left\{\langle \dot{S}_{2(c_2,e_2)_{\frac{4\oplus 5}{2}}}, \dot{S}_{2(c_2,e_2)_{\frac{3\oplus 4}{2}}}\rangle, \langle \ddot{S}_{2(c_2,e_2)_{\frac{4\oplus 6}{2}}}, \ddot{S}_{2(c_2,e_2)_{\frac{3\oplus 7}{2}}}\rangle\right\}}{\left\{\langle \dot{S}_{1(c_2,e_1)_{\frac{1\oplus 4}{2}}}, \dot{S}_{1(c_2,e_1)_{\frac{1\oplus 6}{2}}}\rangle, \langle \ddot{S}_{1(c_2,e_1)_{\frac{3\oplus 4}{2}}}, \ddot{S}_{1(c_2,e_1)_{\frac{1\oplus 3}{2}}}\rangle\right\} \vee \left\{\langle \dot{S}_{2(c_2,e_2)_{\frac{4\oplus 5}{2}}}, \dot{S}_{2(c_2,e_2)_{\frac{3\oplus 4}{2}}}\rangle, \langle \ddot{S}_{2(c_2,e_2)_{\frac{4\oplus 6}{2}}}, \ddot{S}_{2(c_2,e_2)_{\frac{3\oplus 7}{2}}}\rangle\right\}}$$

$$=\left(\langle \dot{s}_{\frac{5}{128}}, \dot{s}_{\frac{251}{48}}\rangle, \langle \ddot{s}_{\frac{1}{12}}, \ddot{s}_{\frac{262}{63}}\rangle\right) \oplus \left(\langle \dot{s}_{\frac{5}{112}}, \dot{s}_{\frac{529}{144}}\rangle, \langle \ddot{s}_{\frac{1}{8}}, \ddot{s}_{\frac{359}{70}}\rangle\right) = \left(\langle \dot{s}_{\frac{19175}{229376}}, \dot{s}_{\frac{132779}{110592}}\rangle, \langle \ddot{s}_{\frac{93}{448}}, \ddot{s}_{\frac{47029}{30870}}\rangle\right),$$

above equation represents the similarity between opinions of experts from selection judgments for t_2. Now to find the overall similarity we use (A)

$$=\frac{\left(\langle \dot{s}_{\frac{1193}{12288}}, \dot{s}_{\frac{2873}{2048}}\rangle, \langle \ddot{s}_{\frac{2739}{21952}}, \ddot{s}_{\frac{66421}{30184}}\rangle\right) \oplus \left(\langle \dot{s}_{\frac{19175}{229376}}, \dot{s}_{\frac{132779}{110592}}\rangle, \langle \ddot{s}_{\frac{93}{448}}, \ddot{s}_{\frac{47029}{30870}}\rangle\right)}{2}$$

$$=\frac{\left(\langle \dot{s}_{\frac{7818611713}{41427140608}}, \dot{s}_{\frac{381474067}{3623878656}}\rangle, \langle \ddot{s}_{\frac{45505785}{137682944}}, \ddot{s}_{\frac{3123713209}{13044921120}}\rangle\right)}{2},$$

this implies that

$$\mathrm{SM}(\mathrm{S}_1, \mathrm{S}_2) = (\langle \dot{s}_{0.0946457456}, \dot{s}_{1.297793748}\rangle, \langle \ddot{s}_{0.1453533018}, \ddot{s}_{1.830960040}\rangle).$$

4.4. TYPE-IV SIMILARITY MEASURE FOR GLIVIFSESs.

Definition. Let $\{z_i : i = 1, . . . , q\}$ where $q \in \mathbb{Z}$ represents a set of experts and $\{w_j : j = 1, . . . , r\}$ with $r \in \mathbb{Z}$ represents the alternatives or proposals and $\{h_v : v = 1, . . . , a\}$ with $a \in \mathbb{Z}$ represents the set of criteria's with general form of generalized linguistic interval-valued intuitionistic fuzzy soft expert sets

$$\mathrm{S}_1 = \left\{\langle \dot{\mathrm{s}}_{[\alpha,\alpha']}, \dot{\mathrm{s}}_{[\beta,\beta']}\rangle , \langle \ddot{\mathrm{s}}_{[\gamma,\gamma']}, \ddot{\mathrm{s}}_{[\delta,\delta']}\rangle\right\},$$
$$\mathrm{S}_2 = \left\{\langle \dot{\mathrm{s}}_{[\alpha_1,\alpha_1']}, \dot{\mathrm{s}}_{[\beta_1,\beta_1']}\rangle , \langle \ddot{\mathrm{s}}_{[\gamma_1,\gamma_1']}, \ddot{\mathrm{s}}_{[\delta_1,\delta_1']}\rangle\right\}.$$

According to Type-IV similarity measure between the opinions of experts for a specific alternative or for a specific proposal

$$\sum_{v=1}^{a}\sum_{i=1}^{q}\frac{\left[\begin{array}{c}\left\{\langle \dot{\mathrm{S}}_{1(h_v,z_i)_{[\alpha,\alpha']}}, \dot{\mathrm{S}}_{1(h_v,z_i)_{[\beta,\beta']}}\rangle , \langle \ddot{\mathrm{S}}_{1(h_v,z_i)_{[\gamma,\gamma']}}, \ddot{\mathrm{S}}_{1(h_v,z_i)_{[\delta,\delta']}}\rangle\right\} \wedge \\ \left\{\langle \dot{\mathrm{S}}_{2(h_v,z_{i+1})_{[\alpha_1,\alpha_1']}}, \dot{\mathrm{S}}_{2(h_v,z_{i+1})_{[\beta_1,\beta_1']}}\rangle , \langle \ddot{\mathrm{S}}_{2(h_v,z_{i+1})_{[\gamma_1,\gamma_1']}}, \ddot{\mathrm{S}}_{2(h_v,z_{i+1})_{[\delta_1,\delta_1']}}\rangle\right\}\end{array}\right]}{\left[\begin{array}{c}\left\{\langle \dot{\mathrm{S}}_{1(h_v,z_i)_{[\alpha,\alpha']}}, \dot{\mathrm{S}}_{1(h_v,z_i)_{[\beta,\beta']}}\rangle , \langle \ddot{\mathrm{S}}_{1(h_v,z_i)_{[\gamma,\gamma']}}, \ddot{\mathrm{S}}_{1(h_v,z_i)_{[\delta,\delta']}}\rangle\right\} \vee \\ \left\{\langle \dot{\mathrm{S}}_{2(h_v,z_{i+1})_{[\alpha_1,\alpha_1']}}, \dot{\mathrm{S}}_{2(h_v,z_{i+1})_{[\beta_1,\beta_1']}}\rangle , \langle \ddot{\mathrm{S}}_{2(h_v,z_{i+1})_{[\gamma_1,\gamma_1']}}, \ddot{\mathrm{S}}_{2(h_v,z_{i+1})_{[\delta_1,\delta_1']}}\rangle\right\}\end{array}\right]} \quad (4.4)$$

now to find the overall similarity between the opinions of experts for all alternatives we will use

$$\mathrm{SM}(\mathrm{S}_1, \mathrm{S}_2) = \frac{\sum_{\mathrm{j}=1}^{\mathrm{r}} S_j(\mathrm{S}_1, \mathrm{S}_2)}{\mathrm{j}}.$$

To illustrate the above we construct an example to briefly discuss the methodology of that similarity measure, but firstly we consider the Example 4.1.1 to observe the difference between similarities by different similarity measures.

EXAMPLE 4.4.1.

Collecting data from Example 4.1.1, the expert's opinions regarding t_1 are as under

$$\mathrm{S}_1(\mathrm{h}_1, \mathrm{a}) = \left\{\langle \dot{\mathrm{s}}_{[2,3]}, \dot{\mathrm{s}}_{[1,2]}\rangle , \langle \ddot{\mathrm{s}}_{[1,2]}, \ddot{\mathrm{s}}_{[1,2]}\rangle\right\},$$

$$\mathrm{S}_1(\mathrm{h}_2,\mathrm{a})=\left\{\langle \dot{\mathrm{s}}_{[3,4]}, \dot{\mathrm{s}}_{[1,2]} \rangle, \langle \ddot{\mathrm{s}}_{[1,2]}, \ddot{\mathrm{s}}_{[1,2]} \rangle\right\},$$
$$\mathrm{S}_2(\mathrm{h}_1,\mathrm{b})=\left\{\langle \dot{\mathrm{s}}_{[1,3]}, \dot{\mathrm{s}}_{[2,3]} \rangle, \langle \ddot{\mathrm{s}}_{[1,2]}, \ddot{\mathrm{s}}_{[1,2]} \rangle\right\},$$
$$\mathrm{S}_2(\mathrm{h}_2,\mathrm{b})=\left\{\langle \dot{\mathrm{s}}_{[1,2]}, \dot{\mathrm{s}}_{[3,4]} \rangle, \langle \ddot{\mathrm{s}}_{[1,2]}, \ddot{\mathrm{s}}_{[1,2]} \rangle\right\},$$

and the opinions of experts regarding t_2 are as under

$$\mathrm{S}_1(\mathrm{h}_1,\mathrm{a})=\left\{\langle \dot{\mathrm{s}}_{[1,2]}, \dot{\mathrm{s}}_{[2,4]} \rangle, \langle \ddot{\mathrm{s}}_{[1,2]}, \ddot{\mathrm{s}}_{[1,2]} \rangle\right\},$$
$$\mathrm{S}_1(\mathrm{h}_2,\mathrm{a})=\left\{\langle \dot{\mathrm{s}}_{[2,4]}, \dot{\mathrm{s}}_{[1,2]} \rangle, \langle \ddot{\mathrm{s}}_{[1,2]}, \ddot{\mathrm{s}}_{[1,2]} \rangle\right\},$$
$$\mathrm{S}_2(\mathrm{h}_1,\mathrm{b})=\left\{\langle \dot{\mathrm{s}}_{[2,3]}, \dot{\mathrm{s}}_{[2,3]} \rangle, \langle \ddot{\mathrm{s}}_{[1,2]}, \ddot{\mathrm{s}}_{[1,2]} \rangle\right\},$$
$$\mathrm{S}_2(\mathrm{h}_2,\mathrm{b})=\left\{\langle \dot{\mathrm{s}}_{[1,2]}, \dot{\mathrm{s}}_{[3,4]} \rangle, \langle \ddot{\mathrm{s}}_{[1,2]}, \ddot{\mathrm{s}}_{[1,2]} \rangle\right\}.$$

Now by applying Type-IV similarity measure to find the similarity between the opinions of experts for t_1 we get

$$\frac{\begin{bmatrix} \left\{\langle \dot{\mathrm{s}}_{1(h_1,a)_{[2,3]}}, \dot{\mathrm{s}}_{1(h_1,a)_{[1,2]}} \rangle, \langle \ddot{\mathrm{s}}_{1(h_1,a)_{[1,2]}}, \ddot{\mathrm{s}}_{1(h_1,a)_{[1,2]}} \rangle\right\} \wedge \\ \left\{\langle \dot{\mathrm{s}}_{2(h_1,b)_{[1,3]}}, \dot{\mathrm{s}}_{2(h_1,b)_{[2,3]}} \rangle, \langle \ddot{\mathrm{s}}_{2(h_1,b)_{[1,2]}}, \ddot{\mathrm{s}}_{2(h_1,b)_{[1,2]}} \rangle\right\} \end{bmatrix}}{\begin{bmatrix} \left\{\langle \dot{\mathrm{s}}_{1(h_1,a)_{[2,3]}}, \dot{\mathrm{s}}_{1(h_1,a)_{[1,2]}} \rangle, \langle \ddot{\mathrm{s}}_{1(h_1,a)_{[1,2]}}, \ddot{\mathrm{s}}_{1(h_1,a)_{[1,2]}} \rangle\right\} \vee \\ \left\{\langle \dot{\mathrm{s}}_{2(h_1,b)_{[1,3]}}, \dot{\mathrm{s}}_{2(h_1,b)_{[2,3]}} \rangle, \langle \ddot{\mathrm{s}}_{2(h_1,b)_{[1,2]}}, \ddot{\mathrm{s}}_{2(h_1,b)_{[1,2]}} \rangle\right\} \end{bmatrix}}$$
$$\oplus \frac{\begin{bmatrix} \left\{\langle \dot{\mathrm{s}}_{1(h_2,a)_{[3,4]}}, \dot{\mathrm{s}}_{1(h_2,a)_{[1,2]}} \rangle, \langle \ddot{\mathrm{s}}_{1(h_2,a)_{[1,2]}}, \ddot{\mathrm{s}}_{1(h_2,a)_{[1,2]}} \rangle\right\} \wedge \\ \left\{\langle \dot{\mathrm{s}}_{2(h_2,b)_{[1,2]}}, \dot{\mathrm{s}}_{2(h_2,b)_{[3,4]}} \rangle, \langle \ddot{\mathrm{s}}_{2(h_2,b)_{[1,2]}}, \ddot{\mathrm{s}}_{2(h_2,b)_{[1,2]}} \rangle\right\} \end{bmatrix}}{\begin{bmatrix} \left\{\langle \dot{\mathrm{s}}_{1(h_2,a)_{[3,4]}}, \dot{\mathrm{s}}_{1(h_2,a)_{[1,2]}} \rangle, \langle \ddot{\mathrm{s}}_{1(h_2,a)_{[1,2]}}, \ddot{\mathrm{s}}_{1(h_2,a)_{[1,2]}} \rangle\right\} \vee \\ \left\{\langle \dot{\mathrm{s}}_{2(h_2,b)_{[1,2]}}, \dot{\mathrm{s}}_{2(h_2,b)_{[3,4]}} \rangle, \langle \ddot{\mathrm{s}}_{2(h_2,b)_{[1,2]}}, \ddot{\mathrm{s}}_{2(h_2,b)_{[1,2]}} \rangle\right\} \end{bmatrix}}$$

after finding intersection and union we get

$$=\begin{bmatrix} \left\{\left(\langle \dot{\mathrm{s}}_{[1,3]}, \dot{\mathrm{s}}_{[2,3]} \rangle, \langle \ddot{\mathrm{s}}_{[1,2]}, \ddot{\mathrm{s}}_{[1,2]} \rangle\right) \otimes \left(\langle \dot{\mathrm{s}}_{\left[\frac{1}{3},\frac{1}{2}\right]}, \dot{\mathrm{s}}_{\left[\frac{1}{2},1\right]} \rangle, \langle \ddot{\mathrm{s}}_{\left[\frac{1}{2},1\right]}, \ddot{\mathrm{s}}_{\left[\frac{1}{2},1\right]} \rangle\right)\right\} \oplus \\ \left\{\left(\langle \dot{\mathrm{s}}_{[1,2]}, \dot{\mathrm{s}}_{[3,4]} \rangle, \langle \ddot{\mathrm{s}}_{[1,2]}, \ddot{\mathrm{s}}_{[1,2]} \rangle\right) \otimes \left(\langle \dot{\mathrm{s}}_{\left[\frac{1}{4},\frac{1}{3}\right]}, \dot{\mathrm{s}}_{\left[\frac{1}{2},1\right]} \rangle, \langle \ddot{\mathrm{s}}_{\left[\frac{1}{2},1\right]}, \ddot{\mathrm{s}}_{\left[\frac{1}{2},1\right]} \rangle\right)\right\} \end{bmatrix}$$
$$=\left(\langle \dot{\mathrm{s}}_{\left[\frac{1}{18},\frac{1}{4}\right]}, \dot{\mathrm{s}}_{\left[\frac{7}{3},\frac{7}{2}\right]} \rangle, \langle \ddot{\mathrm{s}}_{\left[\frac{1}{8},\frac{1}{2}\right]}, \ddot{\mathrm{s}}_{\left[\frac{11}{8},\frac{5}{2}\right]} \rangle\right) \oplus \left(\langle \dot{\mathrm{s}}_{\left[\frac{1}{24},\frac{1}{9}\right]}, \dot{\mathrm{s}}_{\left[\frac{10}{3},\frac{13}{2}\right]} \rangle, \langle \ddot{\mathrm{s}}_{\left[\frac{1}{8},\frac{1}{2}\right]}, \ddot{\mathrm{s}}_{\left[\frac{11}{8},\frac{5}{2}\right]} \rangle\right)$$
$$=\left(\langle \dot{\mathrm{s}}_{\left[\frac{251}{2592},\frac{77}{216}\right]}, \dot{\mathrm{s}}_{\left[\frac{35}{27},\frac{91}{36}\right]} \rangle, \langle \ddot{\mathrm{s}}_{\left[\frac{63}{256},\frac{15}{16}\right]}, \ddot{\mathrm{s}}_{\left[\frac{121}{256},\frac{25}{16}\right]} \rangle\right),$$

This represents the similarity between the opinions of experts regarding the teacher t_1.

Similarly, similarity between opinions of experts in case of t_2 using Type-IV similarity measure

$$\frac{\left[\begin{array}{l}\left\{\left\langle \dot{s}_{1_{(h_1,a)_{[1,2]}}}, \dot{s}_{1(h_1,a)_{[2,4]}}\right\rangle, \left\langle \ddot{s}_{1(h_1,a)_{[1,2]}}, \ddot{s}_{1(h_1,a)_{[1,2]}}\right\rangle\right\} \wedge \\ \left\{\left\langle \dot{s}_{2_{(h_1,b)_{[2,3]}}}, \dot{s}_{2(h_1,b)_{[2,3]}}\right\rangle, \left\langle \ddot{s}_{2(h_1,b)_{[1,2]}}, \ddot{s}_{2(h_1,b)_{[1,2]}}\right\rangle\right\}\end{array}\right]}{\left[\begin{array}{l}\left\{\left\langle \dot{s}_{1_{(h_1,a)_{[1,2]}}}, \dot{s}_{1(h_1,a)_{[2,4]}}\right\rangle, \left\langle \ddot{s}_{1(h_1,a)_{[1,2]}}, \ddot{s}_{1(h_1,a)_{[1,2]}}\right\rangle\right\} \vee \\ \left\{\left\langle \dot{s}_{2_{(h_1,b)_{[2,3]}}}, \dot{s}_{2(h_1,b)_{[2,3]}}\right\rangle, \left\langle \ddot{s}_{2(h_1,b)_{[1,2]}}, \ddot{s}_{2(h_1,b)_{[1,2]}}\right\rangle\right\}\end{array}\right]}$$

$$\oplus \frac{\left[\begin{array}{l}\left\{\left\langle \dot{s}_{1_{(h_2,a)_{[2,4]}}}, \dot{s}_{1(h_2,a)_{[1,2]}}\right\rangle, \left\langle \ddot{s}_{1(h_2,a)_{[1,2]}}, \ddot{s}_{1(h_2,a)_{[1,2]}}\right\rangle\right\} \wedge \\ \left\{\left\langle \dot{s}_{2_{(h_2,b)_{[1,2]}}}, \dot{s}_{2(h_2,b)_{[3,4]}}\right\rangle, \left\langle \ddot{s}_{2(h_2,b)_{[1,2]}}, \ddot{s}_{2(h_2,b)_{[1,2]}}\right\rangle\right\}\end{array}\right]}{\left[\begin{array}{l}\left\{\left\langle \dot{s}_{1_{(h_2,a)_{[2,4]}}}, \dot{s}_{1(h_2,a)_{[1,2]}}\right\rangle, \left\langle \ddot{s}_{1(h_2,a)_{[1,2]}}, \ddot{s}_{1(h_2,a)_{[1,2]}}\right\rangle\right\} \vee \\ \left\{\left\langle \dot{s}_{2_{(h_2,b)_{[1,2]}}}, \dot{s}_{2(h_2,b)_{[3,4]}}\right\rangle, \left\langle \ddot{s}_{2(h_2,b)_{[1,2]}}, \ddot{s}_{2(h_2,b)_{[1,2]}}\right\rangle\right\}\end{array}\right]}$$

$$= \left[\begin{array}{l}\left\{\left(\left\langle \dot{s}_{[1,2]}, \dot{s}_{[2,4]}\right\rangle, \left\langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\right\rangle\right) \otimes \left(\left\langle \dot{s}_{\left[\frac{1}{3},\frac{1}{2}\right]}, \dot{s}_{\left[\frac{1}{3},\frac{1}{2}\right]}\right\rangle, \left\langle \ddot{s}_{\left[\frac{1}{2},1\right]}, \ddot{s}_{\left[\frac{1}{2},1\right]}\right\rangle\right)\right\} \oplus \\ \left\{\left(\left\langle \dot{s}_{[1,2]}, \dot{s}_{[3,4]}\right\rangle, \left\langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\right\rangle\right) \otimes \left(\left\langle \dot{s}_{\left[\frac{1}{4},\frac{1}{2}\right]}, \dot{s}_{\left[\frac{1}{2},1\right]}\right\rangle, \left\langle \ddot{s}_{\left[\frac{1}{2},1\right]}, \ddot{s}_{\left[\frac{1}{2},1\right]}\right\rangle\right)\right\}\end{array}\right]$$

$$= \left(\left\langle \dot{s}_{\left[\frac{1}{18},\frac{1}{6}\right]}, \dot{s}_{\left[\frac{20}{9},\frac{25}{6}\right]}\right\rangle, \left\langle \ddot{s}_{\left[\frac{1}{8},\frac{1}{2}\right]}, \ddot{s}_{\left[\frac{11}{8},\frac{5}{2}\right]}\right\rangle\right) \oplus \left(\left\langle \dot{s}_{\left[\frac{1}{24},\frac{1}{6}\right]}, \dot{s}_{\left[\frac{10}{3},\frac{13}{2}\right]}\right\rangle, \left\langle \ddot{s}_{\left[\frac{1}{8},\frac{1}{2}\right]}, \ddot{s}_{\left[\frac{11}{8},\frac{5}{2}\right]}\right\rangle\right)$$

$$= \left(\left\langle \dot{s}_{\left[\frac{251}{2592},\frac{71}{216}\right]}, \dot{s}_{\left[\frac{100}{81},\frac{325}{108}\right]}\right\rangle, \left\langle \ddot{s}_{\left[\frac{63}{256},\frac{15}{16}\right]}, \ddot{s}_{\left[\frac{121}{256},\frac{25}{16}\right]}\right\rangle\right),$$

this represents the similarity between the opinions of experts for evaluation of teacher t_2.

Now to find the similarity between opinions of experts for both the teachers t_1 and t_2, we use the following rule

$$\mathrm{SM}(\mathrm{S}_1, \mathrm{S}_2) = \frac{\sum_{\mathrm{i}=1}^{2} S_i(\mathrm{S}_1, \mathrm{S}_2)}{2},$$

By substituting these values we get

$$= \frac{\left(\left\langle \dot{s}_{\left[\frac{251}{2592},\frac{77}{216}\right]}, \dot{s}_{\left[\frac{35}{27},\frac{91}{36}\right]}\right\rangle, \left\langle \ddot{s}_{\left[\frac{63}{256},\frac{15}{16}\right]}, \ddot{s}_{\left[\frac{121}{256},\frac{25}{16}\right]}\right\rangle\right) \oplus \left(\left\langle \dot{s}_{\left[\frac{251}{2592},\frac{71}{216}\right]}, \dot{s}_{\left[\frac{100}{81},\frac{325}{108}\right]}\right\rangle, \left\langle \ddot{s}_{\left[\frac{63}{256},\frac{15}{16}\right]}, \ddot{s}_{\left[\frac{121}{256},\frac{25}{16}\right]}\right\rangle\right)}{2}$$

$$= \frac{1}{2}\left(\left\langle \dot{s}_{\left[\frac{7744103}{40310784},\frac{811}{1296}\right]}, \dot{s}_{\left[\frac{1750}{6561},\frac{29575}{23328}\right]}\right\rangle, \left\langle \ddot{s}_{\left[\frac{125055}{262144},\frac{1695}{1024}\right]}, \ddot{s}_{\left[\frac{14641}{262144},\frac{625}{1024}\right]}\right\rangle\right),$$

by applying operation of scalar multiplication we get

$$\mathrm{SM}(\mathrm{S}_1, \mathrm{S}_2) = \left(\left\langle \dot{s}_{\left[\frac{251}{2592},0.3214993\right]}, \dot{s}_{[1.26505565,2.75803167]}\right\rangle, \left\langle \ddot{s}_{\left[\frac{63}{256},\frac{15}{16}\right]}, \ddot{s}_{\left[\frac{121}{256},\frac{25}{16}\right]}\right\rangle\right),$$

that above equation represents the similarity between opinions of experts for all the alternatives (teachers) using Type-IV similarity measure.

Now to compare the similarity results obtained by Type-IV similarity measure with the previous similarity measures results we use the score and accuracy function such that

$$S(\mathrm{SM}(\mathrm{S}_1,\mathrm{S}_2)) = s_{\left[\frac{6+\frac{251}{2592}-1.26505565}{12},\frac{6+0.3214993-2.75803167}{12}\right]\otimes\left[\frac{4+\frac{63}{256}-\frac{121}{256}}{8},\frac{4+\frac{15}{16}-\frac{25}{16}}{8}\right]}$$

$= s_{[0.4026484,0.29695564]\otimes\left[\frac{483}{1024},\frac{27}{64}\right]} = s_{[0.125278160,0.1899210715]} = s_{0.1575996157}$,

this shows that similarity result obtained by Type-IV similarity measure is lowest among previous similarity measures results.

EXAMPLE 4.4.2.

A company announces to allocate the budget to its department if the following conditions will be satisfied

- Department annual performance
- Continuity in work
- Customer's responses

for that purpose the executive committee of that company nominates the experts $\{e_1, e_2\}$ to give suggestions by evaluation of that department, where the decision will be made on the behalf of expert's opinions regarding the fulfillment of conditions seated by company with the restriction on experts that also attach their own knowledge information about that department.

Experts set a set of criteria for judgment of departmental performance based on the conditions imposed by executive committee of company

- Number of employees
- Office scheduled timings

we call them as x_1, x_2 respectively such that $\{x_1, x_2\}$ represent a set of criteria. Now due to unshorten or leak of exactly the right opinion, experts give opinions in the form of intervals from predefined linguistic term set S_t with $0 \leq t \leq 20$ for opinions regarding department and for information regarding the own knowledge the predefined linguistic term set $S_{t'}$ with $0 \leq t' \leq 14$.

The expert's opinions regarding the 'department annual performance'

$$S_1(x_1,e_1)=\left\{\langle \dot{s}_{[1,6]},\dot{s}_{[2,4]}\rangle ,\langle \ddot{s}_{[1,3]} ,\ddot{s}_{[5,8]}\rangle\right\},$$
$$S_1(x_2,e_1)=\left\{\langle \dot{s}_{[2,8]},\dot{s}_{[1,3]}\rangle ,\langle \ddot{s}_{[2,4]} ,\ddot{s}_{[1,5]}\rangle\right\},$$
$$S_2(x_1,e_2)=\left\{\langle \dot{s}_{[5,6]},\dot{s}_{[2,3]}\rangle ,\langle \ddot{s}_{[1,3]} ,\ddot{s}_{[1,2]}\rangle\right\},$$
$$S_2(x_2,e_2)=\left\{\langle \dot{s}_{[5,8]},\dot{s}_{[3,4]}\rangle ,\langle \ddot{s}_{[1,4]} ,\ddot{s}_{[3,4]}\rangle\right\}.$$

The expert's opinions regarding the 'continuity in work'

$$S_1(x_1,e_1)=\left\{\langle \dot{s}_{[4,8]},\dot{s}_{[1,3]}\rangle ,\langle \ddot{s}_{[2,3]} ,\ddot{s}_{[5,6]}\rangle\right\},$$
$$S_1(x_2,e_1)=\left\{\langle \dot{s}_{[7,8]},\dot{s}_{[1,2]}\rangle ,\langle \ddot{s}_{[3,4]} ,\ddot{s}_{[3,5]}\rangle\right\},$$
$$S_2(x_1,e_2)=\left\{\langle \dot{s}_{[5,9]},\dot{s}_{[2,5]}\rangle ,\langle \ddot{s}_{[2,3]} ,\ddot{s}_{[1,3]}\rangle\right\},$$
$$S_2(x_2,e_2)=\left\{\langle \dot{s}_{[5,10]},\dot{s}_{[2,4]}\rangle ,\langle \ddot{s}_{[3,7]} ,\ddot{s}_{[1,2]}\rangle\right\}.$$

The expert's opinions regarding the 'customer's responses'

$$S_1(x_1,e_1)=\left\{\langle \dot{s}_{[3,6]},\dot{s}_{[2,5]}\rangle ,\langle \ddot{s}_{[3,4]} ,\ddot{s}_{[1,2]}\rangle\right\},$$
$$S_1(x_2,e_1)=\left\{\langle \dot{s}_{[6,8]},\dot{s}_{[1,2]}\rangle ,\langle \ddot{s}_{[2,7]} ,\ddot{s}_{[1,3]}\rangle\right\},$$
$$S_2(x_1,e_2)=\left\{\langle \dot{s}_{[5,9]},\dot{s}_{[2,4]}\rangle ,\langle \ddot{s}_{[6,9]} ,\ddot{s}_{[1,4]}\rangle\right\},$$
$$S_2(x_2,e_2)=\left\{\langle \dot{s}_{[5,8]},\dot{s}_{[1,4]}\rangle ,\langle \ddot{s}_{[5,10]} ,\ddot{s}_{[1,3]}\rangle\right\}.$$

Now by applying Type-IV similarity measure to calculate the similarity between opinions of experts in case of 'department annual report' we get

$$\frac{\begin{bmatrix}\left\{\langle \dot{s}_{1_{(x_1,e_1)_{[1,6]}}},\dot{s}_{1(x_1,e_1)_{[2,4]}}\rangle ,\langle \ddot{s}_{1(x_1,e_1)_{[1,3]}} ,\ddot{s}_{1(x_1,e_1)_{[5,8]}}\rangle\right\}\wedge \\ \left\{\langle \dot{s}_{2_{(x_1,e_2)_{[5,6]}}},\dot{s}_{2(x_1,e_2)_{[2,3]}}\rangle ,\langle \ddot{s}_{2(x_1,e_2)_{[1,3]}} ,\ddot{s}_{2(x_1,e_2)_{[1,2]}}\rangle\right\}\end{bmatrix}}{\begin{bmatrix}\left\{\langle \dot{s}_{1_{(x_1,e_1)_{[1,6]}}},\dot{s}_{1(x_1,e_1)_{[2,4]}}\rangle ,\langle \ddot{s}_{1(x_1,e_1)_{[1,3]}} ,\ddot{s}_{1(x_1,e_1)_{[5,8]}}\rangle\right\}\vee \\ \left\{\langle \dot{s}_{2_{(x_1,e_2)_{[5,6]}}},\dot{s}_{2(x_1,e_2)_{[2,3]}}\rangle ,\langle \ddot{s}_{2(x_1,e_2)_{[1,3]}} ,\ddot{s}_{2(x_1,e_2)_{[1,2]}}\rangle\right\}\end{bmatrix}}$$

$$\oplus \frac{\begin{bmatrix}\left\{\langle \dot{s}_{1_{(x_2,e_1)_{[2,8]}}},\dot{s}_{1(x_2,e_1)_{[1,3]}}\rangle ,\langle \ddot{s}_{1(x_2,e_1)_{[2,4]}} ,\ddot{s}_{1(x_2,e_1)_{[1,5]}}\rangle\right\}\wedge \\ \left\{\langle \dot{s}_{2_{(x_2,e_2)_{[5,8]}}},\dot{s}_{2(x_2,e_2)_{[3,4]}}\rangle ,\langle \ddot{s}_{2(x_2,e_2)_{[1,4]}} ,\ddot{s}_{2(x_2,e_2)_{[3,4]}}\rangle\right\}\end{bmatrix}}{\begin{bmatrix}\left\{\langle \dot{s}_{1_{(x_2,e_1)_{[2,8]}}},\dot{s}_{1(x_2,e_1)_{[1,3]}}\rangle ,\langle \ddot{s}_{1(x_2,e_1)_{[2,4]}} ,\ddot{s}_{1(x_2,e_1)_{[1,5]}}\rangle\right\}\vee \\ \left\{\langle \dot{s}_{2_{(x_2,e_2)_{[5,8]}}},\dot{s}_{2(x_2,e_2)_{[3,4]}}\rangle ,\langle \ddot{s}_{2(x_2,e_2)_{[1,4]}} ,\ddot{s}_{2(x_2,e_2)_{[3,4]}}\rangle\right\}\end{bmatrix}}$$

$$=\begin{bmatrix}\left\{\left(\langle \dot{s}_{[1,6]},\dot{s}_{[2,4]}\rangle ,\langle \ddot{s}_{[1,3]} ,\ddot{s}_{[5,8]}\rangle\right)\otimes\left(\langle \dot{s}_{\left[\frac{1}{6},\frac{1}{5}\right]},\dot{s}_{\left[\frac{1}{3},\frac{1}{2}\right]}\rangle ,\langle \ddot{s}_{\left[\frac{1}{3},1\right]} ,\ddot{s}_{\left[\frac{1}{2},1\right]}\rangle\right)\right\}\oplus \\ \left\{\left(\langle \dot{s}_{[2,8]},\dot{s}_{[3,4]}\rangle ,\langle \ddot{s}_{[1,4]} ,\ddot{s}_{[3,5]}\rangle\right)\otimes\left(\langle \dot{s}_{\left[\frac{1}{8},\frac{1}{5}\right]},\dot{s}_{\left[\frac{1}{3},1\right]}\rangle ,\langle \ddot{s}_{\left[\frac{1}{4},\frac{1}{2}\right]} ,\ddot{s}_{\left[\frac{1}{4},1\right]}\rangle\right)\right\}\end{bmatrix}$$

$$=\left(\left\langle \dot{s}_{\left[\frac{1}{120},\frac{3}{50}\right]}, \dot{s}_{\left[\frac{23}{10},\frac{22}{5}\right]}\right\rangle, \left\langle \ddot{s}_{\left[\frac{1}{42},\frac{3}{28}\right]}, \ddot{s}_{\left[\frac{149}{28},\frac{59}{7}\right]}\right\rangle\right) \oplus \left(\left\langle \dot{s}_{\left[\frac{1}{80},\frac{2}{25}\right]}, \dot{s}_{\left[\frac{197}{60},\frac{24}{5}\right]}\right\rangle, \left\langle \ddot{s}_{\left[\frac{1}{56},\frac{1}{7}\right]}, \ddot{s}_{\left[\frac{179}{56},\frac{79}{14}\right]}\right\rangle\right)$$

$$=\left(\left\langle \dot{s}_{\left[\frac{1333}{64000},\frac{1747}{12500}\right]}, \dot{s}_{\left[\frac{4531}{12000},\frac{132}{125}\right]}\right\rangle, \left\langle \ddot{s}_{\left[\frac{457}{10976},\frac{683}{2744}\right]}, \ddot{s}_{\left[\frac{26671}{21952},\frac{4661}{1372}\right]}\right\rangle\right).$$

The similarity between opinions of experts for 'continuity in work' using Type-IV similarity measure is given as

$$\frac{\left[\begin{array}{l}\left\{\left\langle \dot{s}_{1(x_1,e_1)_{[4,8]}}, \dot{s}_{1(x_1,e_1)_{[1,3]}}\right\rangle, \left\langle \ddot{s}_{1(x_1,e_1)_{[2,3]}}, \ddot{s}_{1(x_1,e_1)_{[5,6]}}\right\rangle\right\} \wedge \\ \left\{\left\langle \dot{s}_{2(x_1,e_2)_{[5,9]}}, \dot{s}_{2(x_1,e_2)_{[2,5]}}\right\rangle, \left\langle \ddot{s}_{2(x_1,e_2)_{[2,3]}}, \ddot{s}_{2(x_1,e_2)_{[1,3]}}\right\rangle\right\}\end{array}\right]}{\left[\begin{array}{l}\left\{\left\langle \dot{s}_{1(x_1,e_1)_{[4,8]}}, \dot{s}_{1(x_1,e_1)_{[1,3]}}\right\rangle, \left\langle \ddot{s}_{1(x_1,e_1)_{[2,3]}}, \ddot{s}_{1(x_1,e_1)_{[5,6]}}\right\rangle\right\} \wedge \\ \left\{\left\langle \dot{s}_{2(x_1,e_2)_{[5,9]}}, \dot{s}_{2(x_1,e_2)_{[2,5]}}\right\rangle, \left\langle \ddot{s}_{2(x_1,e_2)_{[2,3]}}, \ddot{s}_{2(x_1,e_2)_{[1,3]}}\right\rangle\right\}\end{array}\right]}$$

$$\oplus \frac{\left[\begin{array}{l}\left\{\left\langle \dot{s}_{1(x_2,e_1)_{[7,8]}}, \dot{s}_{1(x_2,e_1)_{[1,2]}}\right\rangle, \left\langle \ddot{s}_{1(x_2,e_1)_{[3,4]}}, \ddot{s}_{1(x_2,e_1)_{[3,5]}}\right\rangle\right\} \wedge \\ \left\{\left\langle \dot{s}_{2(x_2,e_2)_{[5,10]}}, \dot{s}_{2(x_2,e_2)_{[2,4]}}\right\rangle, \left\langle \ddot{s}_{2(x_2,e_2)_{[3,7]}}, \ddot{s}_{2(x_2,e_2)_{[1,2]}}\right\rangle\right\}\end{array}\right]}{\left[\begin{array}{l}\left\{\left\langle \dot{s}_{1(x_2,e_1)_{[7,8]}}, \dot{s}_{1(x_2,e_1)_{[1,2]}}\right\rangle, \left\langle \ddot{s}_{1(x_2,e_1)_{[3,4]}}, \ddot{s}_{1(x_2,e_1)_{[3,5]}}\right\rangle\right\} \vee \\ \left\{\left\langle \dot{s}_{2(x_2,e_2)_{[5,10]}}, \dot{s}_{2(x_2,e_2)_{[2,4]}}\right\rangle, \left\langle \ddot{s}_{2(x_2,e_2)_{[3,7]}}, \ddot{s}_{2(x_2,e_2)_{[1,2]}}\right\rangle\right\}\end{array}\right]}$$

$$=\left[\begin{array}{l}\left\{\left(\left\langle \dot{s}_{[4,8]}, \dot{s}_{[2,5]}\right\rangle, \left\langle \ddot{s}_{[2,3]}, \ddot{s}_{[5,6]}\right\rangle\right) \otimes \left(\left\langle \dot{s}_{\left[\frac{1}{9},\frac{1}{5}\right]}, \dot{s}_{\left[\frac{1}{3},1\right]}\right\rangle, \left\langle \ddot{s}_{\left[\frac{1}{3},\frac{1}{2}\right]}, \ddot{s}_{\left[\frac{1}{3},1\right]}\right\rangle\right)\right\} \oplus \\ \left\{\left(\left\langle \dot{s}_{[5,8]}, \dot{s}_{[2,4]}\right\rangle, \left\langle \ddot{s}_{[3,4]}, \ddot{s}_{[3,5]}\right\rangle\right) \otimes \left(\left\langle \dot{s}_{\left[\frac{1}{10},\frac{1}{7}\right]}, \dot{s}_{\left[\frac{1}{2},1\right]}\right\rangle, \left\langle \ddot{s}_{\left[\frac{1}{4},\frac{1}{3}\right]}, \ddot{s}_{\left[\frac{1}{2},1\right]}\right\rangle\right)\right\}\end{array}\right]$$

$$=\left(\left\langle \dot{s}_{\left[\frac{1}{45},\frac{2}{25}\right]}, \dot{s}_{\left[\frac{23}{10},\frac{23}{4}\right]}\right\rangle, \left\langle \ddot{s}_{\left[\frac{1}{21},\frac{3}{28}\right]}, \ddot{s}_{\left[\frac{73}{14},\frac{46}{7}\right]}\right\rangle\right) \oplus \left(\left\langle \dot{s}_{\left[\frac{1}{40},\frac{2}{35}\right]}, \dot{s}_{\left[\frac{49}{20},\frac{24}{5}\right]}\right\rangle, \left\langle \ddot{s}_{\left[\frac{3}{56},\frac{2}{21}\right]}, \ddot{s}_{\left[\frac{95}{28},\frac{79}{14}\right]}\right\rangle\right)$$

$$=\left(\left\langle \dot{s}_{\left[\frac{1699}{36000},\frac{5999}{4375}\right]}, \dot{s}_{\left[\frac{1127}{4000},\frac{69}{50}\right]}\right\rangle, \left\langle \ddot{s}_{\left[\frac{1663}{16464},\frac{415}{2058}\right]}, \ddot{s}_{\left[\frac{6935}{5488},\frac{1817}{686}\right]}\right\rangle\right).$$

Similarly, the similarity between the opinions of experts in a case of 'custom's responses'

$$\frac{\begin{bmatrix}\left\{\langle \dot{s}_{1(x_1,e_1)_{[3,6]}}, \dot{s}_{1(x_1,e_1)_{[2,5]}}\rangle, \langle \ddot{s}_{1(x_1,e_1)_{[3,4]}}, \ddot{s}_{1(x_1,e_1)_{[1,2]}}\rangle\right\} \wedge \\ \left\{\langle \dot{s}_{2(x_1,e_2)_{[5,9]}}, \dot{s}_{2(x_1,e_2)_{[2,4]}}\rangle, \langle \ddot{s}_{2(x_1,e_2)_{[6,9]}}, \ddot{s}_{2(x_1,e_2)_{[1,4]}}\rangle\right\}\end{bmatrix}}{\begin{bmatrix}\left\{\langle \dot{s}_{1(x_1,e_1)_{[3,6]}}, \dot{s}_{1(x_1,e_1)_{[2,5]}}\rangle, \langle \ddot{s}_{1(x_1,e_1)_{[3,4]}}, \ddot{s}_{1(x_1,e_1)_{[1,2]}}\rangle\right\} \vee \\ \left\{\langle \dot{s}_{2(x_1,e_2)_{[5,9]}}, \dot{s}_{2(x_1,e_2)_{[2,4]}}\rangle, \langle \ddot{s}_{2(x_1,e_2)_{[6,9]}}, \ddot{s}_{2(x_1,e_2)_{[1,4]}}\rangle\right\}\end{bmatrix}}$$

$$\oplus \frac{\begin{bmatrix}\left\{\langle \dot{s}_{1(x_2,e_1)_{[6,8]}}, \dot{s}_{1(x_2,e_1)_{[1,2]}}\rangle, \langle \ddot{s}_{1(x_2,e_1)_{[2,7]}}, \ddot{s}_{1(x_2,e_1)_{[1,3]}}\rangle\right\} \wedge \\ \left\{\langle \dot{s}_{2(x_2,e_2)_{[5,8]}}, \dot{s}_{2(x_2,e_2)_{[1,4]}}\rangle, \langle \ddot{s}_{2(x_2,e_2)_{[5,10]}}, \ddot{s}_{2(x_2,e_2)_{[1,3]}}\rangle\right\}\end{bmatrix}}{\begin{bmatrix}\left\{\langle \dot{s}_{1(x_2,e_1)_{[6,8]}}, \dot{s}_{1(x_2,e_1)_{[1,2]}}\rangle, \langle \ddot{s}_{1(x_2,e_1)_{[2,7]}}, \ddot{s}_{1(x_2,e_1)_{[1,3]}}\rangle\right\} \vee \\ \left\{\langle \dot{s}_{2(x_2,e_2)_{[5,8]}}, \dot{s}_{2(x_2,e_2)_{[1,4]}}\rangle, \langle \ddot{s}_{2(x_2,e_2)_{[5,10]}}, \ddot{s}_{2(x_2,e_2)_{[1,3]}}\rangle\right\}\end{bmatrix}}$$

$$=\begin{bmatrix}\left\{\left(\langle \dot{s}_{[3,6]}, \dot{s}_{[2,5]}\rangle, \langle \ddot{s}_{[3,4]}, \ddot{s}_{[1,4]}\rangle\right) \otimes \left(\langle \dot{s}_{\left[\frac{1}{9},\frac{1}{5}\right]}, \dot{s}_{\left[\frac{1}{4},\frac{1}{2}\right]}\rangle, \langle \ddot{s}_{\left[\frac{1}{9},\frac{1}{6}\right]}, \ddot{s}_{\left[\frac{1}{2},1\right]}\rangle\right)\right\} \oplus \\ \left\{\left(\langle \dot{s}_{[5,8]}, \dot{s}_{[1,4]}\rangle, \langle \ddot{s}_{[2,7]}, \ddot{s}_{[1,3]}\rangle\right) \otimes \left(\langle \dot{s}_{\left[\frac{1}{8},\frac{1}{6}\right]}, \dot{s}_{\left[\frac{1}{2},1\right]}\rangle, \langle \ddot{s}_{\left[\frac{1}{10},\frac{1}{5}\right]}, \ddot{s}_{\left[\frac{1}{3},1\right]}\rangle\right)\right\}\end{bmatrix}$$

$$=\left(\langle \dot{s}_{\left[\frac{1}{60},\frac{3}{50}\right]}, \dot{s}_{\left[\frac{89}{40},\frac{43}{8}\right]}\rangle, \langle \ddot{s}_{\left[\frac{1}{42},\frac{1}{21}\right]}, \ddot{s}_{\left[\frac{41}{28},\frac{33}{7}\right]}\rangle\right) \oplus \left(\langle \dot{s}_{\left[\frac{1}{32},\frac{1}{15}\right]}, \dot{s}_{\left[\frac{59}{40},\frac{24}{5}\right]}\rangle, \langle \ddot{s}_{\left[\frac{1}{70},\frac{1}{10}\right]}, \ddot{s}_{\left[\frac{55}{42},\frac{53}{14}\right]}\rangle\right)$$

$$=\left(\langle \dot{s}_{\left[\frac{613}{12800},\frac{1897}{15000}\right]}, \dot{s}_{\left[\frac{5251}{32000},\frac{129}{100}\right]}\rangle, \langle \ddot{s}_{\left[\frac{1567}{41160},\frac{433}{2940}\right]}, \ddot{s}_{\left[\frac{2255}{16464},\frac{1749}{1372}\right]}\rangle\right).$$

Now to find the overall similarity between opinions of experts for the departmental evaluation in favor and against them using all the criteria's we use Equation (A)

$$=\frac{1}{3}\begin{bmatrix}\left(\langle \dot{s}_{\left[\frac{1333}{64000},\frac{1747}{12500}\right]}, \dot{s}_{\left[\frac{4531}{12000},\frac{132}{125}\right]}\rangle, \langle \ddot{s}_{\left[\frac{457}{10976},\frac{683}{2744}\right]}, \ddot{s}_{\left[\frac{26671}{21952},\frac{4661}{1372}\right]}\rangle\right) \oplus \\ \left(\langle \dot{s}_{\left[\frac{1699}{36000},\frac{5999}{4375}\right]}, \dot{s}_{\left[\frac{1127}{4000},\frac{69}{50}\right]}\rangle, \langle \ddot{s}_{\left[\frac{1663}{16464},\frac{415}{2058}\right]}, \ddot{s}_{\left[\frac{6935}{5488},\frac{1817}{686}\right]}\rangle\right) \oplus \\ \left(\langle \dot{s}_{\left[\frac{613}{12800},\frac{1897}{15000}\right]}, \dot{s}_{\left[\frac{5251}{32000},\frac{129}{100}\right]}\rangle, \langle \ddot{s}_{\left[\frac{1567}{41160},\frac{433}{2940}\right]}, \ddot{s}_{\left[\frac{2255}{16464},\frac{1749}{1372}\right]}\rangle\right)\end{bmatrix}$$

$$=\frac{\left(\langle \dot{s}_{[0.115698,1.61835]}, \dot{s}_{[0.0000436,0.0046997]}\rangle, \langle \ddot{s}_{[0.18002,0.58955]}, \ddot{s}_{[0.00107,0.058252]}\rangle\right)}{3},$$

$$sm(s_1, s_2) = \left(\langle \dot{s}_{[0.03864,0.55469]}, \dot{s}_{[0.259328,1.23417]}\rangle, \langle \ddot{s}_{[0.06027,0.188854]}, \ddot{s}_{[0.59413,2.25176]}\rangle\right).$$

4.5. TYPE-V SIMILARITY MEASURE FOR GLIVIFSESs.

Type-V similarity measure is same as Type-III similarity measure for generalized linguistic interval-valued intuitionistic fuzzy soft expert sets with variation in place shifting of linguistic terms in conversion from division to multiplication of two generalized linguistic intuitionistic fuzzy soft expert sets.

Definition. Let $U=\{u_i: i=1,...\ ,n\}$ be a set of alternatives and $E=\{e_j: j=1,...\ ,m\}$ represents the set of decision makers and $C=\{c_k: k=1,...\ ,r\}$ represents the set of criteria where n, r, m$\in\mathbb{Z}$ with the property that either ‘n’, ‘r’, ‘m’ are same or different. Suppose

$$S_1=\left\{\langle \dot{s}_{[\alpha,\alpha']}, \dot{s}_{[\beta,\beta']}\rangle\ ,\langle \ddot{s}_{[\gamma,\gamma']}\ ,\ddot{s}_{[\delta,\delta']}\rangle\right\},$$

and

$$S_2=\left\{\langle \dot{s}_{[\alpha_1,\alpha_1']}, \dot{s}_{[\beta_1,\beta_1']}\rangle\ ,\langle \ddot{s}_{[\gamma_1,\gamma_1']}\ ,\ddot{s}_{[\delta_1,\delta_1']}\rangle\right\},$$

the general form of generalized linguistic interval-valued intuitionistic fuzzy soft expert sets.

According to Type-V similarity measure between different GLIVIFSESs we have firstly to calculate the similarity between GLIVIFSESs for a specific alternative, the expression for that calculation is defined as

$$\sum_{j=1}^{m}\sum_{k=1}^{r}\frac{\left[\begin{array}{c}\left\{\langle \dot{S}_1(e_j,c_k)_{\frac{\alpha\oplus\alpha'}{2}}, \dot{S}_1(e_j,c_k)_{\frac{\beta\oplus\beta'}{2}}\rangle, \langle \ddot{S}_1(e_j,c_k)_{\frac{\gamma\oplus\gamma'}{2}}, \ddot{S}_1(e_j,c_k)_{\frac{\delta\oplus\delta'}{2}}\rangle\right\}\wedge \\ \left\{\langle \dot{S}_2(e_{j+1},c_k)_{\frac{\alpha_1\oplus\alpha_1'}{2}}, \dot{S}_2(e_{j+1},c_k)_{\frac{\beta_1\oplus\beta_1'}{2}}\rangle, \langle \ddot{S}_2(e_{j+1},c_k)_{\frac{\gamma_1\oplus\gamma_1'}{2}}, \ddot{S}_2(e_{j+1},c_k)_{\frac{\delta_1\oplus\delta_1'}{2}}\rangle\right\}\end{array}\right]}{\left[\begin{array}{c}\left\{\langle \dot{S}_1(e_j,c_k)_{\frac{\alpha\oplus\alpha'}{2}}, \dot{S}_1(e_j,c_k)_{\frac{\beta\oplus\beta'}{2}}\rangle, \langle \ddot{S}_1(e_j,c_k)_{\frac{\gamma\oplus\gamma'}{2}}, \ddot{S}_1(e_j,c_k)_{\frac{\delta\oplus\delta'}{2}}\rangle\right\}\vee \\ \left\{\langle \dot{S}_2(e_{j+1},c_k)_{\frac{\alpha_1\oplus\alpha_1'}{2}}, \dot{S}_2(e_{j+1},c_k)_{\frac{\beta_1\oplus\beta_1'}{2}}\rangle, \langle \ddot{S}_2(e_{j+1},c_k)_{\frac{\gamma_1\oplus\gamma_1'}{2}}, \ddot{S}_2(e_{j+1},c_k)_{\frac{\delta_1\oplus\delta_1'}{2}}\rangle\right\}\end{array}\right]} \qquad (4.5)$$

which shows the similarity between the opinions of expert’s for specific alternative u_i where ‘i’ range goes from one to ‘n’. Similarly for each alternative we will use the above mentioned formula to calculate the similarity between the opinions of experts.

Now to find the combined similarity in the opinions of experts for all alternatives we will use the following relation

$$SM(S_1,S_2)=\frac{\sum_{i=0}^{n} S_i(S_1,S_2)}{n}.$$

To compare that similarity measure with previous similarity measures firstly we consider Example 4.1.1 and then construct a practical example to briefly discuss the above mentioned strategy to measure the similarity between GLIVIFSESs.

EXAMPLE 4.5.1.

By taking data from Example 4.1.1 we have the following generalized linguistic interval-valued intuitionistic fuzzy soft expert sets

$$
\begin{aligned}
S_1(h_1, a) &= \left\{\left\langle \dot{s}_{[2,3]}, \dot{s}_{[1,2]}\right\rangle, \left\langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\right\rangle\right\},\\
S_1(h_2, a) &= \left\{\left\langle \dot{s}_{[3,4]}, \dot{s}_{[1,2]}\right\rangle, \left\langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\right\rangle\right\},\\
S_2(h_1, b) &= \left\{\left\langle \dot{s}_{[1,3]}, \dot{s}_{[2,3]}\right\rangle, \left\langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\right\rangle\right\},\\
S_2(h_2, b) &= \left\{\left\langle \dot{s}_{[1,2]}, \dot{s}_{[3,4]}\right\rangle, \left\langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\right\rangle\right\},
\end{aligned}
$$

these sets represent the evaluation values by students ‘a’ and ‘b’ for the teacher t_1. Now for the second teacher t_2 the evaluation values are as under

$$
\begin{aligned}
S_1(h_1, a) &= \left\{\left\langle \dot{s}_{[1,2]}, \dot{s}_{[2,4]}\right\rangle, \left\langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\right\rangle\right\},\\
S_1(h_2, a) &= \left\{\left\langle \dot{s}_{[2,4]}, \dot{s}_{[1,2]}\right\rangle, \left\langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\right\rangle\right\},\\
S_2(h_1, b) &= \left\{\left\langle \dot{s}_{[2,3]}, \dot{s}_{[2,3]}\right\rangle, \left\langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\right\rangle\right\},\\
S_2(h_2, b) &= \left\{\left\langle \dot{s}_{[1,2]}, \dot{s}_{[3,4]}\right\rangle, \left\langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\right\rangle\right\}.
\end{aligned}
$$

Now by using the Type-V similarity measure we will firstly find the similarity between opinions of experts in case of t_1

$$
\frac{\left\{\left\langle \dot{S}_{1(h_1,a)_{\frac{2\oplus 3}{2}}}, \dot{S}_{1(h_1,a)_{\frac{1\oplus 2}{2}}}\right\rangle, \left\langle \ddot{S}_{1(h_1,a)_{\frac{1\oplus 2}{2}}}, \ddot{S}_{1(h_1,a)_{\frac{1\oplus 2}{2}}}\right\rangle\right\} \wedge \left\{\left\langle \dot{S}_{2(h_1,b)_{\frac{1\oplus 3}{2}}}, \dot{S}_{2(h_1,b)_{\frac{2\oplus 3}{2}}}\right\rangle, \left\langle \ddot{S}_{2(h_1,b)_{\frac{1\oplus 2}{2}}}, \ddot{S}_{2(h_1,b)_{\frac{1\oplus 2}{2}}}\right\rangle\right\}}{\left\{\left\langle \dot{S}_{1(h_1,a)_{\frac{2\oplus 3}{2}}}, \dot{S}_{1(h_1,a)_{\frac{1\oplus 2}{2}}}\right\rangle, \left\langle \ddot{S}_{1(h_1,a)_{\frac{1\oplus 2}{2}}}, \ddot{S}_{1(h_1,a)_{\frac{1\oplus 2}{2}}}\right\rangle\right\} \wedge \left\{\left\langle \dot{S}_{2(h_1,b)_{\frac{1\oplus 3}{2}}}, \dot{S}_{2(h_1,b)_{\frac{2\oplus 3}{2}}}\right\rangle, \left\langle \ddot{S}_{2(h_1,b)_{\frac{1\oplus 2}{2}}}, \ddot{S}_{2(h_1,b)_{\frac{1\oplus 2}{2}}}\right\rangle\right\}} \oplus
$$

$$
\frac{\left\{\left\langle \dot{S}_{1(h_2,a)_{\frac{3\oplus 4}{2}}}, \dot{S}_{1(h_2,a)_{\frac{1\oplus 2}{2}}}\right\rangle, \left\langle \ddot{S}_{1(h_2,a)_{\frac{1\oplus 2}{2}}}, \ddot{S}_{1(h_2,a)_{\frac{1\oplus 2}{2}}}\right\rangle\right\} \wedge \left\{\left\langle \dot{S}_{2(h_2,b)_{\frac{1\oplus 2}{2}}}, \dot{S}_{2(h_2,b)_{\frac{3\oplus 4}{2}}}\right\rangle, \left\langle \ddot{S}_{2(h_2,b)_{\frac{1\oplus 2}{2}}}, \ddot{S}_{2(h_2,b)_{\frac{1\oplus 2}{2}}}\right\rangle\right\}}{\left\{\left\langle \dot{S}_{1(h_2,a)_{\frac{3\oplus 4}{2}}}, \dot{S}_{1(h_2,a)_{\frac{1\oplus 2}{2}}}\right\rangle, \left\langle \ddot{S}_{1(h_2,a)_{\frac{1\oplus 2}{2}}}, \ddot{S}_{1(h_2,a)_{\frac{1\oplus 2}{2}}}\right\rangle\right\} \wedge \left\{\left\langle \dot{S}_{2(h_2,b)_{\frac{1\oplus 2}{2}}}, \dot{S}_{2(h_2,b)_{\frac{3\oplus 4}{2}}}\right\rangle, \left\langle \ddot{S}_{2(h_2,b)_{\frac{1\oplus 2}{2}}}, \ddot{S}_{2(h_2,b)_{\frac{1\oplus 2}{2}}}\right\rangle\right\}}
$$

$$
= \frac{\left(\left\langle \dot{s}_{2}, \dot{s}_{\frac{3}{2}}\right\rangle, \left\langle \ddot{s}_{\frac{3}{2}}, \ddot{s}_{\frac{3}{2}}\right\rangle\right)}{\left(\left\langle \dot{s}_{\frac{5}{2}}, \dot{s}_{\frac{3}{2}}\right\rangle, \left\langle \ddot{s}_{\frac{3}{2}}, \ddot{s}_{\frac{3}{2}}\right\rangle\right)} \oplus \frac{\left(\left\langle \dot{s}_{\frac{3}{2}}, \dot{s}_{\frac{7}{2}}\right\rangle, \left\langle \ddot{s}_{\frac{3}{2}}, \ddot{s}_{\frac{3}{2}}\right\rangle\right)}{\left(\left\langle \dot{s}_{\frac{7}{2}}, \dot{s}_{\frac{3}{2}}\right\rangle, \left\langle \ddot{s}_{\frac{3}{2}}, \ddot{s}_{\frac{3}{2}}\right\rangle\right)}
$$

$$=\left[\left(\langle \dot{s}_{2}, \dot{s}_{\frac{3}{2}}\rangle, \langle \ddot{s}_{\frac{3}{2}}, \ddot{s}_{\frac{3}{2}}\rangle\right)\otimes\left(\langle \dot{s}_{\frac{2}{5}}, \dot{s}_{\frac{2}{3}}\rangle, \langle \ddot{s}_{\frac{2}{3}}, \ddot{s}_{\frac{2}{3}}\rangle\right)\right]\oplus\left[\left(\langle \dot{s}_{\frac{3}{2}}, \dot{s}_{\frac{7}{2}}\rangle, \langle \ddot{s}_{\frac{3}{2}}, \ddot{s}_{\frac{3}{2}}\rangle\right)\otimes\left(\langle \dot{s}_{\frac{2}{7}}, \dot{s}_{\frac{2}{3}}\rangle, \langle \ddot{s}_{\frac{2}{3}}, \ddot{s}_{\frac{2}{3}}\rangle\right)\right]$$

$$=\left(\langle \dot{s}_{\frac{2}{15}}, \dot{s}_{2}\rangle, \langle \ddot{s}_{\frac{1}{4}}, \ddot{s}_{\frac{23}{12}}\rangle\right)\oplus\left(\langle \dot{s}_{\frac{1}{14}}, \dot{s}_{\frac{34}{9}}\rangle, \langle \ddot{s}_{\frac{1}{4}}, \ddot{s}_{\frac{23}{12}}\rangle\right)=\left(\langle \dot{s}_{\frac{44}{315}}, \dot{s}_{\frac{34}{27}}\rangle, \langle \ddot{s}_{\frac{31}{64}}, \ddot{s}_{\frac{529}{576}}\rangle\right).$$

This represents the similarity between the opinions of experts for the first teacher. Now for the second teacher t_2, by substituting the opinions of expert in expression of Type-V similarity measure we get

$$\frac{\left\{\langle \dot{s}_{1(h_1,a)_{\frac{1\oplus 2}{2}}}, \dot{s}_{1(h_1,a)_{\frac{2\oplus 4}{2}}}\rangle, \langle \ddot{s}_{1(h_1,a)_{\frac{1\oplus 2}{2}}}, \ddot{s}_{1(h_1,a)_{\frac{1\oplus 2}{2}}}\rangle\right\}\wedge\left\{\langle \dot{s}_{2(h_1,b)_{\frac{2\oplus 3}{2}}}, \dot{s}_{2(h_1,b)_{\frac{2\oplus 3}{2}}}\rangle, \langle \ddot{s}_{2(h_1,b)_{\frac{1\oplus 2}{2}}}, \ddot{s}_{2(h_1,b)_{\frac{1\oplus 2}{2}}}\rangle\right\}}{\left\{\langle \dot{s}_{1(h_1,a)_{\frac{1\oplus 2}{2}}}, \dot{s}_{1(h_1,a)_{\frac{2\oplus 4}{2}}}\rangle, \langle \ddot{s}_{1(h_1,a)_{\frac{1\oplus 2}{2}}}, \ddot{s}_{1(h_1,a)_{\frac{1\oplus 2}{2}}}\rangle\right\}\vee\left\{\langle \dot{s}_{2(h_1,b)_{\frac{2\oplus 3}{2}}}, \dot{s}_{2(h_1,b)_{\frac{2\oplus 3}{2}}}\rangle, \langle \ddot{s}_{2(h_1,b)_{\frac{1\oplus 2}{2}}}, \ddot{s}_{2(h_1,b)_{\frac{1\oplus 2}{2}}}\rangle\right\}}\oplus$$

$$\frac{\left\{\langle \dot{s}_{1(h_2,a)_{\frac{2\oplus 4}{2}}}, \dot{s}_{1(h_2,a)_{\frac{1\oplus 2}{2}}}\rangle, \langle \ddot{s}_{1(h_2,a)_{\frac{1\oplus 2}{2}}}, \ddot{s}_{1(h_2,a)_{\frac{1\oplus 2}{2}}}\rangle\right\}\wedge\left\{\langle \dot{s}_{2(h_2,b)_{\frac{1\oplus 2}{2}}}, \dot{s}_{2(h_2,b)_{\frac{3\oplus 4}{2}}}\rangle, \langle \ddot{s}_{2(h_2,b)_{\frac{1\oplus 2}{2}}}, \ddot{s}_{2(h_2,b)_{\frac{1\oplus 2}{2}}}\rangle\right\}}{\left\{\langle \dot{s}_{1(h_2,a)_{\frac{2\oplus 4}{2}}}, \dot{s}_{1(h_2,a)_{\frac{1\oplus 2}{2}}}\rangle, \langle \ddot{s}_{1(h_2,a)_{\frac{1\oplus 2}{2}}}, \ddot{s}_{1(h_2,a)_{\frac{1\oplus 2}{2}}}\rangle\right\}\vee\left\{\langle \dot{s}_{2(h_2,b)_{\frac{1\oplus 2}{2}}}, \dot{s}_{2(h_2,b)_{\frac{3\oplus 4}{2}}}\rangle, \langle \ddot{s}_{2(h_2,b)_{\frac{1\oplus 2}{2}}}, \ddot{s}_{2(h_2,b)_{\frac{1\oplus 2}{2}}}\rangle\right\}}$$

after simplifications we get

$$=\frac{\left(\langle \dot{s}_{\frac{3}{2}}, \dot{s}_{3}\rangle, \langle \ddot{s}_{\frac{3}{2}}, \ddot{s}_{\frac{3}{2}}\rangle\right)}{\left(\langle \dot{s}_{\frac{5}{2}}, \dot{s}_{\frac{5}{2}}\rangle, \langle \ddot{s}_{\frac{3}{2}}, \ddot{s}_{\frac{3}{2}}\rangle\right)}\oplus\frac{\left(\langle \dot{s}_{\frac{3}{2}}, \dot{s}_{\frac{7}{2}}\rangle, \langle \ddot{s}_{\frac{3}{2}}, \ddot{s}_{\frac{3}{2}}\rangle\right)}{\left(\langle \dot{s}_{3}, \dot{s}_{\frac{3}{2}}\rangle, \langle \ddot{s}_{\frac{3}{2}}, \ddot{s}_{\frac{3}{2}}\rangle\right)}$$

$$=\left[\left(\langle \dot{s}_{\frac{3}{2}}, \dot{s}_{3}\rangle, \langle \ddot{s}_{\frac{3}{2}}, \ddot{s}_{\frac{3}{2}}\rangle\right)\otimes\left(\langle \dot{s}_{\frac{2}{5}}, \dot{s}_{\frac{2}{5}}\rangle, \langle \ddot{s}_{\frac{2}{3}}, \ddot{s}_{\frac{2}{3}}\rangle\right)\right]\oplus\left[\left(\langle \dot{s}_{\frac{3}{2}}, \dot{s}_{\frac{7}{2}}\rangle, \langle \ddot{s}_{\frac{3}{2}}, \ddot{s}_{\frac{3}{2}}\rangle\right)\otimes\left(\langle \dot{s}_{3}, \dot{s}_{\frac{2}{3}}\rangle, \langle \ddot{s}_{\frac{2}{3}}, \ddot{s}_{\frac{2}{3}}\rangle\right)\right]$$

$$=\left(\langle \dot{s}_{\frac{1}{10}}, \dot{s}_{\frac{16}{5}}\rangle, \langle \ddot{s}_{\frac{1}{4}}, \ddot{s}_{\frac{23}{12}}\rangle\right)\oplus\left(\langle \dot{s}_{\frac{3}{4}}, \dot{s}_{\frac{34}{9}}\rangle, \langle \ddot{s}_{\frac{1}{4}}, \ddot{s}_{\frac{23}{12}}\rangle\right)=\left(\langle \dot{s}_{\frac{67}{80}}, \dot{s}_{\frac{272}{135}}\rangle, \langle \ddot{s}_{\frac{31}{64}}, \ddot{s}_{\frac{529}{576}}\rangle\right).$$

Now by using Equation (A) we have

$$=\frac{\left(\langle \dot{s}_{\frac{44}{315}}, \dot{s}_{\frac{34}{27}}\rangle, \langle \ddot{s}_{\frac{31}{64}}, \ddot{s}_{\frac{529}{576}}\rangle\right)\oplus\left(\langle \dot{s}_{\frac{67}{80}}, \dot{s}_{\frac{272}{135}}\rangle, \langle \ddot{s}_{\frac{31}{64}}, \ddot{s}_{\frac{529}{576}}\rangle\right)}{2},$$

this implies that

$$\mathrm{SM}(\mathrm{S}_1, \mathrm{S}_2)=\left(\langle \dot{s}_{0.49965}, \dot{s}_{1.59285}\rangle, \langle \ddot{s}_{\frac{31}{64}}, \ddot{s}_{\frac{529}{576}}\rangle\right).$$

Now to compare the result obtained by Type-V similarity measure with the previous similarity measure's results we use score and accuracy function.

According to score function

$$S(\mathrm{SM}(\mathrm{S}_1, \mathrm{S}_2)) = \mathrm{s}_{\left[\frac{6+0.49965-1.59285}{12}\right] \times \left[\frac{4+\frac{31}{64}-\frac{529}{576}}{8}\right]} = \mathrm{s}_{0.1822658},$$

this shows that similarity results obtained by Type-V similarity measure are higher in order from Type-IV similarity measure results with miner difference from Type-III similarity measure results.

Now to illustrate the Type-V similarity measure methodology briefly we consider a practical problem.

EXAMPLE 4.5.2.

Government of A county interested in mining of specific areas in F city where A geological center point out that in these areas there will be in large amount reserves of gold(approximately 1297 tons, which is still under the earth due to lack of special technology and skills to separate it), copper(more than 1352 tons, this amount is largest in all over the world), chromite(which is estimated as 226.5 million metric tons), coal(150 billion tons, in tharparkar recently 3 billion tons high quality coal reserves were found), oil(estimated as 618 billion barrel but no capacity to bring out), gypsum(six billion tons or higher), zinc(24 million tons), marble and granite(297 billion tons but according to report, obtained only 229 tons in 2018 from mountains), iron(1500 million tons), precious stones(according to report published in 2017 A has more than 30% of precious stones present on earth, but unfortunately exports of these stones like other minerals is negligible such that only 0.03%) and so many other mineral's(according to research report in about 600000 square kilometer area in A minerals are present) to take benefits by their exports and spend the money on country peoples, in developing projects and to overcome the country loans which are increasing day by day due to interest rates to remove the financial bearer's in the way of country needs fulfillment.

Now to achieve these objectives government issued a tender notice internationally for mining, with the following conditions to be fulfilled by a company to do mining

- High quality machinery
- Special skills

- Achievements in mining processes up to that

and nominate experts $\{e_1, e_2\}$ to present reports regarding the companies applied for mining processes after their evaluation on the behalf of criteria's fixed by government executive authority with the condition that also give the details regarding own expertise about mining processes, to overcome the inappropriate results regarding companies.

We call the set of conditions as of criteria's with representative set $\{c_1, c_2, c_3\}$. Three companies BGCC, TCC, NRPL applied for mining license in A, we call them as $u_1,\ u_2,\ u_3$ respectively such that $\{u_1,\ u_2,\ u_3\}$ represent a set of alternatives. The strategy of government for issuance of mining license to certain company based on similarity between experts opinions in fever of that company, such that if similarity between opinions of experts is greater than 50% than that company will be considered for license issuance.

Here we consider the predefined linguistic term set S_t for opinions about companies with $0 \le t \le 18$ and linguistic term set $S_{t'}$ for information about own expertise by experts with $0 \le t' \le 16$, here the opinions will be in the form 2-D linguistic interval-valued intuitionistic fuzzy soft expert sets due to uncertain and vague information.

The decisions of experts for u_1 are

$$\mathrm{S}_1(\mathrm{c}_1, e_1)=\left\{\langle \dot{\mathrm{s}}_{[3,4]}, \dot{\mathrm{s}}_{[1,2]}\rangle, \langle \ddot{\mathrm{s}}_{[2,3]}, \ddot{\mathrm{s}}_{[1,3]}\rangle\right\},$$
$$\mathrm{S}_1(\mathrm{c}_2, e_1)=\left\{\langle \dot{\mathrm{s}}_{[5,10]}, \dot{\mathrm{s}}_{[2,5]}\rangle, \langle \ddot{\mathrm{s}}_{[3,5]}, \ddot{\mathrm{s}}_{[3,4]}\rangle\right\},$$
$$\mathrm{S}_1(\mathrm{c}_3, e_1)=\left\{\langle \dot{\mathrm{s}}_{[11,12]}, \dot{\mathrm{s}}_{[4,5]}\rangle, \langle \ddot{\mathrm{s}}_{[9,10]}, \ddot{\mathrm{s}}_{[1,2]}\rangle\right\},$$
$$\mathrm{S}_2(\mathrm{c}_1, e_2)=\left\{\langle \dot{\mathrm{s}}_{[3,5]}, \dot{\mathrm{s}}_{[1,3]}\rangle, \langle \ddot{\mathrm{s}}_{[2,4]}, \ddot{\mathrm{s}}_{[1,3]}\rangle\right\},$$
$$\mathrm{S}_2(\mathrm{c}_2, e_2)=\left\{\langle \dot{\mathrm{s}}_{[5,11]}, \dot{\mathrm{s}}_{[2,5]}\rangle, \langle \ddot{\mathrm{s}}_{[3,6]}, \ddot{\mathrm{s}}_{[2,4]}\rangle\right\},$$
$$\mathrm{S}_2(\mathrm{c}_3, e_2)=\left\{\langle \dot{\mathrm{s}}_{[10,12]}, \dot{\mathrm{s}}_{[4,6]}\rangle, \langle \ddot{\mathrm{s}}_{[9,11]}, \ddot{\mathrm{s}}_{[2,3]}\rangle\right\}.$$

The decisions of experts for u_2 are

$$\mathrm{S}_1(\mathrm{c}_1, e_1)=\left\{\langle \dot{\mathrm{s}}_{[5,7]}, \dot{\mathrm{s}}_{[2,3]}\rangle, \langle \ddot{\mathrm{s}}_{[8,9]}, \ddot{\mathrm{s}}_{[1,2]}\rangle\right\},$$
$$\mathrm{S}_1(\mathrm{c}_2, e_1)=\left\{\langle \dot{\mathrm{s}}_{[8,10]}, \dot{\mathrm{s}}_{[3,4]}\rangle, \langle \ddot{\mathrm{s}}_{[7,8]}, \ddot{\mathrm{s}}_{[2,3]}\rangle\right\},$$
$$\mathrm{S}_1(\mathrm{c}_3, e_1)=\left\{\langle \dot{\mathrm{s}}_{[13,14]}, \dot{\mathrm{s}}_{[2,3]}\rangle, \langle \ddot{\mathrm{s}}_{[11,13]}, \ddot{\mathrm{s}}_{[1,2]}\rangle\right\},$$
$$\mathrm{S}_2(\mathrm{c}_1, e_2)=\left\{\langle \dot{\mathrm{s}}_{[6,7]}, \dot{\mathrm{s}}_{[2,3]}\rangle, \langle \ddot{\mathrm{s}}_{[7,9]}, \ddot{\mathrm{s}}_{[1,3]}\rangle\right\},$$

$$\mathrm{S}_2(\mathrm{c}_2, e_2)=\{\langle \dot{\mathrm{s}}_{[8,10]}, \dot{\mathrm{s}}_{[3,5]}\rangle, \langle \ddot{\mathrm{s}}_{[7,9]}, \ddot{\mathrm{s}}_{[1,3]}\rangle\},$$
$$\mathrm{S}_2(\mathrm{c}_3, e_2)=\{\langle \dot{\mathrm{s}}_{[12,13]}, \dot{\mathrm{s}}_{[2,4]}\rangle, \langle \ddot{\mathrm{s}}_{[11,14]}, \ddot{\mathrm{s}}_{[1,2]}\rangle\}.$$

The decisions of experts for u_3 are

$$\mathrm{S}_1(\mathrm{c}_1, e_1)=\{\langle \dot{\mathrm{s}}_{[2,3]}, \dot{\mathrm{s}}_{[5,9]}\rangle, \langle \ddot{\mathrm{s}}_{[7,8]}, \ddot{\mathrm{s}}_{[1,2]}\rangle\},$$
$$\mathrm{S}_1(\mathrm{c}_2, e_1)=\{\langle \dot{\mathrm{s}}_{[3,4]}, \dot{\mathrm{s}}_{[4,7]}\rangle, \langle \ddot{\mathrm{s}}_{[7,9]}, \ddot{\mathrm{s}}_{[1,3]}\rangle\},$$
$$\mathrm{S}_1(\mathrm{c}_3, e_1)=\{\langle \dot{\mathrm{s}}_{[5,7]}, \dot{\mathrm{s}}_{[2,3]}\rangle, \langle \ddot{\mathrm{s}}_{[6,9]}, \ddot{\mathrm{s}}_{[3,4]}\rangle\},$$
$$\mathrm{S}_2(\mathrm{c}_1, e_2)=\{\langle \dot{\mathrm{s}}_{[7,8]}, \dot{\mathrm{s}}_{[3,4]}\rangle, \langle \ddot{\mathrm{s}}_{[6,10]}, \ddot{\mathrm{s}}_{[3,4]}\rangle\},$$
$$\mathrm{S}_2(\mathrm{c}_2, e_2)=\{\langle \dot{\mathrm{s}}_{[5,9]}, \dot{\mathrm{s}}_{[3,8]}\rangle, \langle \ddot{\mathrm{s}}_{[5,7]}, \ddot{\mathrm{s}}_{[3,4]}\rangle\},$$
$$\mathrm{S}_2(\mathrm{c}_3, e_2)=\{\langle \dot{\mathrm{s}}_{[1,3]}, \dot{\mathrm{s}}_{[8,9]}\rangle, \langle \ddot{\mathrm{s}}_{[3,5]}, \ddot{\mathrm{s}}_{[1,2]}\rangle\}.$$

Now to find the similarity between the opinions of experts in case of u_1 using Type-V similarity measure we have

$$
\left[\frac{\left\{\left\langle \dot{s}_{1(c_1,e_1)_{\frac{3\oplus 4}{2}}}, \dot{s}_{1(c_1,e_1)_{\frac{1\oplus 2}{2}}}\right\rangle, \left\langle \ddot{s}_{1(c_1,e_1)_{\frac{2\oplus 3}{2}}}, \ddot{s}_{1(c_1,e_1)_{\frac{1\oplus 3}{2}}}\right\rangle\right\} \wedge \left\{\left\langle \dot{s}_{2(c_1,e_2)_{\frac{3\oplus 5}{2}}}, \dot{s}_{2(c_1,e_2)_{\frac{1\oplus 3}{2}}}\right\rangle, \left\langle \ddot{s}_{2(c_1,e_2)_{\frac{2\oplus 4}{2}}}, \ddot{s}_{2(c_1,e_2)_{\frac{1\oplus 3}{2}}}\right\rangle\right\}}{\left\{\left\langle \dot{s}_{1(c_1,e_1)_{\frac{3\oplus 4}{2}}}, \dot{s}_{1(c_1,e_1)_{\frac{1\oplus 2}{2}}}\right\rangle, \left\langle \ddot{s}_{1(c_1,e_1)_{\frac{2\oplus 3}{2}}}, \ddot{s}_{1(c_1,e_1)_{\frac{1\oplus 3}{2}}}\right\rangle\right\} \vee \left\{\left\langle \dot{s}_{2(c_1,e_2)_{\frac{3\oplus 5}{2}}}, \dot{s}_{2(c_1,e_2)_{\frac{1\oplus 3}{2}}}\right\rangle, \left\langle \ddot{s}_{2(c_1,e_2)_{\frac{2\oplus 4}{2}}}, \ddot{s}_{2(c_1,e_2)_{\frac{1\oplus 3}{2}}}\right\rangle\right\}}\right]
$$

$$
\oplus\left[\frac{\left\{\left\langle \dot{s}_{1(c_2,e_1)_{\frac{5\oplus 10}{2}}}, \dot{s}_{1(c_2,e_1)_{\frac{2\oplus 5}{2}}}\right\rangle, \left\langle \ddot{s}_{1(c_2,e_1)_{\frac{3\oplus 5}{2}}}, \ddot{s}_{1(c_2,e_1)_{\frac{3\oplus 4}{2}}}\right\rangle\right\} \wedge \left\{\left\langle \dot{s}_{2(c_2,e_2)_{\frac{5\oplus 11}{2}}}, \dot{s}_{2(c_2,e_2)_{\frac{2\oplus 5}{2}}}\right\rangle, \left\langle \ddot{s}_{2(c_2,e_2)_{\frac{3\oplus 6}{2}}}, \ddot{s}_{2(c_2,e_2)_{\frac{2\oplus 4}{2}}}\right\rangle\right\}}{\left\{\left\langle \dot{s}_{1(c_2,e_1)_{\frac{5\oplus 10}{2}}}, \dot{s}_{1(c_2,e_1)_{\frac{2\oplus 5}{2}}}\right\rangle, \left\langle \ddot{s}_{1(c_2,e_1)_{\frac{3\oplus 5}{2}}}, \ddot{s}_{1(c_2,e_1)_{\frac{3\oplus 4}{2}}}\right\rangle\right\} \vee \left\{\left\langle \dot{s}_{2(c_2,e_2)_{\frac{5\oplus 11}{2}}}, \dot{s}_{2(c_2,e_2)_{\frac{2\oplus 5}{2}}}\right\rangle, \left\langle \ddot{s}_{2(c_2,e_2)_{\frac{3\oplus 6}{2}}}, \ddot{s}_{2(c_2,e_2)_{\frac{2\oplus 4}{2}}}\right\rangle\right\}}\right]
$$

$$
\oplus\left[\frac{\left\{\left\langle \dot{s}_{1(c_3,e_1)_{\frac{11\oplus 12}{2}}}, \dot{s}_{1(c_3,e_1)_{\frac{4\oplus 5}{2}}}\right\rangle, \left\langle \ddot{s}_{1(c_3,e_1)_{\frac{9\oplus 10}{2}}}, \ddot{s}_{1(c_3,e_1)_{\frac{1\oplus 2}{2}}}\right\rangle\right\} \wedge \left\{\left\langle \dot{s}_{2(c_3,e_2)_{\frac{10\oplus 12}{2}}}, \dot{s}_{2(c_3,e_2)_{\frac{4\oplus 6}{2}}}\right\rangle, \left\langle \ddot{s}_{2(c_3,e_2)_{\frac{9\oplus 11}{2}}}, \ddot{s}_{2(c_3,e_2)_{\frac{2\oplus 3}{2}}}\right\rangle\right\}}{\left\{\left\langle \dot{s}_{1(c_3,e_1)_{\frac{11\oplus 12}{2}}}, \dot{s}_{1(c_3,e_1)_{\frac{4\oplus 5}{2}}}\right\rangle, \left\langle \ddot{s}_{1(c_3,e_1)_{\frac{9\oplus 10}{2}}}, \ddot{s}_{1(c_3,e_1)_{\frac{1\oplus 2}{2}}}\right\rangle\right\} \vee \left\{\left\langle \dot{s}_{2(c_3,e_2)_{\frac{10\oplus 12}{2}}}, \dot{s}_{2(c_3,e_2)_{\frac{4\oplus 6}{2}}}\right\rangle, \left\langle \ddot{s}_{2(c_3,e_2)_{\frac{9\oplus 11}{2}}}, \ddot{s}_{2(c_3,e_2)_{\frac{2\oplus 3}{2}}}\right\rangle\right\}}\right]
$$

$=(\langle \dot{s}_{0.153397}, \dot{s}_{0.15403}\rangle, \langle \ddot{s}_{0.166436}, \ddot{s}_{0.10965}\rangle)$.

Now to find the similarity between the opinions of experts in case of u_2 using Type-V similarity measure we have

$$\left[\frac{\left\{\left\langle \dot{s}_{1(c_1,e_1)_{\frac{5\oplus 7}{2}}}, \dot{s}_{1(c_1,e_1)_{\frac{2\oplus 3}{2}}}\right\rangle, \left\langle \ddot{s}_{1(c_1,e_1)_{\frac{8\oplus 9}{2}}}, \ddot{s}_{1(c_1,e_1)_{\frac{1\oplus 2}{2}}}\right\rangle\right\} \wedge \left\{\left\langle \dot{s}_{2(c_1,e_2)_{\frac{6\oplus 7}{2}}}, \dot{s}_{2(c_1,e_2)_{\frac{2\oplus 3}{2}}}\right\rangle, \left\langle \ddot{s}_{2(c_1,e_2)_{\frac{7\oplus 9}{2}}}, \ddot{s}_{2(c_1,e_2)_{\frac{1\oplus 3}{2}}}\right\rangle\right\}}{\left\{\left\langle \dot{s}_{1(c_1,e_1)_{\frac{5\oplus 7}{2}}}, \dot{s}_{1(c_1,e_1)_{\frac{2\oplus 3}{2}}}\right\rangle, \left\langle \ddot{s}_{1(c_1,e_1)_{\frac{8\oplus 9}{2}}}, \ddot{s}_{1(c_1,e_1)_{\frac{1\oplus 2}{2}}}\right\rangle\right\} \vee \left\{\left\langle \dot{s}_{2(c_1,e_2)_{\frac{6\oplus 7}{2}}}, \dot{s}_{2(c_1,e_2)_{\frac{2\oplus 3}{2}}}\right\rangle, \left\langle \ddot{s}_{2(c_1,e_2)_{\frac{7\oplus 9}{2}}}, \ddot{s}_{2(c_1,e_2)_{\frac{1\oplus 3}{2}}}\right\rangle\right\}}\right]$$

$$\oplus \left[\frac{\left\{\left\langle \dot{s}_{1(c_2,e_1)_{\frac{8\oplus 10}{2}}}, \dot{s}_{1(c_2,e_1)_{\frac{3\oplus 4}{2}}}\right\rangle, \left\langle \ddot{s}_{1(c_2,e_1)_{\frac{7\oplus 8}{2}}}, \ddot{s}_{1(c_2,e_1)_{\frac{2\oplus 3}{2}}}\right\rangle\right\} \wedge \left\{\left\langle \dot{s}_{2(c_2,e_2)_{\frac{8\oplus 10}{2}}}, \dot{s}_{2(c_2,e_2)_{\frac{3\oplus 5}{2}}}\right\rangle, \left\langle \ddot{s}_{2(c_2,e_2)_{\frac{7\oplus 9}{2}}}, \ddot{s}_{2(c_2,e_2)_{\frac{1\oplus 3}{2}}}\right\rangle\right\}}{\left\{\left\langle \dot{s}_{1(c_2,e_1)_{\frac{8\oplus 10}{2}}}, \dot{s}_{1(c_2,e_1)_{\frac{3\oplus 4}{2}}}\right\rangle, \left\langle \ddot{s}_{1(c_2,e_1)_{\frac{7\oplus 8}{2}}}, \ddot{s}_{1(c_2,e_1)_{\frac{2\oplus 3}{2}}}\right\rangle\right\} \vee \left\{\left\langle \dot{s}_{2(c_2,e_2)_{\frac{8\oplus 10}{2}}}, \dot{s}_{2(c_2,e_2)_{\frac{3\oplus 5}{2}}}\right\rangle, \left\langle \ddot{s}_{2(c_2,e_2)_{\frac{7\oplus 9}{2}}}, \ddot{s}_{2(c_2,e_2)_{\frac{1\oplus 3}{2}}}\right\rangle\right\}}\right]$$

$$\oplus \left[\frac{\left\{\left\langle \dot{s}_{1(c_3,e_1)_{\frac{13\oplus 14}{2}}}, \dot{s}_{1(c_3,e_1)_{\frac{2\oplus 3}{2}}}\right\rangle, \left\langle \ddot{s}_{1(c_3,e_1)_{\frac{11\oplus 13}{2}}}, \ddot{s}_{1(c_3,e_1)_{\frac{1\oplus 2}{2}}}\right\rangle\right\} \wedge \left\{\left\langle \dot{s}_{2(c_3,e_2)_{\frac{12\oplus 13}{2}}}, \dot{s}_{2(c_3,e_2)_{\frac{2\oplus 4}{2}}}\right\rangle, \left\langle \ddot{s}_{2(c_3,e_2)_{\frac{11\oplus 14}{2}}}, \ddot{s}_{2(c_3,e_2)_{\frac{1\oplus 2}{2}}}\right\rangle\right\}}{\left\{\left\langle \dot{s}_{1(c_3,e_1)_{\frac{13\oplus 14}{2}}}, \dot{s}_{1(c_3,e_1)_{\frac{2\oplus 3}{2}}}\right\rangle, \left\langle \ddot{s}_{1(c_3,e_1)_{\frac{11\oplus 13}{2}}}, \ddot{s}_{1(c_3,e_1)_{\frac{1\oplus 2}{2}}}\right\rangle\right\} \vee \left\{\left\langle \dot{s}_{2(c_3,e_2)_{\frac{12\oplus 13}{2}}}, \dot{s}_{2(c_3,e_2)_{\frac{2\oplus 4}{2}}}\right\rangle, \left\langle \ddot{s}_{2(c_3,e_2)_{\frac{11\oplus 14}{2}}}, \ddot{s}_{2(c_3,e_2)_{\frac{1\oplus 2}{2}}}\right\rangle\right\}}\right]$$

$=(\langle \dot{s}_{0.157815}, \dot{s}_{012359}\rangle, \langle \ddot{s}_{0.17676}, \ddot{s}_{0.06204}\rangle).$

Now to find the similarity between the opinions of experts in case of u_3 using Type-V similarity measure we have

$$
\left[\frac{\left\{\left\langle \dot{s}_{1(c_1,e_1)_{\frac{2\oplus 3}{2}}}, \dot{s}_{1(c_1,e_1)_{\frac{5\oplus 9}{2}}}\right\rangle, \left\langle \ddot{s}_{1(c_1,e_1)_{\frac{7\oplus 8}{2}}}, \ddot{s}_{1(c_1,e_1)_{\frac{1\oplus 2}{2}}}\right\rangle\right\} \wedge \left\{\left\langle \dot{s}_{2(c_1,e_2)_{\frac{7\oplus 8}{2}}}, \dot{s}_{2(c_1,e_2)_{\frac{3\oplus 4}{2}}}\right\rangle, \left\langle \ddot{s}_{2(c_1,e_2)_{\frac{6\oplus 10}{2}}}, \ddot{s}_{2(c_1,e_2)_{\frac{3\oplus 4}{2}}}\right\rangle\right\}}{\left\{\left\langle \dot{s}_{1(c_1,e_1)_{\frac{2\oplus 3}{2}}}, \dot{s}_{1(c_1,e_1)_{\frac{5\oplus 9}{2}}}\right\rangle, \left\langle \ddot{s}_{1(c_1,e_1)_{\frac{7\oplus 8}{2}}}, \ddot{s}_{1(c_1,e_1)_{\frac{1\oplus 2}{2}}}\right\rangle\right\} \vee \left\{\left\langle \dot{s}_{2(c_1,e_2)_{\frac{7\oplus 8}{2}}}, \dot{s}_{2(c_1,e_2)_{\frac{3\oplus 4}{2}}}\right\rangle, \left\langle \ddot{s}_{2(c_1,e_2)_{\frac{6\oplus 10}{2}}}, \ddot{s}_{2(c_1,e_2)_{\frac{3\oplus 4}{2}}}\right\rangle\right\}}\right]
$$

$$
\oplus \left[\frac{\left\{\left\langle \dot{s}_{1(c_2,e_1)_{\frac{3\oplus 4}{2}}}, \dot{s}_{1(c_2,e_1)_{\frac{4\oplus 7}{2}}}\right\rangle, \left\langle \ddot{s}_{1(c_2,e_1)_{\frac{7\oplus 9}{2}}}, \ddot{s}_{1(c_2,e_1)_{\frac{1\oplus 3}{2}}}\right\rangle\right\} \wedge \left\{\left\langle \dot{s}_{2(c_2,e_2)_{\frac{5\oplus 9}{2}}}, \dot{s}_{2(c_2,e_2)_{\frac{3\oplus 8}{2}}}\right\rangle, \left\langle \ddot{s}_{2(c_2,e_2)_{\frac{5\oplus 7}{2}}}, \ddot{s}_{2(c_2,e_2)_{\frac{3\oplus 4}{2}}}\right\rangle\right\}}{\left\{\left\langle \dot{s}_{1(c_2,e_1)_{\frac{3\oplus 4}{2}}}, \dot{s}_{1(c_2,e_1)_{\frac{4\oplus 7}{2}}}\right\rangle, \left\langle \ddot{s}_{1(c_2,e_1)_{\frac{7\oplus 9}{2}}}, \ddot{s}_{1(c_2,e_1)_{\frac{1\oplus 3}{2}}}\right\rangle\right\} \vee \left\{\left\langle \dot{s}_{2(c_2,e_2)_{\frac{5\oplus 9}{2}}}, \dot{s}_{2(c_2,e_2)_{\frac{3\oplus 8}{2}}}\right\rangle, \left\langle \ddot{s}_{2(c_2,e_2)_{\frac{5\oplus 7}{2}}}, \ddot{s}_{2(c_2,e_2)_{\frac{3\oplus 4}{2}}}\right\rangle\right\}}\right]
$$

$$
\oplus \left[\frac{\left\{\left\langle \dot{s}_{1(c_3,e_1)_{\frac{5\oplus 7}{2}}}, \dot{s}_{1(c_3,e_1)_{\frac{2\oplus 3}{2}}}\right\rangle, \left\langle \ddot{s}_{1(c_3,e_1)_{\frac{6\oplus 9}{2}}}, \ddot{s}_{1(c_3,e_1)_{\frac{3\oplus 4}{2}}}\right\rangle\right\} \wedge \left\{\left\langle \dot{s}_{2(c_3,e_2)_{\frac{1\oplus 3}{2}}}, \dot{s}_{2(c_3,e_2)_{\frac{8\oplus 9}{2}}}\right\rangle, \left\langle \ddot{s}_{2(c_3,e_2)_{\frac{3\oplus 5}{2}}}, \ddot{s}_{2(c_3,e_2)_{\frac{1\oplus 2}{2}}}\right\rangle\right\}}{\left\{\left\langle \dot{s}_{1(c_3,e_1)_{\frac{5\oplus 7}{2}}}, \dot{s}_{1(c_3,e_1)_{\frac{2\oplus 3}{2}}}\right\rangle, \left\langle \ddot{s}_{1(c_3,e_1)_{\frac{6\oplus 9}{2}}}, \ddot{s}_{1(c_3,e_1)_{\frac{3\oplus 4}{2}}}\right\rangle\right\} \vee \left\{\left\langle \dot{s}_{2(c_3,e_2)_{\frac{1\oplus 3}{2}}}, \dot{s}_{2(c_3,e_2)_{\frac{8\oplus 9}{2}}}\right\rangle, \left\langle \ddot{s}_{2(c_3,e_2)_{\frac{3\oplus 5}{2}}}, \ddot{s}_{2(c_3,e_2)_{\frac{1\oplus 2}{2}}}\right\rangle\right\}}\right]
$$

$$=(\langle \dot{s}_{0.06474}, \dot{s}_{1.08529}\rangle, \langle \ddot{s}_{0.138411}, \ddot{s}_{0.2457}\rangle).$$

From above similarity results obtained by Type-V similarity measure shows inappropriate similarity between the opinions of experts, as their opinions are close to each other but similarity between them is too low. To overcome that we now modify Type-V similarity measure.

Firstly we consider the opinions of experts which are too different from each other to observe the similarity in that case. Here we take imaginary GLIVIFSESs containing opinions of experts.

The decisions of experts for u_1 are

$$S_1(c_1,e_1)=\{\langle \dot{s}_{[0,11]},\dot{s}_{[5,6]}\rangle,\langle \ddot{s}_{[5,7]},\ddot{s}_{[7,8]}\rangle\},$$
$$S_1(c_2,e_1)=\{\langle \dot{s}_{[0,1]},\dot{s}_{[1,2]}\rangle,\langle \ddot{s}_{[7,8]},\ddot{s}_{[5,7]}\rangle\},$$
$$S_1(c_3,e_1)=\{\langle \dot{s}_{[2,3]},\dot{s}_{[7,9]}\rangle,\langle \ddot{s}_{[1,3]},\ddot{s}_{[5,10]}\rangle\},$$
$$S_2(c_1,e_2)=\{\langle \dot{s}_{[3,5]},\dot{s}_{[1,3]}\rangle,\langle \ddot{s}_{[2,4]},\ddot{s}_{[1,3]}\rangle\},$$
$$S_2(c_2,e_2)=\{\langle \dot{s}_{[5,11]},\dot{s}_{[2,5]}\rangle,\langle \ddot{s}_{[3,6]},\ddot{s}_{[2,4]}\rangle\},$$
$$S_2(c_3,e_2)=\{\langle \dot{s}_{[10,12]},\dot{s}_{[4,6]}\rangle,\langle \ddot{s}_{[9,11]},\ddot{s}_{[2,3]}\rangle\}.$$

Now to find the similarity between the opinions of experts in case of u_1 using Type-V similarity measure we have

$$\left[\frac{\left\{\left\langle \dot{s}_{1(c_1,e_1)_{\frac{0\oplus 11}{2}}},\dot{s}_{1(c_1,e_1)_{\frac{5\oplus 6}{2}}}\right\rangle,\left\langle \ddot{s}_{1(c_1,e_1)_{\frac{5\oplus 7}{2}}},\ddot{s}_{1(c_1,e_1)_{\frac{7\oplus 8}{2}}}\right\rangle\right\}\wedge\left\{\left\langle \dot{s}_{2(c_1,e_2)_{\frac{3\oplus 5}{2}}},\dot{s}_{2(c_1,e_2)_{\frac{1\oplus 3}{2}}}\right\rangle,\left\langle \ddot{s}_{2(c_1,e_2)_{\frac{2\oplus 4}{2}}},\ddot{s}_{2(c_1,e_2)_{\frac{1\oplus 3}{2}}}\right\rangle\right\}}{\left\{\left\langle \dot{s}_{1(c_1,e_1)_{\frac{0\oplus 11}{2}}},\dot{s}_{1(c_1,e_1)_{\frac{5\oplus 6}{2}}}\right\rangle,\left\langle \ddot{s}_{1(c_1,e_1)_{\frac{5\oplus 7}{2}}},\ddot{s}_{1(c_1,e_1)_{\frac{7\oplus 8}{2}}}\right\rangle\right\}\vee\left\{\left\langle \dot{s}_{2(c_1,e_2)_{\frac{3\oplus 5}{2}}},\dot{s}_{2(c_1,e_2)_{\frac{1\oplus 3}{2}}}\right\rangle,\left\langle \ddot{s}_{2(c_1,e_2)_{\frac{2\oplus 4}{2}}},\ddot{s}_{2(c_1,e_2)_{\frac{1\oplus 3}{2}}}\right\rangle\right\}}\right]$$
$$\oplus\left[\frac{\left\{\left\langle \dot{s}_{1(c_2,e_1)_{\frac{0\oplus 1}{2}}},\dot{s}_{1(c_2,e_1)_{\frac{1\oplus 2}{2}}}\right\rangle,\left\langle \ddot{s}_{1(c_2,e_1)_{\frac{7\oplus 8}{2}}},\ddot{s}_{1(c_2,e_1)_{\frac{5\oplus 7}{2}}}\right\rangle\right\}\wedge\left\{\left\langle \dot{s}_{2(c_2,e_2)_{\frac{5\oplus 11}{2}}},\dot{s}_{2(c_2,e_2)_{\frac{2\oplus 5}{2}}}\right\rangle,\left\langle \ddot{s}_{2(c_2,e_2)_{\frac{3\oplus 6}{2}}},\ddot{s}_{2(c_2,e_2)_{\frac{2\oplus 4}{2}}}\right\rangle\right\}}{\left\{\left\langle \dot{s}_{1(c_2,e_1)_{\frac{0\oplus 1}{2}}},\dot{s}_{1(c_2,e_1)_{\frac{1\oplus 2}{2}}}\right\rangle,\left\langle \ddot{s}_{1(c_2,e_1)_{\frac{7\oplus 8}{2}}},\ddot{s}_{1(c_2,e_1)_{\frac{5\oplus 7}{2}}}\right\rangle\right\}\vee\left\{\left\langle \dot{s}_{2(c_2,e_2)_{\frac{5\oplus 11}{2}}},\dot{s}_{2(c_2,e_2)_{\frac{2\oplus 5}{2}}}\right\rangle,\left\langle \ddot{s}_{2(c_2,e_2)_{\frac{3\oplus 6}{2}}},\ddot{s}_{2(c_2,e_2)_{\frac{2\oplus 4}{2}}}\right\rangle\right\}}\right]$$
$$\oplus\left[\frac{\left\{\left\langle \dot{s}_{1(c_3,e_1)_{\frac{2\oplus 3}{2}}},\dot{s}_{1(c_3,e_1)_{\frac{7\oplus 9}{2}}}\right\rangle,\left\langle \ddot{s}_{1(c_3,e_1)_{\frac{1\oplus 3}{2}}},\ddot{s}_{1(c_3,e_1)_{\frac{5\oplus 10}{2}}}\right\rangle\right\}\wedge\left\{\left\langle \dot{s}_{2(c_3,e_2)_{\frac{10\oplus 12}{2}}},\dot{s}_{2(c_3,e_2)_{\frac{4\oplus 6}{2}}}\right\rangle,\left\langle \ddot{s}_{2(c_3,e_2)_{\frac{9\oplus 11}{2}}},\ddot{s}_{2(c_3,e_2)_{\frac{2\oplus 3}{2}}}\right\rangle\right\}}{\left\{\left\langle \dot{s}_{1(c_3,e_1)_{\frac{2\oplus 3}{2}}},\dot{s}_{1(c_3,e_1)_{\frac{7\oplus 9}{2}}}\right\rangle,\left\langle \ddot{s}_{1(c_3,e_1)_{\frac{1\oplus 3}{2}}},\ddot{s}_{1(c_3,e_1)_{\frac{5\oplus 10}{2}}}\right\rangle\right\}\vee\left\{\left\langle \dot{s}_{2(c_3,e_2)_{\frac{10\oplus 12}{2}}},\dot{s}_{2(c_3,e_2)_{\frac{4\oplus 6}{2}}}\right\rangle,\left\langle \ddot{s}_{2(c_3,e_2)_{\frac{9\oplus 11}{2}}},\ddot{s}_{2(c_3,e_2)_{\frac{2\oplus 3}{2}}}\right\rangle\right\}}\right]$$
$$=(\langle \dot{s}_{0.056465},\dot{s}_{0.59094}\rangle,\langle \ddot{s}_{0.081126},\ddot{s}_{1.452468}\rangle).$$

By comparing that with previous GLIVIFSES $\left(\langle \dot{s}_{0.153397}, \dot{s}_{0.15403}\rangle, \langle \ddot{s}_{0.166436}, \ddot{s}_{0.10965}\rangle\right)$ which is obtained when opinions of experts are close to each other, we can observe that similarity in case of 1-D slightly decreases but similarity in case of 2-D increases significantly.

4.6. MODIFIED TYPE-V SIMILARITY MEASURE FOR GLIVIFSESs.

Definition. Let $U = \{u_i : i = 1, . . . , n\}$ be a set of alternatives and $E = \{e_j : j = 1, . . . , m\}$ represents the set of decision makers and $C = \{c_k : k = 1, . . . , r\}$ represents the set of criteria where n, r, m∈ $\mathbb{Z}$ with the property that either 'n', 'r', 'm' are same or different. Suppose

$$S_1=\left\{\langle \dot{s}_{[\alpha,\alpha']}, \dot{s}_{[\beta,\beta']}\rangle , \langle \ddot{s}_{[\gamma,\gamma']} , \ddot{s}_{[\delta,\delta']}\rangle\right\},$$

and

$$S_2=\left\{\langle \dot{s}_{[\alpha_1,\alpha_1']}, \dot{s}_{[\beta_1,\beta_1']}\rangle , \langle \ddot{s}_{[\gamma_1,\gamma_1']} , \ddot{s}_{[\delta_1,\delta_1']}\rangle\right\},$$

the general form of generalized linguistic interval-valued intuitionistic fuzzy soft expert sets. According to modified Type-V similarity measure between different GLIVIFSESs we have firstly to calculate the similarity between GLIVIFSESs for a specific alternative, which is defined mathematically as under,

$$\sum_{j=1}^{m}\sum_{k=1}^{r}\left[\begin{array}{c}\left\{\langle \dot{s}_{1(e_j,c_k)_{\frac{\alpha\oplus\alpha'}{2}}}, \dot{s}_{1(e_j,c_k)_{\frac{\beta\oplus\beta'}{2}}}\rangle , \langle \ddot{s}_{1(e_j,c_k)_{\frac{\gamma\oplus\gamma'}{2}}} , \ddot{s}_{1(e_j,c_k)_{\frac{\delta\oplus\delta'}{2}}}\rangle\right\}\wedge \\ \left\{\langle \dot{s}_{2(e_{j+1},c_k)_{\frac{\alpha_1\oplus\alpha_1'}{2}}}, \dot{s}_{2(e_{j+1},c_k)_{\frac{\beta_1\oplus\beta_1'}{2}}}\rangle , \langle \ddot{s}_{2(e_{j+1},c_k)_{\frac{\gamma_1\oplus\gamma_1'}{2}}} , \ddot{s}_{2(e_{j+1},c_k)_{\frac{\delta_1\oplus\delta_1'}{2}}}\rangle\right\}\end{array}\right] \qquad (4.6)$$

which shows the similarity between the opinions of expert's for specific alternative u_i where 'i' range goes from one to 'n'. Similarly for each alternative we will use the above mentioned formula to calculate the similarity between the opinions of experts.

Now to find the combined similarity in the opinions of experts for all alternatives we will use the following relation

$$SM(S_1, S_2) = \frac{\sum_{i=0}^{n} S_i(S_1, S_2)}{n}.$$

Now to compare that similarity measure with Type-V similarity measure we consider the data of Example 4.5.2.

EXAMPLE 4.6.1.

By taking data presented in Example 4.5.2 with experts opinions, now by applying modified Type-V similarity measure to find the similarity between the opinions of experts in case of u_1. We have

$$\begin{bmatrix}\left\{\langle \dot{S}_{1(c_1,e_1)_{\frac{3\oplus 4}{2}}}, \dot{S}_{1(c_1,e_1)_{\frac{1\oplus 2}{2}}}\rangle, \langle \ddot{S}_{1(c_1,e_1)_{\frac{2\oplus 3}{2}}}, \ddot{S}_{1(c_1,e_1)_{\frac{1\oplus 3}{2}}}\rangle\right\}\wedge \\ \left\{\langle \dot{S}_{2(c_1,e_2)_{\frac{3\oplus 5}{2}}}, \dot{S}_{2(c_1,e_2)_{\frac{1\oplus 3}{2}}}\rangle, \langle \ddot{S}_{2(c_1,e_2)_{\frac{2\oplus 4}{2}}}, \ddot{S}_{2(c_1,e_2)_{\frac{1\oplus 3}{2}}}\rangle\right\}\end{bmatrix}$$

$$\oplus\begin{bmatrix}\left\{\langle \dot{s}_{1(c_2,e_1)_{\frac{5\oplus 10}{2}}}, \dot{s}_{1(c_2,e_1)_{\frac{2\oplus 5}{2}}}\rangle, \langle \ddot{s}_{1(c_2,e_1)_{\frac{3\oplus 5}{2}}}, \ddot{s}_{1(c_2,e_1)_{\frac{3\oplus 4}{2}}}\rangle\right\}\wedge \\ \left\{\langle \dot{s}_{2(c_2,e_2)_{\frac{5\oplus 11}{2}}}, \dot{s}_{2(c_2,e_2)_{\frac{2\oplus 5}{2}}}\rangle, \langle \ddot{s}_{2(c_2,e_2)_{\frac{3\oplus 6}{2}}}, \ddot{s}_{2(c_2,e_2)_{\frac{2\oplus 4}{2}}}\rangle\right\}\end{bmatrix}$$

$$\oplus\begin{bmatrix}\left\{\langle \dot{s}_{1(c_3,e_1)_{\frac{11\oplus 12}{2}}}, \dot{s}_{1(c_3,e_1)_{\frac{4\oplus 5}{2}}}\rangle, \langle \ddot{s}_{1(c_3,e_1)_{\frac{9\oplus 10}{2}}}, \ddot{s}_{1(c_3,e_1)_{\frac{1\oplus 2}{2}}}\rangle\right\}\wedge \\ \left\{\langle \dot{s}_{2(c_3,e_2)_{\frac{10\oplus 12}{2}}}, \dot{s}_{2(c_3,e_2)_{\frac{4\oplus 6}{2}}}\rangle, \langle \ddot{s}_{2(c_3,e_2)_{\frac{9\oplus 11}{2}}}, \ddot{s}_{2(c_3,e_2)_{\frac{2\oplus 3}{2}}}\rangle\right\}\end{bmatrix}$$

$$= \left(\langle \dot{s}_{\frac{7}{2}}, \dot{s}_2\rangle, \langle \ddot{s}_{\frac{5}{2}}, \ddot{s}_2\rangle\right) \oplus \left(\langle \dot{s}_{\frac{15}{2}}, \dot{s}_{\frac{7}{2}}\rangle, \langle \ddot{s}_4, \ddot{s}_{\frac{7}{2}}\rangle\right) \oplus \left(\langle \dot{s}_{11}, \dot{s}_5\rangle, \langle \ddot{s}_{\frac{19}{2}}, \ddot{s}_{\frac{5}{2}}\rangle\right),$$

after evaluating the addition operator we get

$$S_{u_1}(S_1, S_2) = (\langle \dot{s}_{14.71069}, \dot{s}_{0.10802}\rangle, \langle \ddot{s}_{11.88672}, \ddot{s}_{0.06836}\rangle).$$

Now in case of u_2 the similarity between opinions of experts using modified Type-V similarity measure, we have

$$\begin{bmatrix} \left\{\langle \dot{s}_{1(c_1,e_1)_{\frac{5\oplus 7}{2}}}, \dot{s}_{1(c_1,e_1)_{\frac{2\oplus 3}{2}}}\rangle, \langle \ddot{s}_{1(c_1,e_1)_{\frac{8\oplus 9}{2}}}, \ddot{s}_{1(c_1,e_1)_{\frac{1\oplus 2}{2}}}\rangle\right\} \wedge \\ \left\{\langle \dot{s}_{2(c_1,e_2)_{\frac{6\oplus 7}{2}}}, \dot{s}_{2(c_1,e_2)_{\frac{2\oplus 3}{2}}}\rangle, \langle \ddot{s}_{2(c_1,e_2)_{\frac{7\oplus 9}{2}}}, \ddot{s}_{2(c_1,e_2)_{\frac{1\oplus 3}{2}}}\rangle\right\} \end{bmatrix}$$

$$\oplus \begin{bmatrix} \left\{\langle \dot{s}_{1(c_2,e_1)_{\frac{3\oplus 4}{2}}}, \dot{s}_{1(c_2,e_1)_{\frac{4\oplus 7}{2}}}\rangle, \langle \ddot{s}_{1(c_2,e_1)_{\frac{7\oplus 9}{2}}}, \ddot{s}_{1(c_2,e_1)_{\frac{1\oplus 3}{2}}}\rangle\right\} \wedge \\ \left\{\langle \dot{s}_{2(c_2,e_2)_{\frac{5\oplus 9}{2}}}, \dot{s}_{2(c_2,e_2)_{\frac{3\oplus 8}{2}}}\rangle, \langle \ddot{s}_{2(c_2,e_2)_{\frac{5\oplus 7}{2}}}, \ddot{s}_{2(c_2,e_2)_{\frac{3\oplus 4}{2}}}\rangle\right\} \end{bmatrix}$$

$$\oplus \begin{bmatrix} \left\{\langle \dot{s}_{1(c_3,e_1)_{\frac{13\oplus 14}{2}}}, \dot{s}_{1(c_3,e_1)_{\frac{2\oplus 3}{2}}}\rangle, \langle \ddot{s}_{1(c_3,e_1)_{\frac{11\oplus 13}{2}}}, \ddot{s}_{1(c_3,e_1)_{\frac{1\oplus 2}{2}}}\rangle\right\} \wedge \\ \left\{\langle \dot{s}_{2(c_3,e_2)_{\frac{12\oplus 13}{2}}}, \dot{s}_{2(c_3,e_2)_{\frac{2\oplus 4}{2}}}\rangle, \langle \ddot{s}_{2(c_3,e_2)_{\frac{11\oplus 14}{2}}}, \ddot{s}_{2(c_3,e_2)_{\frac{1\oplus 2}{2}}}\rangle\right\} \end{bmatrix}$$

$= \left(\langle \dot{s}_6, \dot{s}_{\frac{5}{2}}\rangle, \langle \ddot{s}_8, \ddot{s}_2\rangle\right) \oplus \left(\langle \dot{s}_9, \dot{s}_4\rangle, \langle \ddot{s}_{\frac{15}{2}}, \ddot{s}_{\frac{5}{2}}\rangle\right) \oplus \left(\langle \dot{s}_{\frac{25}{2}}, \dot{s}_3\rangle, \langle \ddot{s}_{12}, \ddot{s}_{\frac{3}{2}}\rangle\right),$

after evaluating addition operator we get

$$S_{u_2}(S_1, S_2) = (\langle \dot{s}_{16.1667}, \dot{s}_{0.09259}\rangle, \langle \ddot{s}_{14.9375}, \ddot{s}_{0.029297}\rangle).$$

Now to find the similarity between the opinions of experts using modified Type-V similarity measure we have

$$\begin{bmatrix} \left\{\langle \dot{s}_{1(c_1,e_1)_{\frac{2\oplus 3}{2}}}, \dot{s}_{1(c_1,e_1)_{\frac{5\oplus 9}{2}}}\rangle, \langle \ddot{s}_{1(c_1,e_1)_{\frac{7\oplus 8}{2}}}, \ddot{s}_{1(c_1,e_1)_{\frac{1\oplus 2}{2}}}\rangle\right\} \wedge \\ \left\{\langle \dot{s}_{2(c_1,e_2)_{\frac{7\oplus 8}{2}}}, \dot{s}_{2(c_1,e_2)_{\frac{3\oplus 4}{2}}}\rangle, \langle \ddot{s}_{2(c_1,e_2)_{\frac{6\oplus 10}{2}}}, \ddot{s}_{2(c_1,e_2)_{\frac{3\oplus 4}{2}}}\rangle\right\} \end{bmatrix}$$

$$\oplus \begin{bmatrix} \left\{\langle \dot{s}_{1(c_2,e_1)_{\frac{3\oplus 4}{2}}}, \dot{s}_{1(c_2,e_1)_{\frac{4\oplus 7}{2}}}\rangle, \langle \ddot{s}_{1(c_2,e_1)_{\frac{7\oplus 9}{2}}}, \ddot{s}_{1(c_2,e_1)_{\frac{1\oplus 3}{2}}}\rangle\right\} \wedge \\ \left\{\langle \dot{s}_{2(c_2,e_2)_{\frac{5\oplus 9}{2}}}, \dot{s}_{2(c_2,e_2)_{\frac{3\oplus 8}{2}}}\rangle, \langle \ddot{s}_{2(c_2,e_2)_{\frac{5\oplus 7}{2}}}, \ddot{s}_{2(c_2,e_2)_{\frac{3\oplus 4}{2}}}\rangle\right\} \end{bmatrix}$$

$$\oplus \begin{bmatrix} \left\{\langle \dot{s}_{1(c_3,e_1)_{\frac{5\oplus 7}{2}}}, \dot{s}_{1(c_3,e_1)_{\frac{2\oplus 3}{2}}}\rangle, \langle \ddot{s}_{1(c_3,e_1)_{\frac{6\oplus 9}{2}}}, \ddot{s}_{1(c_3,e_1)_{\frac{3\oplus 4}{2}}}\rangle\right\} \wedge \\ \left\{\langle \dot{s}_{2(c_3,e_2)_{\frac{1\oplus 3}{2}}}, \dot{s}_{2(c_3,e_2)_{\frac{8\oplus 9}{2}}}\rangle, \langle \ddot{s}_{2(c_3,e_2)_{\frac{3\oplus 5}{2}}}, \ddot{s}_{2(c_3,e_2)_{\frac{1\oplus 2}{2}}}\rangle\right\} \end{bmatrix}$$

$= \left(\langle \dot{s}_{\frac{5}{2}}, \dot{s}_7\rangle, \langle \ddot{s}_{\frac{15}{2}}, \ddot{s}_{\frac{7}{2}}\rangle\right) \oplus \left(\langle \dot{s}_{\frac{7}{2}}, \dot{s}_{\frac{11}{2}}\rangle, \langle \ddot{s}_6, \ddot{s}_{\frac{7}{2}}\rangle\right) \oplus \left(\langle \dot{s}_2, \dot{s}_{\frac{17}{2}}\rangle, \langle \ddot{s}_4, \ddot{s}_{\frac{7}{2}}\rangle\right),$

after evaluating addition operator we get

$$S_{u_3}(S_1, S_2) = (\langle \dot{s}_{6.90123}, \dot{s}_{1.01003}\rangle, \langle \ddot{s}_{12.015625}, \ddot{s}_{0.16748}\rangle).$$

These similarity results show a vast difference between Type-V and modified Type-V similarity measures.

From the above similarity results, we can observe that the similarity between the opinions of experts in case of u_1 and u_2 is greater than 50% in fever of these alternatives but further in case of u_2 similarity in fever goes to 89% which is higher than the case of u_1. So, from above calculations it is clear that the nomination by experts for mining license in A is u_2 which is tethyan copper company (TCC).

4.7. MAX-MIN SIMILARITY MEASURE FOR GLIVIFSESs.

Max-min similarity measure for GLIVIFSESs uses the operation of minimum and maximum for similarity reasons which we take from [8] in some sense because in that paper Karacapilidis et al. Uses that operator in finding similarity measure for simple fuzzy sets and then Wen-Liang Hung et al. [9] extended that idea for intuitionistic fuzzy sets.

Definition. Let $U = \{u_i : i = 1, . . . \ , n\}$ be a set of alternatives and $E = \{e_j : j = 1, . . . \ , m\}$ represents the set of decision makers and $C = \{c_k : k = 1, . . . \ , r\}$ represents the set of criteria where n, r, m$\in \mathbb{Z}$ with the property that either ‘n’, ‘r’, ‘m’ are same or different. Suppose

$$S_1=\left\{\langle \dot{s}_{[\alpha,\alpha']}, \dot{s}_{[\beta,\beta']}\rangle \ , \langle \ddot{s}_{[\gamma,\gamma']} \ , \ddot{s}_{[\delta,\delta']}\rangle\right\},$$

and

$$S_2=\left\{\langle \dot{s}_{[\alpha_1,\alpha_1']}, \dot{s}_{[\beta_1,\beta_1']}\rangle \ , \langle \ddot{s}_{[\gamma_1,\gamma_1']} \ , \ddot{s}_{[\delta_1,\delta_1']}\rangle\right\},$$

the general form of generalized linguistic interval-valued intuitionistic fuzzy soft expert sets. According to max-min similarity measure for GLIVIFSESs we have

$$= \frac{\sum_{i=1}^{n}\left[\sum_{j=1}^{m}\sum_{k=1}^{r}\left[\begin{array}{c}\left\{\langle \dot{s}_{1(e_j,c_k)_{[\alpha,\alpha']}}, \dot{s}_{1(e_j,c_k)_{[\beta,\beta']}}\rangle, \langle \ddot{s}_{1(e_j,c_k)_{[\gamma,\gamma']}}, \ddot{s}_{1(e_j,c_k)_{[\delta,\delta']}}\rangle\right\} \wedge \\ \left\{\langle \dot{s}_{2(e_{j+1},c_k)_{[\alpha_1,\alpha_1']}}, \dot{s}_{2(e_{j+1},c_k)_{[\beta_1,\beta_1']}}\rangle, \langle \ddot{s}_{2(e_{j+1},c_k)_{[\gamma_1,\gamma_1']}}, \ddot{s}_{2(e_{j+1},c_k)_{[\delta_1,\delta_1']}}\rangle\right\}\end{array}\right]\right]}{\sum_{i=1}^{n}\left[\sum_{j=1}^{m}\sum_{k=1}^{r}\left[\begin{array}{c}\left\{\langle \dot{s}_{1(e_j,c_k)_{[\alpha,\alpha']}}, \dot{s}_{1(e_j,c_k)_{[\beta,\beta']}}\rangle, \langle \ddot{s}_{1(e_j,c_k)_{[\gamma,\gamma']}}, \ddot{s}_{1(e_j,c_k)_{[\delta,\delta']}}\rangle\right\} \vee \\ \left\{\langle \dot{s}_{2(e_{j+1},c_k)_{[\alpha_1,\alpha_1']}}, \dot{s}_{2(e_{j+1},c_k)_{[\beta_1,\beta_1']}}\rangle, \langle \ddot{s}_{2(e_{j+1},c_k)_{[\gamma_1,\gamma_1']}}, \ddot{s}_{2(e_{j+1},c_k)_{[\delta_1,\delta_1']}}\rangle\right\}\end{array}\right]\right]} \quad \text{(a)}$$

Such that for a specific alternative i.e. $i = g$ where $g \in \{1,2, . . . , r\}$ we have

$$= \frac{\sum_{j=1}^{m}\sum_{k=1}^{r}\left[\begin{array}{c}\left\{\langle \dot{s}_{1(e_j,c_k)_{[\alpha,\alpha']}}, \dot{s}_{1(e_j,c_k)_{[\beta,\beta']}}\rangle, \langle \ddot{s}_{1(e_j,c_k)_{[\gamma,\gamma']}}, \ddot{s}_{1(e_j,c_k)_{[\delta,\delta']}}\rangle\right\} \wedge \\ \left\{\langle \dot{s}_{2(e_{j+1},c_k)_{[\alpha_1,\alpha_1']}}, \dot{s}_{2(e_{j+1},c_k)_{[\beta_1,\beta_1']}}\rangle, \langle \ddot{s}_{2(e_{j+1},c_k)_{[\gamma_1,\gamma_1']}}, \ddot{s}_{2(e_{j+1},c_k)_{[\delta_1,\delta_1']}}\rangle\right\}\end{array}\right]}{\sum_{j=1}^{m}\sum_{k=1}^{r}\left[\begin{array}{c}\left\{\langle \dot{s}_{1(e_j,c_k)_{[\alpha,\alpha']}}, \dot{s}_{1(e_j,c_k)_{[\beta,\beta']}}\rangle, \langle \ddot{s}_{1(e_j,c_k)_{[\gamma,\gamma']}}, \ddot{s}_{1(e_j,c_k)_{[\delta,\delta']}}\rangle\right\} \vee \\ \left\{\langle \dot{s}_{2(e_{j+1},c_k)_{[\alpha_1,\alpha_1']}}, \dot{s}_{2(e_{j+1},c_k)_{[\beta_1,\beta_1']}}\rangle, \langle \ddot{s}_{2(e_{j+1},c_k)_{[\gamma_1,\gamma_1']}}, \ddot{s}_{2(e_{j+1},c_k)_{[\delta_1,\delta_1']}}\rangle\right\}\end{array}\right]} \quad \text{(b)}$$

here point to notice that we can change aggregation in above Equation (b) in combing opinions when experts and criteria's changes.

Now to illustrate the above listed methodology we construct a practical problem but firstly consider data from Example 4.1.1 for comparison reasons with previously discussed similarity measures.

EXAMPLE 4.7.1.

By taking data from Example 4.1.1 we have the following GLIVIFSESs, which basically represent the opinions of experts (students) regarding their teachers (alternatives).

$$S_1(h_1, a)=\{\langle \dot{s}_{[2,3]}, \dot{s}_{[1,2]}\rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\rangle\},$$
$$S_1(h_2, a)=\{\langle \dot{s}_{[3,4]}, \dot{s}_{[1,2]}\rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\rangle\},$$
$$S_2(h_1, b)=\{\langle \dot{s}_{[1,3]}, \dot{s}_{[2,3]}\rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\rangle\},$$

$$\mathrm{S}_2(\mathrm{h}_2,\mathrm{b})=\left\{\langle \dot{\mathrm{s}}_{[1,2]}, \dot{\mathrm{s}}_{[3,4]}\rangle, \langle \ddot{\mathrm{s}}_{[1,2]}, \ddot{\mathrm{s}}_{[1,2]}\rangle\right\}.$$

These sets represent the evaluation values by students ‘a’ and ‘b’ for the teacher t_1. Now for the second teacher t_2 the evaluation values are as under

$$\mathrm{S}_1(\mathrm{h}_1,\mathrm{a})=\left\{\langle \dot{\mathrm{s}}_{[1,2]}, \dot{\mathrm{s}}_{[2,4]}\rangle, \langle \ddot{\mathrm{s}}_{[1,2]}, \ddot{\mathrm{s}}_{[1,2]}\rangle\right\},$$
$$\mathrm{S}_1(\mathrm{h}_2,\mathrm{a})=\left\{\langle \dot{\mathrm{s}}_{[2,4]}, \dot{\mathrm{s}}_{[1,2]}\rangle, \langle \ddot{\mathrm{s}}_{[1,2]}, \ddot{\mathrm{s}}_{[1,2]}\rangle\right\},$$
$$\mathrm{S}_2(\mathrm{h}_1,\mathrm{b})=\left\{\langle \dot{\mathrm{s}}_{[2,3]}, \dot{\mathrm{s}}_{[2,3]}\rangle, \langle \ddot{\mathrm{s}}_{[1,2]}, \ddot{\mathrm{s}}_{[1,2]}\rangle\right\},$$
$$\mathrm{S}_2(\mathrm{h}_2,\mathrm{b})=\left\{\langle \dot{\mathrm{s}}_{[1,2]}, \dot{\mathrm{s}}_{[3,4]}\rangle, \langle \ddot{\mathrm{s}}_{[1,2]}, \ddot{\mathrm{s}}_{[1,2]}\rangle\right\}.$$

Now by applying max-min similarity measure for aggregating opinions regarding t_1

$$=\frac{\begin{bmatrix}\left\{\langle \dot{\mathrm{S}}_{1(a,h_1)_{[2,3]}}, \dot{\mathrm{S}}_{1(a,h_1)_{[1,2]}}\rangle, \langle \ddot{\mathrm{S}}_{1(a,h_1)_{[1,2]}}, \ddot{\mathrm{S}}_{1(a,h_1)_{[1,2]}}\rangle\right\}\wedge \\ \left\{\langle \dot{\mathrm{S}}_{2(b,h_1)_{[1,3]}}, \dot{\mathrm{S}}_{2(b,h_1)_{[2,3]}}\rangle, \langle \ddot{\mathrm{S}}_{2(b,h_1)_{[1,2]}}, \ddot{\mathrm{S}}_{2(b,h_1)_{[1,2]}}\rangle\right\}\end{bmatrix} \oplus \begin{bmatrix}\left\{\langle \dot{\mathrm{S}}_{1(a,h_2)_{[3,4]}}, \dot{\mathrm{S}}_{1(a,h_2)_{[1,2]}}\rangle, \langle \ddot{\mathrm{S}}_{1(a,h_2)_{[1,2]}}, \ddot{\mathrm{S}}_{1(a,h_2)_{[1,2]}}\rangle\right\}\wedge \\ \left\{\langle \dot{\mathrm{S}}_{2(b,h_2)_{[1,2]}}, \dot{\mathrm{S}}_{2(b,h_2)_{[3,4]}}\rangle, \langle \ddot{\mathrm{S}}_{2(b,h_2)_{[1,2]}}, \ddot{\mathrm{S}}_{2(b,h_2)_{[1,2]}}\rangle\right\}\end{bmatrix}}{\begin{bmatrix}\left\{\langle \dot{\mathrm{S}}_{1(a,h_1)_{[2,3]}}, \dot{\mathrm{S}}_{1(a,h_1)_{[1,2]}}\rangle, \langle \ddot{\mathrm{S}}_{1(a,h_1)_{[1,2]}}, \ddot{\mathrm{S}}_{1(a,h_1)_{[1,2]}}\rangle\right\}\vee \\ \left\{\langle \dot{\mathrm{S}}_{2(b,h_1)_{[1,3]}}, \dot{\mathrm{S}}_{2(b,h_1)_{[2,3]}}\rangle, \langle \ddot{\mathrm{S}}_{2(b,h_1)_{[1,2]}}, \ddot{\mathrm{S}}_{2(b,h_1)_{[1,2]}}\rangle\right\}\end{bmatrix} \oplus \begin{bmatrix}\left\{\langle \dot{\mathrm{S}}_{1(a,h_2)_{[3,4]}}, \dot{\mathrm{S}}_{1(a,h_2)_{[1,2]}}\rangle, \langle \ddot{\mathrm{S}}_{1(a,h_2)_{[1,2]}}, \ddot{\mathrm{S}}_{1(a,h_2)_{[1,2]}}\rangle\right\}\vee \\ \left\{\langle \dot{\mathrm{S}}_{2(b,h_2)_{[1,2]}}, \dot{\mathrm{S}}_{2(b,h_2)_{[3,4]}}\rangle, \langle \ddot{\mathrm{S}}_{2(b,h_2)_{[1,2]}}, \ddot{\mathrm{S}}_{2(b,h_2)_{[1,2]}}\rangle\right\}\end{bmatrix}}$$

$$=\frac{\left\langle \dot{s}_{\left[\frac{11}{6},4\right]}, \dot{s}_{[1,2]}\right\rangle, \left\langle \ddot{s}_{\left[\frac{7}{4},3\right]}, \ddot{s}_{\left[\frac{1}{4},1\right]}\right\rangle}{\left\langle \dot{s}_{[4,5]}, \dot{s}_{\left[\frac{1}{6},\frac{2}{3}\right]}\right\rangle, \left\langle \ddot{s}_{\left[\frac{7}{4},3\right]}, \ddot{s}_{\left[\frac{1}{4},1\right]}\right\rangle}.$$

Also, by applying max-min similarity measure for aggregating opinions regarding t_2 we get

$$=\frac{\begin{bmatrix}\left\{\langle \dot{\mathrm{S}}_{1(a,h_1)_{[1,2]}}, \dot{\mathrm{S}}_{1(a,h_1)_{[2,4]}}\rangle, \langle \ddot{\mathrm{S}}_{1(a,h_1)_{[1,2]}}, \ddot{\mathrm{S}}_{1(a,h_1)_{[1,2]}}\rangle\right\}\wedge \\ \left\{\langle \dot{\mathrm{S}}_{2(b,h_1)_{[2,3]}}, \dot{\mathrm{S}}_{2(b,h_1)_{[2,3]}}\rangle, \langle \ddot{\mathrm{S}}_{2(b,h_1)_{[1,2]}}, \ddot{\mathrm{S}}_{2(b,h_1)_{[1,2]}}\rangle\right\}\end{bmatrix} \oplus \begin{bmatrix}\left\{\langle \dot{\mathrm{S}}_{1(a,h_2)_{[2,4]}}, \dot{\mathrm{S}}_{1(a,h_2)_{[1,2]}}\rangle, \langle \ddot{\mathrm{S}}_{1(a,h_2)_{[1,2]}}, \ddot{\mathrm{S}}_{1(a,h_2)_{[1,2]}}\rangle\right\}\wedge \\ \left\{\langle \dot{\mathrm{S}}_{2(b,h_2)_{[1,2]}}, \dot{\mathrm{S}}_{2(b,h_2)_{[3,4]}}\rangle, \langle \ddot{\mathrm{S}}_{2(b,h_2)_{[1,2]}}, \ddot{\mathrm{S}}_{2(b,h_2)_{[1,2]}}\rangle\right\}\end{bmatrix}}{\begin{bmatrix}\left\{\langle \dot{\mathrm{S}}_{1(a,h_1)_{[1,2]}}, \dot{\mathrm{S}}_{1(a,h_1)_{[2,4]}}\rangle, \langle \ddot{\mathrm{S}}_{1(a,h_1)_{[1,2]}}, \ddot{\mathrm{S}}_{1(a,h_1)_{[1,2]}}\rangle\right\}\vee \\ \left\{\langle \dot{\mathrm{S}}_{2(b,h_1)_{[2,3]}}, \dot{\mathrm{S}}_{2(b,h_1)_{[2,3]}}\rangle, \langle \ddot{\mathrm{S}}_{2(b,h_1)_{[1,2]}}, \ddot{\mathrm{S}}_{2(b,h_1)_{[1,2]}}\rangle\right\}\end{bmatrix} \oplus \begin{bmatrix}\left\{\langle \dot{\mathrm{S}}_{1(a,h_2)_{[2,4]}}, \dot{\mathrm{S}}_{1(a,h_2)_{[1,2]}}\rangle, \langle \ddot{\mathrm{S}}_{1(a,h_2)_{[1,2]}}, \ddot{\mathrm{S}}_{1(a,h_2)_{[1,2]}}\rangle\right\}\vee \\ \left\{\langle \dot{\mathrm{S}}_{2(b,h_2)_{[1,2]}}, \dot{\mathrm{S}}_{2(b,h_2)_{[3,4]}}\rangle, \langle \ddot{\mathrm{S}}_{2(b,h_2)_{[1,2]}}, \ddot{\mathrm{S}}_{2(b,h_2)_{[1,2]}}\rangle\right\}\end{bmatrix}}$$

$$=\frac{\left\langle \dot{s}_{\left[\frac{11}{6},\frac{10}{3}\right]}, \dot{s}_{\left[1,\frac{8}{3}\right]}\right\rangle, \left\langle \ddot{s}_{\left[\frac{7}{4},3\right]}, \ddot{s}_{\left[\frac{1}{4},1\right]}\right\rangle}{\left\langle \dot{s}_{\left[\frac{23}{6},5\right]}, \dot{s}_{\left[\frac{1}{3},1\right]}\right\rangle, \left\langle \ddot{s}_{\left[\frac{7}{4},3\right]}, \ddot{s}_{\left[\frac{1}{4},1\right]}\right\rangle}.$$

Now by substituting these values in equation (a) we get

$$=\frac{\left(\left\langle \dot{s}_{\left[\frac{11}{6},4\right]}, \dot{s}_{[1,2]}\right\rangle, \left\langle \ddot{s}_{\left[\frac{7}{4},3\right]}, \ddot{s}_{\left[\frac{1}{4},1\right]}\right\rangle\right) \oplus \left(\left\langle \dot{s}_{\left[\frac{11}{6},\frac{10}{3}\right]}, \dot{s}_{\left[1,\frac{8}{3}\right]}\right\rangle, \left\langle \ddot{s}_{\left[\frac{7}{4},3\right]}, \ddot{s}_{\left[\frac{1}{4},1\right]}\right\rangle\right)}{\left(\left\langle \dot{s}_{[4,5]}, \dot{s}_{\left[\frac{1}{6},\frac{2}{3}\right]}\right\rangle, \left\langle \ddot{s}_{\left[\frac{7}{4},3\right]}, \ddot{s}_{\left[\frac{1}{4},1\right]}\right\rangle\right) \oplus \left(\left\langle \dot{s}_{\left[\frac{23}{6},5\right]}, \dot{s}_{\left[\frac{1}{3},1\right]}\right\rangle, \left\langle \ddot{s}_{\left[\frac{7}{4},3\right]}, \ddot{s}_{\left[\frac{1}{4},1\right]}\right\rangle\right)}$$

$$=\frac{\left\langle \dot{s}_{\left[\frac{671}{216},\frac{49}{9}\right]}, \dot{s}_{\left[\frac{1}{6},\frac{8}{9}\right]}\right\rangle, \left\langle \ddot{s}_{\left[\frac{175}{64},\frac{15}{4}\right]}, \ddot{s}_{\left[\frac{1}{64},\frac{1}{4}\right]}\right\rangle}{\left\langle \dot{s}_{\left[\frac{95}{18},\frac{35}{6}\right]}, \dot{s}_{\left[\frac{1}{108},\frac{1}{6}\right]}\right\rangle, \left\langle \ddot{s}_{\left[\frac{175}{64},\frac{15}{4}\right]}, \ddot{s}_{\left[\frac{1}{64},\frac{1}{4}\right]}\right\rangle}$$

$$=\left(\left\langle \dot{s}_{\left[\frac{671}{216},\frac{49}{9}\right]}, \dot{s}_{\left[\frac{1}{6},\frac{8}{9}\right]}\right\rangle, \left\langle \ddot{s}_{\left[\frac{175}{64},\frac{15}{4}\right]}, \ddot{s}_{\left[\frac{1}{64},\frac{1}{4}\right]}\right\rangle\right) \otimes \left(\left\langle \dot{s}_{\left[\frac{6}{35},\frac{18}{95}\right]}, \dot{s}_{\left[\frac{1}{108},\frac{1}{6}\right]}\right\rangle, \left\langle \ddot{s}_{\left[\frac{4}{15},\frac{64}{175}\right]}, \ddot{s}_{\left[\frac{1}{64},\frac{1}{4}\right]}\right\rangle\right),$$

here notice that due to violation of $[0, t]$ and $[0, t']$ we take out the intervals for non-membership for 1-D and 2-D as it is. Now by multiplying these GLIVIFSESs we get

$$=\left(\left\langle \dot{s}_{\left[\frac{671}{7560},\frac{49}{285}\right]}, \dot{s}_{\left[\frac{683}{3888},\frac{167}{162}\right]}\right\rangle, \left\langle \ddot{s}_{\left[\frac{35}{192},\frac{12}{35}\right]}, \ddot{s}_{\left[\frac{511}{16384},\frac{31}{64}\right]}\right\rangle\right).$$

Now to compare that similarity result with previous similarity results we use score function, according to score function

$$= s_{[0.49276, 0.4284]\times[0.51889, 0.48231]} = s_{[0.2066, 0.255689]} = s_{0.23114},$$

but if we take score function for GLIVIFSESs as

$$= s_{\left[\frac{t-(\alpha-\beta)}{2t},\frac{t-(\alpha'-\beta')}{2t}\right]\times\left[\frac{t'-(\gamma-\delta)}{2t'},\frac{t'-(\gamma'-\delta')}{2t'}\right]},$$

rather than the score function

$$= s_{\left[\frac{t+\alpha-\beta}{2t},\frac{t+\alpha'-\beta'}{2t}\right]\times\left[\frac{t'+\gamma-\delta}{2t'},\frac{t'+\gamma'-\delta'}{2t'}\right]},$$

which is defined for GLIVIFSESs, then we get intervals without violation of their property of infimum and supremum such that

$$= s_{[0.50724, 0.571578]\times[0.481112, 0.51769]} = s_{[0.24404, 0.2959]},$$

for comparison reasons we take average value from interval such that the value of score function is given as

$$= s_{0.26997}.$$

Which shows that similarity result obtained by max-min similarity measure is greater than the similarity results obtained by Type-I, Type-III, Type-IV and Type-V similarity measures.

EXAMPLE 4.7.2.

Higher education commission of Pakistan (HEC) and tempus public foundation of hungry (TPF) signed a document of understanding (DOU), according to which Hungarian scholarship program will provide stipend to Pakistani students for undergraduate, graduate, one-tier master degree and doctoral level programs for study in Hungarian specified universities after evaluation of higher education commission and tempus public foundation on merit bases for the award of scholarships to deserving students. The final decision for award will be made on similarity bases between the opinions of HEC and TPF with the restriction that similarity would be greater than 30%.

For simplicity reasons we take $\{e_1, e_2\}$ as authorities of HEC and TPF respectively for making decisions about candidates. We take candidates as $\{a, b, c\}$ with criteria's fixed for their nomination as

- English proficiency
- Innovative work
- Performance/marks in previous study levels

we call these criteria's as $\{c_1, c_2, c_3\}$ respectively. To avoid any inconvenience the authorities $\{e_1, e_2\}$ of HEC and TPF are directed that while giving opinions about specific candidates also give the experience details with specific alternatives.

We take linguistic term set S_t for linguistic terms related to candidate selection with variation of $'t'$ as $0 \leq t \leq 16$ and linguistic term set $S_{t'}$ for linguistic terms related to experience with variation of $'t''$ as $0 \leq t' \leq 14$.

From above discussed problem it is clear that opinions of authorities will be in the form of GLIVIFSESs, such that opinions will be in the form of intervals rather than a fixed linguistic terms due to lack of perfectness in assigning any linguistic term, similarly in case of rejection.

The opinions of authorities regarding candidate 'a' are as under

$$\mathrm{S}_1(\mathrm{c}_1, \mathrm{e}_1)=\left\{\langle \dot{\mathrm{s}}_{[5,7]}, \dot{\mathrm{s}}_{[4,5]} \rangle, \langle \ddot{\mathrm{s}}_{[6,7]}, \ddot{\mathrm{s}}_{[2,3]} \rangle\right\},$$
$$\mathrm{S}_1(\mathrm{c}_2, \mathrm{e}_1)=\left\{\langle \dot{\mathrm{s}}_{[3,4]}, \dot{\mathrm{s}}_{[5,7]} \rangle, \langle \ddot{\mathrm{s}}_{[7,9]}, \ddot{\mathrm{s}}_{[1,2]} \rangle\right\},$$
$$\mathrm{S}_1(\mathrm{c}_3, \mathrm{e}_1)=\left\{\langle \dot{\mathrm{s}}_{[5,5.6]}, \dot{\mathrm{s}}_{[3,4]} \rangle, \langle \ddot{\mathrm{s}}_{[6,7]}, \ddot{\mathrm{s}}_{[2.2,3]} \rangle\right\},$$
$$\mathrm{S}_2(\mathrm{c}_1, \mathrm{e}_2)=\left\{\langle \dot{\mathrm{s}}_{[5.5,7]}, \dot{\mathrm{s}}_{[4,5.5]} \rangle, \langle \ddot{\mathrm{s}}_{[6,6.5]}, \ddot{\mathrm{s}}_{[2,4]} \rangle\right\},$$
$$\mathrm{S}_2(\mathrm{c}_2, \mathrm{e}_2)=\left\{\langle \dot{\mathrm{s}}_{[3,5]}, \dot{\mathrm{s}}_{[5,6.5]} \rangle, \langle \ddot{\mathrm{s}}_{[8,9]}, \ddot{\mathrm{s}}_{[1,2]} \rangle\right\},$$
$$\mathrm{S}_2(\mathrm{c}_3, \mathrm{e}_2)=\left\{\langle \dot{\mathrm{s}}_{[5,6]}, \dot{\mathrm{s}}_{[3,4]} \rangle, \langle \ddot{\mathrm{s}}_{[6,7]}, \ddot{\mathrm{s}}_{[2,3]} \rangle\right\}.$$

Now by applying max-min similarity measure for specific case defined in equation (b) we get

$$=\frac{\left[\begin{array}{c}[\{\langle\dot{s}_{[5,7]},\dot{s}_{[4,5]}\rangle,\langle\ddot{s}_{[6,7]},\ddot{s}_{[2,3]}\rangle\}\wedge\{\langle\dot{s}_{[5.5,7]},\dot{s}_{[4,5.5]}\rangle,\langle\ddot{s}_{[6,6.5]},\ddot{s}_{[2,4]}\rangle\}]\oplus[\{\langle\dot{s}_{[3,4]},\dot{s}_{[5,7]}\rangle,\langle\ddot{s}_{[7,9]},\ddot{s}_{[1,2]}\rangle\}\wedge\{\langle\dot{s}_{[3,5]},\dot{s}_{[5,6.5]}\rangle,\langle\ddot{s}_{[8,9]},\ddot{s}_{[1,2]}\rangle\}]\\ \oplus[\{\langle\dot{s}_{[5,5.6]},\dot{s}_{[3,4]}\rangle,\langle\ddot{s}_{[6,7]},\ddot{s}_{[2.2,3]}\rangle\}\wedge\{\langle\dot{s}_{[5,6]},\dot{s}_{[3,4]}\rangle,\langle\ddot{s}_{[6,7]},\ddot{s}_{[2,3]}\rangle\}]\end{array}\right]}{\left[\begin{array}{c}[\{\langle\dot{s}_{[5,7]},\dot{s}_{[4,5]}\rangle,\langle\ddot{s}_{[6,7]},\ddot{s}_{[2,3]}\rangle\}\vee\{\langle\dot{s}_{[5.5,7]},\dot{s}_{[4,5.5]}\rangle,\langle\ddot{s}_{[6,6.5]},\ddot{s}_{[2,4]}\rangle\}]\oplus[\{\langle\dot{s}_{[3,4]},\dot{s}_{[5,7]}\rangle,\langle\ddot{s}_{[7,9]},\ddot{s}_{[1,2]}\rangle\}\vee\{\langle\dot{s}_{[3,5]},\dot{s}_{[5,6.5]}\rangle,\langle\ddot{s}_{[8,9]},\ddot{s}_{[1,2]}\rangle\}]\\ \oplus[\{\langle\dot{s}_{[5,5.6]},\dot{s}_{[3,4]}\rangle,\langle\ddot{s}_{[6,7]},\ddot{s}_{[2.2,3]}\rangle\}\vee\{\langle\dot{s}_{[5,6]},\dot{s}_{[3,4]}\rangle,\langle\ddot{s}_{[6,7]},\ddot{s}_{[2,3]}\rangle\}]\end{array}\right]}$$

$$=\frac{\left\{\langle\dot{s}_{\left[\frac{2523}{256},\frac{929}{80}\right]},\dot{s}_{\left[\frac{15}{64},\frac{77}{128}\right]}\rangle,\langle\ddot{s}_{\left[\frac{82}{7},\frac{709}{56}\right]},\ddot{s}_{\left[\frac{11}{490},\frac{6}{49}\right]}\rangle\right\}}{\left\{\langle\dot{s}_{\left[\frac{4991}{512},\frac{1553}{128}\right]},\dot{s}_{\left[\frac{15}{64},\frac{65}{128}\right]}\rangle,\langle\ddot{s}_{\left[\frac{590}{49},\frac{51}{4}\right]},\ddot{s}_{\left[\frac{1}{49},\frac{6}{49}\right]}\rangle\right\}} \tag{b1}$$

$$=\left\{\langle\dot{s}_{[0.0508,0.07445]},\dot{s}_{[2.17476,4.7078]}\rangle,\langle\ddot{s}_{[0.0656,0.07511]},\ddot{s}_{[0.2446,8.2177]}\rangle\right\}.$$

The opinions of experts regarding candidate ‘b’ are

$$S_1(c_1,e_1)=\left\{\langle\dot{s}_{[11,12]},\dot{s}_{[2,3]}\rangle,\langle\ddot{s}_{[7,8]},\ddot{s}_{[2,4]}\rangle\right\},$$
$$S_1(c_2,e_1)=\left\{\langle\dot{s}_{[8,9]},\dot{s}_{[3,4]}\rangle,\langle\ddot{s}_{[5,7]},\ddot{s}_{[2,4]}\rangle\right\},$$
$$S_1(c_3,e_1)=\left\{\langle\dot{s}_{[10,11]},\dot{s}_{[1,3]}\rangle,\langle\ddot{s}_{[9,10]},\ddot{s}_{[1,2]}\rangle\right\},$$
$$S_2(c_1,e_2)=\left\{\langle\dot{s}_{[2,3]},\dot{s}_{[2,2.5]}\rangle,\langle\ddot{s}_{[7,8.5]},\ddot{s}_{[1,3]}\rangle\right\},$$
$$S_2(c_2,e_2)=\left\{\langle\dot{s}_{[1,3]},\dot{s}_{[3,4]}\rangle,\langle\ddot{s}_{[6,8]},\ddot{s}_{[1,2]}\rangle\right\},$$
$$S_2(c_3,e_2)=\left\{\langle\dot{s}_{[0,2]},\dot{s}_{[2,4]}\rangle,\langle\ddot{s}_{[8,10]},\ddot{s}_{[2,3]}\rangle\right\},$$

By applying max-min similarity measure for ‘b’ we get

$$=\frac{\left[\begin{array}{c}[\{\langle\dot{s}_{[11,12]},\dot{s}_{[2,3]}\rangle,\langle\ddot{s}_{[7,8]},\ddot{s}_{[2,4]}\rangle\}\wedge\{\langle\dot{s}_{[2,3]},\dot{s}_{[2,2.5]}\rangle,\langle\ddot{s}_{[7,8.5]},\ddot{s}_{[1,3]}\rangle\}]\oplus[\{\langle\dot{s}_{[8,9]},\dot{s}_{[3,4]}\rangle,\langle\ddot{s}_{[5,7]},\ddot{s}_{[2,4]}\rangle\}\wedge\{\langle\dot{s}_{[1,3]},\dot{s}_{[3,4]}\rangle,\langle\ddot{s}_{[6,8]},\ddot{s}_{[1,2]}\rangle\}]\\ \oplus[\{\langle\dot{s}_{[10,11]},\dot{s}_{[1,3]}\rangle,\langle\ddot{s}_{[9,10]},\ddot{s}_{[1,2]}\rangle\}\wedge\{\langle\dot{s}_{[0,2]},\dot{s}_{[2,4]}\rangle,\langle\ddot{s}_{[8,10]},\ddot{s}_{[2,3]}\rangle\}]\end{array}\right]}{\left[\begin{array}{c}[\{\langle\dot{s}_{[11,12]},\dot{s}_{[2,3]}\rangle,\langle\ddot{s}_{[7,8]},\ddot{s}_{[2,4]}\rangle\}\vee\{\langle\dot{s}_{[2,3]},\dot{s}_{[2,2.5]}\rangle,\langle\ddot{s}_{[7,8.5]},\ddot{s}_{[1,3]}\rangle\}]\oplus[\{\langle\dot{s}_{[8,9]},\dot{s}_{[3,4]}\rangle,\langle\ddot{s}_{[5,7]},\ddot{s}_{[2,4]}\rangle\}\vee\{\langle\dot{s}_{[1,3]},\dot{s}_{[3,4]}\rangle,\langle\ddot{s}_{[6,8]},\ddot{s}_{[1,2]}\rangle\}]\\ \oplus[\{\langle\dot{s}_{[10,11]},\dot{s}_{[1,3]}\rangle,\langle\ddot{s}_{[9,10]},\ddot{s}_{[1,2]}\rangle\}\vee\{\langle\dot{s}_{[0,2]},\dot{s}_{[2,4]}\rangle,\langle\ddot{s}_{[8,10]},\ddot{s}_{[2,3]}\rangle\}]\end{array}\right]}$$

$$=\frac{\left\{\langle\dot{s}_{\left[\frac{23}{8},\frac{865}{128}\right]},\dot{s}_{\left[\frac{3}{64},\frac{3}{16}\right]}\rangle,\langle\ddot{s}_{\left[\frac{169}{14},\frac{92}{7}\right]},\ddot{s}_{\left[\frac{2}{49},\frac{12}{49}\right]}\rangle\right\}}{\left\{\langle\dot{s}_{\left[\frac{241}{16},\frac{989}{64}\right]},\dot{s}_{\left[\frac{3}{128},\frac{15}{128}\right]}\rangle,\langle\ddot{s}_{\left[\frac{88}{7},\frac{653}{49}\right]},\ddot{s}_{\left[\frac{1}{196},\frac{1}{98}\right]}\rangle\right\}} \tag{b2}$$

$$=\left\{\langle\dot{s}_{[0.01163,0.02804]},\dot{s}_{[0.07024,8.620833]}\rangle,\langle\ddot{s}_{[0.0647,0.07468]},\ddot{s}_{[0.0459,0.2549]}\rangle\right\}.$$

The opinions of experts for candidate ‘c’ are as under

$$S_1(c_1,e_1)=\left\{\langle\dot{s}_{[2,3]},\dot{s}_{[6,8]}\rangle,\langle\ddot{s}_{[9,11]},\ddot{s}_{[1,2]}\rangle\right\},$$
$$S_1(c_2,e_1)=\left\{\langle\dot{s}_{[3,4]},\dot{s}_{[6,7]}\rangle,\langle\ddot{s}_{[6,8]},\ddot{s}_{[1,4]}\rangle\right\},$$
$$S_1(c_3,e_1)=\left\{\langle\dot{s}_{[2,3]},\dot{s}_{[5,6]}\rangle,\langle\ddot{s}_{[4,5]},\ddot{s}_{[1,3]}\rangle\right\},$$
$$S_2(c_1,e_2)=\left\{\langle\dot{s}_{[1,3]},\dot{s}_{[10,12]}\rangle,\langle\ddot{s}_{[7,9]},\ddot{s}_{[1,4]}\rangle\right\},$$
$$S_2(c_2,e_2)=\left\{\langle\dot{s}_{[2,3]},\dot{s}_{[3,4]}\rangle,\langle\ddot{s}_{[5,7]},\ddot{s}_{[1,3]}\rangle\right\},$$
$$S_2(c_3,e_2)=\left\{\langle\dot{s}_{[4,5]},\dot{s}_{[6,7]}\rangle,\langle\ddot{s}_{[5,6]},\ddot{s}_{[2,3]}\rangle\right\}.$$

Now by applying max-min similarity measure for specific alternative ‘c’

$$= \frac{\begin{bmatrix}[\{\langle \dot{s}_{[2,3]},\dot{s}_{[6,8]}\rangle ,\langle \ddot{s}_{[9,11]} ,\ddot{s}_{[1,2]}\rangle\}\wedge\{\langle \dot{s}_{[1,3]},\dot{s}_{[10,12]}\rangle ,\langle \ddot{s}_{[7,9]} ,\ddot{s}_{[1,4]}\rangle\}]\oplus[\{\langle \dot{s}_{[3,4]},\dot{s}_{[6,7]}\rangle ,\langle \ddot{s}_{[6,8]} ,\ddot{s}_{[1,4]}\rangle\}\wedge\{\langle \dot{s}_{[2,3]},\dot{s}_{[3,4]}\rangle ,\langle \ddot{s}_{[5,7]} ,\ddot{s}_{[1,3]}\rangle\}] \\ \oplus[\{\langle \dot{s}_{[2,3]},\dot{s}_{[5,6]}\rangle ,\langle \ddot{s}_{[4,5]} ,\ddot{s}_{[1,3]}\rangle\}\wedge\{\langle \dot{s}_{[4,5]},\dot{s}_{[6,7]}\rangle ,\langle \ddot{s}_{[5,6]} ,\ddot{s}_{[2,3]}\rangle\}]\end{bmatrix}}{\begin{bmatrix}[\{\langle \dot{s}_{[2,3]},\dot{s}_{[6,8]}\rangle ,\langle \ddot{s}_{[9,11]} ,\ddot{s}_{[1,2]}\rangle\}\vee\{\langle \dot{s}_{[1,3]},\dot{s}_{[10,12]}\rangle ,\langle \ddot{s}_{[7,9]} ,\ddot{s}_{[1,4]}\rangle\}]\oplus[\{\langle \dot{s}_{[3,4]},\dot{s}_{[6,7]}\rangle ,\langle \ddot{s}_{[6,8]} ,\ddot{s}_{[1,4]}\rangle\}\vee\{\langle \dot{s}_{[2,3]},\dot{s}_{[3,4]}\rangle ,\langle \ddot{s}_{[5,7]} ,\ddot{s}_{[1,3]}\rangle\}] \\ \oplus[\{\langle \dot{s}_{[2,3]},\dot{s}_{[5,6]}\rangle ,\langle \ddot{s}_{[4,5]} ,\ddot{s}_{[1,3]}\rangle\}\vee\{\langle \dot{s}_{[4,5]},\dot{s}_{[6,7]}\rangle ,\langle \ddot{s}_{[5,6]} ,\ddot{s}_{[2,3]}\rangle\}]\end{bmatrix}}$$

$$= \frac{\left\{\langle \dot{s}_{\left[\frac{289}{64},\frac{1899}{256}\right]}, \dot{s}_{\left[\frac{45}{32},\frac{147}{64}\right]}\rangle ,\langle \ddot{s}_{\left[\frac{151}{14},\frac{347}{28}\right]} ,\ddot{s}_{\left[\frac{1}{98},\frac{12}{49}\right]}\rangle\right\}}{\left\{\langle \dot{s}_{\left[\frac{239}{32},\frac{595}{64}\right]}, \dot{s}_{\left[\frac{45}{128},\frac{3}{4}\right]}\rangle ,\langle \ddot{s}_{\left[\frac{596}{49},\frac{650}{49}\right]} ,\ddot{s}_{\left[\frac{1}{56},\frac{9}{98}\right]}\rangle\right\}} \qquad \text{(b3)}$$

$$= \left\{\langle \dot{s}_{[0.03036,0.06208]}, \dot{s}_{[2.622396,4.73297]}\rangle ,\langle \ddot{s}_{[0.058077,0.07278]} ,\ddot{s}_{[0.02805,10.9433]}\rangle\right\}.$$

As from calculations from equations (b1), (b2) and (b3) it is clear that results obtained doesn’t fulfill the similarity between the opinions of experts because as we can observe that opinions of experts are not too different from each other but the similarity results obtained by max-min similarity measure are approximately approaches s_0.

To overcome this inappropriateness we now modify the max-min similarity measure for GLIVIFSESs.

4.8. MODIFIED MAX-MIN SIMILARITY MEASURE FOR GLIVIFSESs.

Definition. Let $U = \{u_i: i = 1, . . . \ , n\}$ be a set of alternatives and $E = \{e_j: j = 1, . . . \ , m\}$ represents the set of decision makers and $C = \{c_k: k = 1, . . . \ , r\}$ represents the set of criteria where n, r, m∈ ℤ with the property that either ‘n’, ‘r’, ‘m’ are same or different. Suppose

$$S_1=\left\{\langle \dot{s}_{[\alpha,\alpha']}, \dot{s}_{[\beta,\beta']}\rangle ,\langle \ddot{s}_{[\gamma,\gamma']} ,\ddot{s}_{[\delta,\delta']}\rangle\right\},$$

and

$$S_2=\left\{\langle \dot{s}_{[\alpha_1,\alpha_1']}, \dot{s}_{[\beta_1,\beta_1']}\rangle ,\langle \ddot{s}_{[\gamma_1,\gamma_1']} ,\ddot{s}_{[\delta_1,\delta_1']}\rangle\right\},$$

the general form of generalized linguistic interval-valued intuitionistic fuzzy soft expert sets. According to modified max-min similarity measure for GLIVIFSESs

$$
= \sum_{i=1}^{n} \left[\begin{array}{c} \left[\sum_{j=1}^{m} \sum_{k=1}^{r} \left[\begin{array}{c} \left\{ \left\langle \dot{s}_{1(e_j,c_k)_{[\alpha,\alpha']}}, \dot{s}_{1(e_j,c_k)_{[\beta,\beta']}} \right\rangle , \left\langle \ddot{s}_{1(e_j,c_k)_{[\gamma,\gamma']}} , \ddot{s}_{1(e_j,c_k)_{[\delta,\delta']}} \right\rangle \right\} \wedge \\ \left\{ \left\langle \dot{s}_{2(e_{j+1},c_k)_{[\alpha_1,\alpha_1']}}, \dot{s}_{2(e_{j+1},c_k)_{[\beta_1,\beta_1']}} \right\rangle , \left\langle \ddot{s}_{2(e_{j+1},c_k)_{[\gamma_1,\gamma_1']}} , \ddot{s}_{2(e_{j+1},c_k)_{[\delta_1,\delta_1']}} \right\rangle \right\} \end{array} \right] \right] \otimes \\ \left[\sum_{j=1}^{m} \sum_{k=1}^{r} \left[\begin{array}{c} \left\{ \left\langle \dot{s}_{1(e_j,c_k)_{[\alpha,\alpha']}}, \dot{s}_{1(e_j,c_k)_{[\beta,\beta']}} \right\rangle , \left\langle \ddot{s}_{1(e_j,c_k)_{[\gamma,\gamma']}} , \ddot{s}_{1(e_j,c_k)_{[\delta,\delta']}} \right\rangle \right\} \vee \\ \left\{ \left\langle \dot{s}_{2(e_{j+1},c_k)_{[\alpha_1,\alpha_1']}}, \dot{s}_{2(e_{j+1},c_k)_{[\beta_1,\beta_1']}} \right\rangle , \left\langle \ddot{s}_{2(e_{j+1},c_k)_{[\gamma_1,\gamma_1']}} , \ddot{s}_{2(e_{j+1},c_k)_{[\delta_1,\delta_1']}} \right\rangle \right\} \end{array} \right] \right] \end{array} \right] \tag{4.8}
$$

now by applying modified max-min similarity measure on Example 4.7.2.

EXAMPLE 4.8.1.

By taking data from Example 4.7.2 we have

the opinions of authorities regarding candidate 'a' are as under

$$
\begin{gathered}
S_1(c_1, e_1) = \{\langle \dot{s}_{[5,7]}, \dot{s}_{[4,5]} \rangle , \langle \ddot{s}_{[6,7]} , \ddot{s}_{[2,3]} \rangle\}, \\
S_1(c_2, e_1) = \{\langle \dot{s}_{[3,4]}, \dot{s}_{[5,7]} \rangle , \langle \ddot{s}_{[7,9]} , \ddot{s}_{[1,2]} \rangle\}, \\
S_1(c_3, e_1) = \{\langle \dot{s}_{[5,5.6]}, \dot{s}_{[3,4]} \rangle , \langle \ddot{s}_{[6,7]} , \ddot{s}_{[2.2,3]} \rangle\}, \\
S_2(c_1, e_2) = \{\langle \dot{s}_{[5.5,7]}, \dot{s}_{[4,5.5]} \rangle , \langle \ddot{s}_{[6,6.5]} , \ddot{s}_{[2,4]} \rangle\}, \\
S_2(c_2, e_2) = \{\langle \dot{s}_{[3,5]}, \dot{s}_{[5,6.5]} \rangle , \langle \ddot{s}_{[8,9]} , \ddot{s}_{[1,2]} \rangle\}, \\
S_2(c_3, e_2) = \{\langle \dot{s}_{[5,6]}, \dot{s}_{[3,4]} \rangle , \langle \ddot{s}_{[6,7]} , \ddot{s}_{[2,3]} \rangle\}.
\end{gathered}
$$

Now by applying modified max-min similarity measure for specific case we get

$$
= \left[\begin{array}{c} \left[\begin{array}{c} [\{\langle \dot{s}_{[5,7]}, \dot{s}_{[4,5]} \rangle , \langle \ddot{s}_{[6,7]} , \ddot{s}_{[2,3]} \rangle\} \wedge \{\langle \dot{s}_{[5.5,7]}, \dot{s}_{[4,5.5]} \rangle , \langle \ddot{s}_{[6,6.5]} , \ddot{s}_{[2,4]} \rangle\}] \oplus [\{\langle \dot{s}_{[3,4]}, \dot{s}_{[5,7]} \rangle , \langle \ddot{s}_{[7,9]} , \ddot{s}_{[1,2]} \rangle\} \wedge \{\langle \dot{s}_{[3,5]}, \dot{s}_{[5,6.5]} \rangle , \langle \ddot{s}_{[8,9]} , \ddot{s}_{[1,2]} \rangle\}] \\ \oplus [\{\langle \dot{s}_{[5,5.6]}, \dot{s}_{[3,4]} \rangle , \langle \ddot{s}_{[6,7]} , \ddot{s}_{[2.2,3]} \rangle\} \wedge \{\langle \dot{s}_{[5,6]}, \dot{s}_{[3,4]} \rangle , \langle \ddot{s}_{[6,7]} , \ddot{s}_{[2,3]} \rangle\}] \end{array} \right] \\ \otimes \left[\begin{array}{c} [\{\langle \dot{s}_{[5,7]}, \dot{s}_{[4,5]} \rangle , \langle \ddot{s}_{[6,7]} , \ddot{s}_{[2,3]} \rangle\} \vee \{\langle \dot{s}_{[5.5,7]}, \dot{s}_{[4,5.5]} \rangle , \langle \ddot{s}_{[6,6.5]} , \ddot{s}_{[2,4]} \rangle\}] \oplus [\{\langle \dot{s}_{[3,4]}, \dot{s}_{[5,7]} \rangle , \langle \ddot{s}_{[7,9]} , \ddot{s}_{[1,2]} \rangle\} \vee \{\langle \dot{s}_{[3,5]}, \dot{s}_{[5,6.5]} \rangle , \langle \ddot{s}_{[8,9]} , \ddot{s}_{[1,2]} \rangle\}] \\ \oplus [\{\langle \dot{s}_{[5,5.6]}, \dot{s}_{[3,4]} \rangle , \langle \ddot{s}_{[6,7]} , \ddot{s}_{[2.2,3]} \rangle\} \vee \{\langle \dot{s}_{[5,6]}, \dot{s}_{[3,4]} \rangle , \langle \ddot{s}_{[6,7]} , \ddot{s}_{[2,3]} \rangle\}] \end{array} \right] \end{array} \right]
$$

$$
= \left\{ \left\langle \dot{s}_{\left[\frac{2523}{256}, \frac{929}{80}\right]}, \dot{s}_{\left[\frac{15}{64}, \frac{77}{128}\right]} \right\rangle , \left\langle \ddot{s}_{\left[\frac{82}{7}, \frac{709}{56}\right]} , \ddot{s}_{\left[\frac{11}{490}, \frac{6}{49}\right]} \right\rangle \right\} \otimes \left\{ \left\langle \dot{s}_{\left[\frac{4991}{512}, \frac{1553}{128}\right]}, \dot{s}_{\left[\frac{15}{64}, \frac{65}{128}\right]} \right\rangle , \left\langle \ddot{s}_{\left[\frac{590}{49}, \frac{51}{4}\right]} , \ddot{s}_{\left[\frac{1}{49}, \frac{6}{49}\right]} \right\rangle \right\}
$$

$$
= \{\langle \dot{s}_{[6.0045,8.8056]}, \dot{s}_{[0.4653,1.09028]} \rangle , \langle \ddot{s}_{[10.07497,11.53029]} , \ddot{s}_{[0.0428,0.2438]} \rangle\}.
$$

By comparing it with previous similarity result obtained by max-min similarity measure we can observe that there is vast difference between the similarity results in opinions of experts. According to similarity between authorities in favor of candidate 'a' is approximately 46%.

The opinions of experts regarding candidate ‘b’ are

$$S_1(c_1,e_1)=\left\{\langle \dot{s}_{[11,12]},\dot{s}_{[2,3]}\rangle,\langle \ddot{s}_{[7,8]},\ddot{s}_{[2,4]}\rangle\right\},$$
$$S_1(c_2,e_1)=\left\{\langle \dot{s}_{[8,9]},\dot{s}_{[3,4]}\rangle,\langle \ddot{s}_{[5,7]},\ddot{s}_{[2,4]}\rangle\right\},$$
$$S_1(c_3,e_1)=\left\{\langle \dot{s}_{[10,11]},\dot{s}_{[1,3]}\rangle,\langle \ddot{s}_{[9,10]},\ddot{s}_{[1,2]}\rangle\right\},$$
$$S_2(c_1,e_2)=\left\{\langle \dot{s}_{[2,3]},\dot{s}_{[2,2.5]}\rangle,\langle \ddot{s}_{[7,8.5]},\ddot{s}_{[1,3]}\rangle\right\},$$
$$S_2(c_2,e_2)=\left\{\langle \dot{s}_{[1,3]},\dot{s}_{[3,4]}\rangle,\langle \ddot{s}_{[6,8]},\ddot{s}_{[1,2]}\rangle\right\},$$
$$S_2(c_3,e_2)=\left\{\langle \dot{s}_{[0,2]},\dot{s}_{[2,4]}\rangle,\langle \ddot{s}_{[8,10]},\ddot{s}_{[2,3]}\rangle\right\}.$$

By applying modified max-min similarity measure for ‘b’ we get

$$=\left[\begin{array}{c}\left[\begin{array}{c}\left[\left\{\langle \dot{s}_{[11,12]},\dot{s}_{[2,3]}\rangle,\langle \ddot{s}_{[7,8]},\ddot{s}_{[2,4]}\rangle\right\}\wedge\left\{\langle \dot{s}_{[2,3]},\dot{s}_{[2,2.5]}\rangle,\langle \ddot{s}_{[7,8.5]},\ddot{s}_{[1,3]}\rangle\right\}\right]\oplus\left[\left\{\langle \dot{s}_{[8,9]},\dot{s}_{[3,4]}\rangle,\langle \ddot{s}_{[5,7]},\ddot{s}_{[2,4]}\rangle\right\}\wedge\left\{\langle \dot{s}_{[1,3]},\dot{s}_{[3,4]}\rangle,\langle \ddot{s}_{[6,8]},\ddot{s}_{[1,2]}\rangle\right\}\right]\\ \oplus\left[\left\{\langle \dot{s}_{[10,11]},\dot{s}_{[1,3]}\rangle,\langle \ddot{s}_{[9,10]},\ddot{s}_{[1,2]}\rangle\right\}\wedge\left\{\langle \dot{s}_{[0,2]},\dot{s}_{[2,4]}\rangle,\langle \ddot{s}_{[8,10]},\ddot{s}_{[2,3]}\rangle\right\}\right]\end{array}\right]\\ \otimes\left[\begin{array}{c}\left[\left\{\langle \dot{s}_{[11,12]},\dot{s}_{[2,3]}\rangle,\langle \ddot{s}_{[7,8]},\ddot{s}_{[2,4]}\rangle\right\}\vee\left\{\langle \dot{s}_{[2,3]},\dot{s}_{[2,2.5]}\rangle,\langle \ddot{s}_{[7,8.5]},\ddot{s}_{[1,3]}\rangle\right\}\right]\oplus\left[\left\{\langle \dot{s}_{[8,9]},\dot{s}_{[3,4]}\rangle,\langle \ddot{s}_{[5,7]},\ddot{s}_{[2,4]}\rangle\right\}\vee\left\{\langle \dot{s}_{[1,3]},\dot{s}_{[3,4]}\rangle,\langle \ddot{s}_{[6,8]},\ddot{s}_{[1,2]}\rangle\right\}\right]\\ \oplus\left[\left\{\langle \dot{s}_{[10,11]},\dot{s}_{[1,3]}\rangle,\langle \ddot{s}_{[9,10]},\ddot{s}_{[1,2]}\rangle\right\}\vee\left\{\langle \dot{s}_{[0,2]},\dot{s}_{[2,4]}\rangle,\langle \ddot{s}_{[8,10]},\ddot{s}_{[2,3]}\rangle\right\}\right]\end{array}\right]\end{array}\right]$$

$$=\left\{\langle \dot{s}_{\left[\frac{23}{8},\frac{865}{128}\right]},\dot{s}_{\left[\frac{3}{64},\frac{3}{16}\right]}\rangle,\langle \ddot{s}_{\left[\frac{169}{14},\frac{92}{7}\right]},\ddot{s}_{\left[\frac{2}{49},\frac{12}{49}\right]}\rangle\right\}\otimes\left\{\langle \dot{s}_{\left[\frac{241}{16},\frac{989}{64}\right]},\dot{s}_{\left[\frac{3}{128},\frac{15}{128}\right]}\rangle,\langle \ddot{s}_{\left[\frac{88}{7},\frac{653}{49}\right]},\ddot{s}_{\left[\frac{1}{196},\frac{1}{98}\right]}\rangle\right\}$$

$$=\left\{\langle \dot{s}_{[2.70654,6.5268]},\dot{s}_{[0.07024,0.30423]}\rangle,\langle \ddot{s}_{[10.83965,12.5106]},\ddot{s}_{[0.0459,0.2549]}\rangle\right\}.$$

By comparing it with previous similarity measure result we can observe too much difference between results. The similarity between opinions of authorities in favor of candidate ‘b’ is approximately 29%.

The opinions of experts for candidate ‘c’ are as under

$$S_1(c_1,e_1)=\left\{\langle \dot{s}_{[2,3]},\dot{s}_{[6,8]}\rangle,\langle \ddot{s}_{[9,11]},\ddot{s}_{[1,2]}\rangle\right\},$$
$$S_1(c_2,e_1)=\left\{\langle \dot{s}_{[3,4]},\dot{s}_{[6,7]}\rangle,\langle \ddot{s}_{[6,8]},\ddot{s}_{[1,4]}\rangle\right\},$$
$$S_1(c_3,e_1)=\left\{\langle \dot{s}_{[2,3]},\dot{s}_{[5,6]}\rangle,\langle \ddot{s}_{[4,5]},\ddot{s}_{[1,3]}\rangle\right\},$$
$$S_2(c_1,e_2)=\left\{\langle \dot{s}_{[1,3]},\dot{s}_{[10,12]}\rangle,\langle \ddot{s}_{[7,9]},\ddot{s}_{[1,4]}\rangle\right\},$$
$$S_2(c_2,e_2)=\left\{\langle \dot{s}_{[2,3]},\dot{s}_{[3,4]}\rangle,\langle \ddot{s}_{[5,7]},\ddot{s}_{[1,3]}\rangle\right\},$$
$$S_2(c_3,e_2)=\left\{\langle \dot{s}_{[4,5]},\dot{s}_{[6,7]}\rangle,\langle \ddot{s}_{[5,6]},\ddot{s}_{[2,3]}\rangle\right\}.$$

Now by applying modified max-min similarity measure for specific alternative ‘c’

$$=\left[\begin{array}{c}\left[\left\{\langle \dot{s}_{[2,3]},\dot{s}_{[6,8]}\rangle,\langle \ddot{s}_{[9,11]},\ddot{s}_{[1,2]}\rangle\right\}\wedge\left\{\langle \dot{s}_{[1,3]},\dot{s}_{[10,12]}\rangle,\langle \ddot{s}_{[7,9]},\ddot{s}_{[1,4]}\rangle\right\}\right]\oplus\left[\left\{\langle \dot{s}_{[3,4]},\dot{s}_{[6,7]}\rangle,\langle \ddot{s}_{[6,8]},\ddot{s}_{[1,4]}\rangle\right\}\wedge\left\{\langle \dot{s}_{[2,3]},\dot{s}_{[3,4]}\rangle,\langle \ddot{s}_{[5,7]},\ddot{s}_{[1,3]}\rangle\right\}\right]\\ \oplus\left[\left\{\langle \dot{s}_{[2,3]},\dot{s}_{[5,6]}\rangle,\langle \ddot{s}_{[4,5]},\ddot{s}_{[1,3]}\rangle\right\}\wedge\left\{\langle \dot{s}_{[4,5]},\dot{s}_{[6,7]}\rangle,\langle \ddot{s}_{[5,6]},\ddot{s}_{[2,3]}\rangle\right\}\right]\\ \left[\begin{array}{c}\left[\left\{\langle \dot{s}_{[2,3]},\dot{s}_{[6,8]}\rangle,\langle \ddot{s}_{[9,11]},\ddot{s}_{[1,2]}\rangle\right\}\vee\left\{\langle \dot{s}_{[1,3]},\dot{s}_{[10,12]}\rangle,\langle \ddot{s}_{[7,9]},\ddot{s}_{[1,4]}\rangle\right\}\right]\oplus\left[\left\{\langle \dot{s}_{[3,4]},\dot{s}_{[6,7]}\rangle,\langle \ddot{s}_{[6,8]},\ddot{s}_{[1,4]}\rangle\right\}\vee\left\{\langle \dot{s}_{[2,3]},\dot{s}_{[3,4]}\rangle,\langle \ddot{s}_{[5,7]},\ddot{s}_{[1,3]}\rangle\right\}\right]\\ \oplus\left[\left\{\langle \dot{s}_{[2,3]},\dot{s}_{[5,6]}\rangle,\langle \ddot{s}_{[4,5]},\ddot{s}_{[1,3]}\rangle\right\}\vee\left\{\langle \dot{s}_{[4,5]},\dot{s}_{[6,7]}\rangle,\langle \ddot{s}_{[5,6]},\ddot{s}_{[2,3]}\rangle\right\}\right]\end{array}\right]\end{array}\right]$$

$$=\left\{\langle \dot{s}_{\left[\frac{289}{64},\frac{1899}{256}\right]},\dot{s}_{\left[\frac{45}{32},\frac{147}{64}\right]}\rangle,\langle \ddot{s}_{\left[\frac{151}{14},\frac{347}{28}\right]},\ddot{s}_{\left[\frac{1}{98},\frac{12}{49}\right]}\rangle\right\}\otimes\left\{\langle \dot{s}_{\left[\frac{239}{32},\frac{595}{64}\right]},\dot{s}_{\left[\frac{45}{128},\frac{3}{4}\right]}\rangle,\langle \ddot{s}_{\left[\frac{596}{49},\frac{650}{49}\right]},\ddot{s}_{\left[\frac{1}{56},\frac{9}{98}\right]}\rangle\right\}$$

$= \{\langle \dot{s}_{[2.10788,4.92599]}, \dot{s}_{[1.7269,2.9392]} \rangle, \langle \ddot{s}_{[9.37068,11.7425]}, \ddot{s}_{[0.02805,0.335]} \rangle\}$,
thus the similarity between the opinions of experts in favor in candidate 'c' case is approximately 22%.

Now from above calculations for similarity between opinions of authorities of HEC & TPF it is clear that only in case of candidate 'a' similarity is $46\% > 30\%$, thus the candidate 'a' is nominated for scholarship from a set of three candidates.

Now we apply modified max-min similarity measure on Example 4.1.1 to compare this measure with previous similarity measures.

EXAMPLE 4.8.2.

By taking data from Example 4.1.1 we have

$$S_1(h_1, a)=\{\langle \dot{s}_{[2,3]}, \dot{s}_{[1,2]} \rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]} \rangle\},$$
$$S_1(h_2, a)=\{\langle \dot{s}_{[3,4]}, \dot{s}_{[1,2]} \rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]} \rangle\},$$
$$S_2(h_1, b)=\{\langle \dot{s}_{[1,3]}, \dot{s}_{[2,3]} \rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]} \rangle\},$$
$$S_2(h_2, b)=\{\langle \dot{s}_{[1,2]}, \dot{s}_{[3,4]} \rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]} \rangle\},$$

these sets represent the evaluation values by students 'a' and 'b' for the teacher t_1. Now for the second teacher t_2 the evaluation values are as under

$$S_1(h_1, a)=\{\langle \dot{s}_{[1,2]}, \dot{s}_{[2,4]} \rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]} \rangle\},$$
$$S_1(h_2, a)=\{\langle \dot{s}_{[2,4]}, \dot{s}_{[1,2]} \rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]} \rangle\},$$
$$S_2(h_1, b)=\{\langle \dot{s}_{[2,3]}, \dot{s}_{[2,3]} \rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]} \rangle\},$$
$$S_2(h_2, b)=\{\langle \dot{s}_{[1,2]}, \dot{s}_{[3,4]} \rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]} \rangle\}.$$

Now by applying modified max-min similarity measure for aggregating opinions regarding t_1.

$$
= \left[\begin{array}{c}
\left[\left[\begin{array}{l}\left\{\langle \dot{s}_{1(a,h_1)_{[2,3]}}, \dot{s}_{1(a,h_1)_{[1,2]}}\rangle, \langle \ddot{s}_{1(a,h_1)_{[1,2]}}, \ddot{s}_{1(a,h_1)_{[1,2]}}\rangle\right\} \wedge \\ \left\{\langle \dot{s}_{2(b,h_1)_{[1,3]}}, \dot{s}_{2(b,h_1)_{[2,3]}}\rangle, \langle \ddot{s}_{2(b,h_1)_{[1,2]}}, \ddot{s}_{2(b,h_1)_{[1,2]}}\rangle\right\}\end{array}\right] \oplus \left[\begin{array}{l}\left\{\langle \dot{s}_{1(a,h_2)_{[3,4]}}, \dot{s}_{1(a,h_2)_{[1,2]}}\rangle, \langle \ddot{s}_{1(a,h_2)_{[1,2]}}, \ddot{s}_{1(a,h_2)_{[1,2]}}\rangle\right\} \wedge \\ \left\{\langle \dot{s}_{2(b,h_2)_{[1,2]}}, \dot{s}_{2(b,h_2)_{[3,4]}}\rangle, \langle \ddot{s}_{2(b,h_2)_{[1,2]}}, \ddot{s}_{2(b,h_2)_{[1,2]}}\rangle\right\}\end{array}\right]\right] \otimes \\
\left[\left[\begin{array}{l}\left\{\langle \dot{s}_{1(a,h_1)_{[2,3]}}, \dot{s}_{1(a,h_1)_{[1,2]}}\rangle, \langle \ddot{s}_{1(a,h_1)_{[1,2]}}, \ddot{s}_{1(a,h_1)_{[1,2]}}\rangle\right\} \vee \\ \left\{\langle \dot{s}_{2(b,h_1)_{[1,3]}}, \dot{s}_{2(b,h_1)_{[2,3]}}\rangle, \langle \ddot{s}_{2(b,h_1)_{[1,2]}}, \ddot{s}_{2(b,h_1)_{[1,2]}}\rangle\right\}\end{array}\right] \oplus \left[\begin{array}{l}\left\{\langle \dot{s}_{1(a,h_2)_{[3,4]}}, \dot{s}_{1(a,h_2)_{[1,2]}}\rangle, \langle \ddot{s}_{1(a,h_2)_{[1,2]}}, \ddot{s}_{1(a,h_2)_{[1,2]}}\rangle\right\} \vee \\ \left\{\langle \dot{s}_{2(b,h_2)_{[1,2]}}, \dot{s}_{2(b,h_2)_{[3,4]}}\rangle, \langle \ddot{s}_{2(b,h_2)_{[1,2]}}, \ddot{s}_{2(b,h_2)_{[1,2]}}\rangle\right\}\end{array}\right]\right]
\end{array}\right]
$$

$$
= \left\{\langle \dot{s}_{\left[\frac{11}{6},4\right]}, \dot{s}_{[1,2]}\rangle, \langle \ddot{s}_{\left[\frac{7}{4},3\right]}, \ddot{s}_{\left[\frac{1}{4},1\right]}\rangle\right\} \otimes \left\{\langle \dot{s}_{[4,5]}, \dot{s}_{\left[\frac{1}{6},\frac{2}{3}\right]}\rangle, \langle \ddot{s}_{\left[\frac{7}{4},3\right]}, \ddot{s}_{\left[\frac{1}{4},1\right]}\rangle\right\}
$$

$$
= \left\{\langle \dot{s}_{\left[\frac{11}{9},\frac{10}{3}\right]}, \dot{s}_{\left[\frac{41}{36},\frac{22}{9}\right]}\rangle, \langle \ddot{s}_{\left[\frac{49}{64},\frac{9}{4}\right]}, \ddot{s}_{\left[\frac{31}{64},\frac{7}{4}\right]}\rangle\right\}.
$$

Now in case of t_2, the similarity between opinions of experts from max-min similarity measure is given as

$$
= \left[\begin{array}{c}
\left[\left[\begin{array}{l}\left\{\langle \dot{s}_{1(a,h_1)_{[1,2]}}, \dot{s}_{1(a,h_1)_{[2,4]}}\rangle, \langle \ddot{s}_{1(a,h_1)_{[1,2]}}, \ddot{s}_{1(a,h_1)_{[1,2]}}\rangle\right\} \wedge \\ \left\{\langle \dot{s}_{2(b,h_1)_{[2,3]}}, \dot{s}_{2(b,h_1)_{[2,3]}}\rangle, \langle \ddot{s}_{2(b,h_1)_{[1,2]}}, \ddot{s}_{2(b,h_1)_{[1,2]}}\rangle\right\}\end{array}\right] \oplus \left[\begin{array}{l}\left\{\langle \dot{s}_{1(a,h_2)_{[2,4]}}, \dot{s}_{1(a,h_2)_{[1,2]}}\rangle, \langle \ddot{s}_{1(a,h_2)_{[1,2]}}, \ddot{s}_{1(a,h_2)_{[1,2]}}\rangle\right\} \wedge \\ \left\{\langle \dot{s}_{2(b,h_2)_{[1,2]}}, \dot{s}_{2(b,h_2)_{[3,4]}}\rangle, \langle \ddot{s}_{2(b,h_2)_{[1,2]}}, \ddot{s}_{2(b,h_2)_{[1,2]}}\rangle\right\}\end{array}\right]\right] \otimes \\
\left[\left[\begin{array}{l}\left\{\langle \dot{s}_{1(a,h_1)_{[1,2]}}, \dot{s}_{1(a,h_1)_{[2,4]}}\rangle, \langle \ddot{s}_{1(a,h_1)_{[1,2]}}, \ddot{s}_{1(a,h_1)_{[1,2]}}\rangle\right\} \vee \\ \left\{\langle \dot{s}_{2(b,h_1)_{[2,3]}}, \dot{s}_{2(b,h_1)_{[2,3]}}\rangle, \langle \ddot{s}_{2(b,h_1)_{[1,2]}}, \ddot{s}_{2(b,h_1)_{[1,2]}}\rangle\right\}\end{array}\right] \oplus \left[\begin{array}{l}\left\{\langle \dot{s}_{1(a,h_2)_{[2,4]}}, \dot{s}_{1(a,h_2)_{[1,2]}}\rangle, \langle \ddot{s}_{1(a,h_2)_{[1,2]}}, \ddot{s}_{1(a,h_2)_{[1,2]}}\rangle\right\} \vee \\ \left\{\langle \dot{s}_{2(b,h_2)_{[1,2]}}, \dot{s}_{2(b,h_2)_{[3,4]}}\rangle, \langle \ddot{s}_{2(b,h_2)_{[1,2]}}, \ddot{s}_{2(b,h_2)_{[1,2]}}\rangle\right\}\end{array}\right]\right]
\end{array}\right]
$$

$$
= \left\{\langle \dot{s}_{\left[\frac{11}{6},\frac{10}{3}\right]}, \dot{s}_{\left[1,\frac{8}{3}\right]}\rangle, \langle \ddot{s}_{\left[\frac{7}{4},3\right]}, \ddot{s}_{\left[\frac{1}{4},1\right]}\rangle\right\} \otimes \left\{\langle \dot{s}_{\left[\frac{23}{6},5\right]}, \dot{s}_{\left[\frac{1}{3},1\right]}\rangle, \langle \ddot{s}_{\left[\frac{7}{4},3\right]}, \ddot{s}_{\left[\frac{1}{4},1\right]}\rangle\right\}
$$

$$
= \left\{\langle \dot{s}_{\left[\frac{253}{216},\frac{25}{9}\right]}, \dot{s}_{\left[\frac{23}{18},\frac{29}{9}\right]}\rangle, \langle \ddot{s}_{\left[\frac{49}{64},\frac{9}{4}\right]}, \ddot{s}_{\left[\frac{31}{64},\frac{7}{4}\right]}\rangle\right\}.
$$

Now by adding these similarity results for teacher t_1 and t_2 respectively to find the overall similarity between the opinions of experts for all alternatives. Thus the modified max-min similarity measure proceed as under

$$
= \left\{\langle \dot{s}_{\left[\frac{11}{9},\frac{10}{3}\right]}, \dot{s}_{\left[\frac{41}{36},\frac{22}{9}\right]}\rangle, \langle \ddot{s}_{\left[\frac{49}{64},\frac{9}{4}\right]}, \ddot{s}_{\left[\frac{31}{64},\frac{7}{4}\right]}\rangle\right\} \oplus \left\{\langle \dot{s}_{\left[\frac{253}{216},\frac{25}{9}\right]}, \dot{s}_{\left[\frac{23}{18},\frac{29}{9}\right]}\rangle, \langle \ddot{s}_{\left[\frac{49}{64},\frac{9}{4}\right]}, \ddot{s}_{\left[\frac{31}{64},\frac{7}{4}\right]}\rangle\right\}
$$

$$
= \left\{\langle \dot{s}_{\left[\frac{25135}{11664},\frac{370}{81}\right]}, \dot{s}_{\left[\frac{943}{3888},\frac{319}{243}\right]}\rangle, \langle \ddot{s}_{\left[\frac{22687}{16384},\frac{207}{64}\right]}, \ddot{s}_{\left[\frac{961}{16384},\frac{49}{64}\right]}\rangle\right\}.
$$

Now to compare similarity result obtained by modified max-min similarity measure we use score function. According to score function we have

$$
= s_{[0.6594,0.77126]\times[0.66576,0.8086]} = s_{[0.439,0.623641]}.
$$

For comparison reasons we take average value from interval such that the value of score function is given as

$=s_{0.5313}$.

Which shows that similarity result obtained by modified max-min similarity measure is greater than the similarity results obtained by Type-I, Type-II, Type-III, Type-IV, Type-V, Max-Min similarity measures and from correlation for GLIVIFSESs result.

CHAPTER 5

DISTANCE BASED SIMILARITY MEASURES ON GENERALIZED LINGUISTIC INTERVAL VALUED INTUITIONISTIC FUZZY SOFT EXPERT SETS (GLIVIFSESs)

In this chapter we discus about distance based similarity measures for under-study structure and propose some new distance based similarity measures. Such as Modified Hamming distance, in which we firstly consider Hamming distance which is already defined for fuzzy sets, and then extend that idea by considering different parameters and different criteria's for assigning expert opinion. From the point that distance and similarity are converse of each other, we draw similarity measure on Modified Hamming distance by taking converse of linguistic terms subscripts with the restriction that as we have both member-ship and non-membership intervals so we take converse as subtraction of subscript from half of extreme value our mod on it. Further we construct practical problem example regarding the construction of well-equipped car that fulfills customer's demands. Also we apply data of Example 4.1.1 under Modified Hamming distance based similarity measure and compare its result with previous similarity measures results. Further we extend the idea of Euclidean distance into Modified Euclidean distance measure for under-study structure and then into Modified Euclidean distance-based similarity measure. And then consider the data of Example 4.1.1 and from car construction problem, to compare that similarity measure with previous/other similarity measures results.

Further we take into account the concept of Entropy measure, which is used to measure the fuzziness of fuzzy objects. And extend that idea along with linguistic approach and propose Entropy based similarity measure for under-study structure. And consider data of Example 4.1.1 under Entropy based similarity measure. Further we propose the idea of Dissimilarity measure for under-study structure and construct a practical problem example regarding the improvement of school education department. Later on we propose the concept of correlation for under-study structure, which is used to measure the similarity. And then extend the data of Example 4.1.1 under correlation, for comparison reasons. Lastly in this section we introduce the idea of linguistic fuzzy implication for measuring distance between under-study structures. And take data of Example 4.1.1 under linguistic fuzzy implication distance measure to check the validity of that distance measure.

DISTANCE BASED SIMILARITY MEASURES FOR GLIVIFSESs.

To measure the distance between any two GLIVIFSESs, or in generalized case to measure the distance between finite number of these systems/sets we use the well-known distance measures such as Hamming distance and generalized Hamming distance [16], Normalized Hamming distance, Euclidean distance [17], and Normalized Euclidean distance [18]. These distance measures have been widely used for fuzzy sets and to further extensions of fuzzy sets, now we modify/extend these measures for linguistic approach in fuzzy theory.

According to Hamming distance for fuzzy sets

$$d_H(A,B) = \sum_{i=1}^{n} |\alpha_A(x_i) - \alpha_B(x_i)|,$$

here "n" represents order of set of alternatives/universe of discourse where $\alpha_A(x_i)$ and $\alpha_B(x_i)$ represents membership functions of fuzzy sets A and B respectively.

Now in case of linguistic with 2-D IVIFSESs that distance measure takes the following form

$$d_{MH}(S_1,S_2) = \sum_{i=1}^{n} \left(\frac{\sum_{j=1}^{r} s_1(c_j,x_i)}{r} - \frac{\sum_{j=1}^{r} s_2(c_j,x_i)}{r} \right) \qquad (5.0)$$

here c_j represents a specific alternative with "r" represents order of set of criteria's.

Where S_1 and S_2 are generally defined as

$$S_1 = \left\{ \langle \dot{s}_{[\alpha,\alpha']}, \dot{s}_{[\beta,\beta']} \rangle , \langle \ddot{s}_{[\gamma,\gamma']} , \ddot{s}_{[\delta,\delta']} \rangle \right\},$$

$$S_2 = \left\{ \langle \dot{s}_{[\alpha_1,\alpha_1']}, \dot{s}_{[\beta_1,\beta_1']} \rangle , \langle \ddot{s}_{[\gamma_1,\gamma_1']} , \ddot{s}_{[\delta_1,\delta_1']} \rangle \right\},$$

with normalized case is defined by dividing the expression of modified hamming distance by order of set of alternatives

$$d_{NMH}(S_1,S_2) = \frac{\sum_{i=1}^{n} \left(\frac{\sum_{j=1}^{r} s_1(c_j,x_i)}{r} - \frac{\sum_{j=1}^{r} s_2(c_j,x_i)}{r} \right)}{n}.$$

Now we can extend that to finite number of GLIVIFSESs, such that instead of finding the distance between any two GLIVIFSESs we can find the overall distance between any finite orders of these systems, which is defined as under

$$d_{MH}(S_1,S_2,S_3,\dots,S_m)=\sum_{i=1}^{n}\left[\sum_{k=1}^{m-1}\left(\frac{\sum_{j=1}^{r}S_k(c_j,x_i,e_k)}{r}-\frac{\sum_{j=1}^{r}S_{k+1}(c_j,x_i,e_{k+1})}{r}\right)\right],$$

with

$$d_{NMH}(S_1,S_2,S_3,\dots,S_m)=\frac{\sum_{i=1}^{n}\left[\sum_{k=1}^{m-1}\left(\frac{\sum_{j=1}^{r}S_k(c_j,x_i,e_k)}{r}-\frac{\sum_{j=1}^{r}S_{k+1}(c_j,x_i,e_{k+1})}{r}\right)\right]}{n}.$$

Now by using the modified Hamming distance measure we define a distance based similarity measure keeping in view the basic idea that if distance between two sets is lowest than the similarity between these sets will be highest.

5.1. MODIFIED HAMMING DISTANCE BASED SIMILARITY MEASURE FOR GLIVIFSESs.

Suppose $C=\{c_j : j=1,2,3,\dots,r\}$ represents a set of criteria's & $X=\{x_i : i=1,2,3,\dots,n\}$ represents a set of alternatives with

$$S_1=\left\{\langle \dot{s}_{[\alpha,\alpha']},\dot{s}_{[\beta,\beta']}\rangle\ ,\langle \ddot{s}_{[\gamma,\gamma']}\ ,\ddot{s}_{[\delta,\delta']}\rangle\right\},$$
$$S_2=\left\{\langle \dot{s}_{[\alpha_1,\alpha_1']},\dot{s}_{[\beta_1,\beta_1']}\rangle\ ,\langle \ddot{s}_{[\gamma_1,\gamma_1']}\ ,\ddot{s}_{[\delta_1,\delta_1']}\rangle\right\},$$

be two any general GLIVIFSESs. Now we also take $d_{MH}(S_1,S_2)$ with arbitrary values such that

$$d_{MH}(S_1,S_2)=\left\{\langle \dot{s}_{[\rho,\rho']},\dot{s}_{[\sigma,\sigma']}\rangle\ ,\langle \ddot{s}_{[\varphi,\varphi']}\ ,\ddot{s}_{[\omega,\omega']}\rangle\right\},$$

then the similarity measure based on modified hamming distance is

$$S_{d_{MH}}(S_1,S_2)=\left\{\langle \dot{s}_{\left[\left|\frac{t}{2}-\rho'\right|,\left|\frac{t}{2}-\rho\right|\right]},\dot{s}_{\left[\left|\frac{t}{2}-\sigma'\right|,\left|\frac{t}{2}-\sigma\right|\right]}\rangle\ ,\langle \ddot{s}_{\left[\left|\frac{t'}{2}-\varphi'\right|,\left|\frac{t'}{2}-\varphi\right|\right]}\ ,\ddot{s}_{\left[\left|\frac{t'}{2}-\omega'\right|,\left|\frac{t'}{2}-\omega\right|\right]}\rangle\right\} \qquad (5.1)$$

The above expression shows that value of similarity measure is complement of value of distance measure. But in above expression we used $\frac{t}{2}$ rather than $'t'$ since we are countering both the membership and non-membership at the same time with the restriction that their sum will be less than or equal to s_t. Such that if distance is s_0 then similarity will be s_t and conversely if distance between GLIVIFSESs will be highest then similarity between them will be lowest.

To compare the above listed similarity measure we firstly consider Example 4.1.1 and then for illustration purposes we construct a practical problem and apply that similarity measure to check the reliability that similarity measure.

EXAMPLE 5.1.1.

By taking data from Example 4.1.1 we have following GLIVIFSESs

$$\mathrm{S}_1(\mathrm{h}_1,\mathrm{a})=\{\langle\dot{\mathrm{s}}_{[2,3]},\dot{\mathrm{s}}_{[1,2]}\rangle,\langle\ddot{\mathrm{s}}_{[1,2]},\ddot{\mathrm{s}}_{[1,2]}\rangle\},$$
$$\mathrm{S}_1(\mathrm{h}_2,\mathrm{a})=\{\langle\dot{\mathrm{s}}_{[3,4]},\dot{\mathrm{s}}_{[1,2]}\rangle,\langle\ddot{\mathrm{s}}_{[1,2]},\ddot{\mathrm{s}}_{[1,2]}\rangle\},$$
$$\mathrm{S}_2(\mathrm{h}_1,\mathrm{b})=\{\langle\dot{\mathrm{s}}_{[1,3]},\dot{\mathrm{s}}_{[2,3]}\rangle,\langle\ddot{\mathrm{s}}_{[1,2]},\ddot{\mathrm{s}}_{[1,2]}\rangle\},$$
$$\mathrm{S}_2(\mathrm{h}_2,\mathrm{b})=\{\langle\dot{\mathrm{s}}_{[1,2]},\dot{\mathrm{s}}_{[3,4]}\rangle,\langle\ddot{\mathrm{s}}_{[1,2]},\ddot{\mathrm{s}}_{[1,2]}\rangle\}.$$

These sets represent the evaluation values by students 'a' and 'b' for the teacher t_1. Now for the second teacher t_2 the evaluation values are as under

$$\mathrm{S}_1(\mathrm{h}_1,\mathrm{a})=\{\langle\dot{\mathrm{s}}_{[1,2]},\dot{\mathrm{s}}_{[2,4]}\rangle,\langle\ddot{\mathrm{s}}_{[1,2]},\ddot{\mathrm{s}}_{[1,2]}\rangle\},$$
$$\mathrm{S}_1(\mathrm{h}_2,\mathrm{a})=\{\langle\dot{\mathrm{s}}_{[2,4]},\dot{\mathrm{s}}_{[1,2]}\rangle,\langle\ddot{\mathrm{s}}_{[1,2]},\ddot{\mathrm{s}}_{[1,2]}\rangle\},$$
$$\mathrm{S}_2(\mathrm{h}_1,\mathrm{b})=\{\langle\dot{\mathrm{s}}_{[2,3]},\dot{\mathrm{s}}_{[2,3]}\rangle,\langle\ddot{\mathrm{s}}_{[1,2]},\ddot{\mathrm{s}}_{[1,2]}\rangle\},$$
$$\mathrm{S}_2(\mathrm{h}_2,\mathrm{b})=\{\langle\dot{\mathrm{s}}_{[1,2]},\dot{\mathrm{s}}_{[3,4]}\rangle,\langle\ddot{\mathrm{s}}_{[1,2]},\ddot{\mathrm{s}}_{[1,2]}\rangle\}.$$

Now

$$d_{MH}(S_1,S_2)=\sum_{i=1}^{2}\left(\frac{\sum_{j=1}^{2}S_1(h_j,t_i)}{2}-\frac{\sum_{j=1}^{2}S_2(h_j,t_i)}{2}\right),$$
$$=\left[\{\langle\dot{\mathrm{s}}_{[2.5359,3.5505]},\dot{\mathrm{s}}_{[1,2]}\rangle,\langle\ddot{\mathrm{s}}_{[1,2]},\ddot{\mathrm{s}}_{[1,2]}\rangle\}-\{\langle\dot{\mathrm{s}}_{[1,2.5359]},\dot{\mathrm{s}}_{[2.449,3.4641]}\rangle,\langle\ddot{\mathrm{s}}_{[1,2]},\ddot{\mathrm{s}}_{[1,2]}\rangle\}\right]+$$
$$\left[\{\langle\dot{\mathrm{s}}_{[1.5279,3.17157]},\dot{\mathrm{s}}_{[1.4142,2.8284]}\rangle,\langle\ddot{\mathrm{s}}_{[1,2]},\ddot{\mathrm{s}}_{[1,2]}\rangle\}-\{\langle\dot{\mathrm{s}}_{[1.5279,2.5359]},\dot{\mathrm{s}}_{[2.449,3.4641]}\rangle,\langle\ddot{\mathrm{s}}_{[1,2]},\ddot{\mathrm{s}}_{[1,2]}\rangle\}\right]$$
$$=\left\{\langle\dot{\mathrm{s}}_{[1.95855,2.51522]},\dot{\mathrm{s}}_{[0.40817,1.1547]}\rangle,\langle\ddot{\mathrm{s}}_{[\frac{1}{4},1]},\ddot{\mathrm{s}}_{[\frac{1}{4},1]}\rangle\right\}+\left\{\langle\dot{\mathrm{s}}_{[0.3891,1.97613]},\dot{\mathrm{s}}_{[0.57723,1.63298]}\rangle,\langle\ddot{\mathrm{s}}_{[\frac{1}{4},1]},\ddot{\mathrm{s}}_{[\frac{1}{4},1]}\rangle\right\}$$

$= \{\langle \dot{s}_{[2.220638,3.66295]}, \dot{s}_{[0.039268,0.31427]}\rangle, \langle \ddot{s}_{[0.484375,1.75]}, \ddot{s}_{[0.015625,0.25]}\rangle\}.$

The above calculated set represents the distance between GLIVIFSESs. Now to calculate distance based similarity between S_1 & S_2 we have

$$S_{d_{MH}}(S_1, S_2) = \left\{ \langle \dot{s}_{\left[\left|\frac{t}{2}-\rho'\right|,\left|\frac{t}{2}-\rho\right|\right]}, \dot{s}_{\left[\left|\frac{t}{2}-\sigma'\right|,\left|\frac{t}{2}-\sigma\right|\right]} \rangle, \langle \ddot{s}_{\left[\left|\frac{t'}{2}-\varphi'\right|,\left|\frac{t'}{2}-\varphi\right|\right]}, \ddot{s}_{\left[\left|\frac{t'}{2}-\omega'\right|,\left|\frac{t'}{2}-\omega\right|\right]} \rangle \right\},$$

by substituting required values in above equation we get

$$S_{d_{MH}}(S_1, S_2) = \{\langle \dot{s}_{[0.66295,0.779362]}, \dot{s}_{[2.68573,2.960732]}\rangle, \langle \ddot{s}_{[0.25,1.515625]}, \ddot{s}_{[1.75,1.984375]}\rangle\}.$$

Which represents distance based similarity between S_1 and S_2. Now to compare that similarity result with previous similarity measures results on the same data sets we use score function for GLIVIFSESs.

According to score function (here we use score function $s_{\left[\frac{t+\alpha+\beta}{2t}, \frac{t+\alpha'+\beta'}{2t}\right] \times \left[\frac{t'+\gamma+\delta}{2t'}, \frac{t'+\gamma'+\delta'}{2t'}\right]}$ to sustain the upper bound and lower bound property of intervals)

$$S^{\$}(S_{d_{MH}}) = s_{[0.7791,0.8116745] \times [0.75,0.9375]} = s_{[0.584325,0.760945]},$$

for comparison reasons we take the arithmetic mean of extreme values of interval such that

$$S^{\$}(S_{d_{MH}}) = s_{0.672635}.$$

By comparing that similarity result with previous similarity results we can observe that this value is greater than Type-I, Type-II, Type-III, Type-IV, Type-V, Correlation, Max-Min similarity measure.

Now to demonstrate that distance based similarity measure we construct a practical problem.

EXAMPLE 5.1.2.

Executive authority of Auto Car Company is interested to manufacture a car which includes all the possible features for different personality's peoples. To fulfill that willing they take the opinions of Engineers for wearing a cloth of reality on that and propose some models of cars, with criteria's set as general favorable qualities in a car a user can demand it. For simplicity reasons we take criteria set with three quantities $C = \{c_i : i = 1,2,3\}$ namely

- c_1 = flexibility in size
- c_2 = quality assurance
- c_3 = suitable for any type of way

and the set of experts/engineers $E = \{e_j : j = 1,2,3\}$ with set of proposed models of cars $U = \{u_k : k = 1,2\}$. Here the restriction imposed on engineers is that while categorizing the models also mentions their own expertise regarding these mechanisms.

From the above listed situation it is clear that opinions will be in the form of linguistic terms rather than numerical quantities because it is difficult to assign a numerical number to any quality of any object, with interval on subscripts rather than a fixed value due to hesitation, along with similar data for opposition. For that purpose we take as generally a term set $S = \{s_t : t = 1,2,3, \dots, 15\}$ also for linguistic approach towards expertise we take term set $S' = \{s_{t'} : t' = 1,2,3, \dots, 17\}$ such that the overall opinion will be in the form of 2-DLIVIFSES/GLIVIFSES.

Now the opinions of engineers regarding the proposal u_1 are

$$\mathrm{S}_1(c_1, e_1) = \{\langle \dot{s}_{[4,5]}, \dot{s}_{[6,7]} \rangle, \langle \ddot{s}_{[10,12]}, \ddot{s}_{[1,3]} \rangle\},$$
$$\mathrm{S}_1(c_2, e_1) = \{\langle \dot{s}_{[3,4]}, \dot{s}_{[5,7]} \rangle, \langle \ddot{s}_{[8,9]}, \ddot{s}_{[3,5]} \rangle\},$$
$$\mathrm{S}_1(c_3, e_1) = \{\langle \dot{s}_{[6,10]}, \dot{s}_{[2,3]} \rangle, \langle \ddot{s}_{[11,14]}, \ddot{s}_{[1,2]} \rangle\},$$
$$\mathrm{S}_2(c_1, e_2) = \{\langle \dot{s}_{[7,8]}, \dot{s}_{[3,4]} \rangle, \langle \ddot{s}_{[9,12]}, \ddot{s}_{[1,3]} \rangle\},$$
$$\mathrm{S}_2(c_2, e_2) = \{\langle \dot{s}_{[5,7]}, \dot{s}_{[7,8]} \rangle, \langle \ddot{s}_{[9,10]}, \ddot{s}_{[4,5]} \rangle\},$$
$$\mathrm{S}_2(c_3, e_2) = \{\langle \dot{s}_{[6,10]}, \dot{s}_{[2,3]} \rangle, \langle \ddot{s}_{[7,8]}, \ddot{s}_{[5,6]} \rangle\},$$
$$\mathrm{S}_3(c_1, e_3) = \{\langle \dot{s}_{[4,7]}, \dot{s}_{[5,6]} \rangle, \langle \ddot{s}_{[8,9]}, \ddot{s}_{[2,3]} \rangle\},$$
$$\mathrm{S}_3(c_2, e_3) = \{\langle \dot{s}_{[7,8]}, \dot{s}_{[4,6]} \rangle, \langle \ddot{s}_{[9,10]}, \ddot{s}_{[4,5]} \rangle\},$$
$$\mathrm{S}_3(c_3, e_3) = \{\langle \dot{s}_{[5,7]}, \dot{s}_{[7,8]} \rangle, \langle \ddot{s}_{[8,12]}, \ddot{s}_{[4,5]} \rangle\}.$$

Similarly, in case of proposed model u_2 the opinions of experts are

$$\mathrm{S}_1(c_1, e_1) = \{\langle \dot{\mathrm{S}}_{[2,5]}, \dot{\mathrm{S}}_{[5,7]} \rangle, \langle \ddot{\mathrm{S}}_{[10,11]}, \ddot{\mathrm{S}}_{[1,4]} \rangle\},$$
$$\mathrm{S}_1(c_2, e_1) = \{\langle \dot{\mathrm{S}}_{[6,7]}, \dot{\mathrm{S}}_{[5,7]} \rangle, \langle \ddot{\mathrm{S}}_{[8,10]}, \ddot{\mathrm{S}}_{[3,4]} \rangle\},$$

$$S_1(c_3,e_1)=\{\langle \dot{s}_{[9,10]}, \dot{s}_{[3,5]}\rangle, \langle \ddot{s}_{[11,13]}, \ddot{s}_{[2,4]}\rangle\},$$
$$S_2(c_1,e_2)=\{\langle \dot{s}_{[7,9]}, \dot{s}_{[3,5]}\rangle, \langle \ddot{s}_{[9,11]}, \ddot{s}_{[4,5]}\rangle\},$$
$$S_2(c_2,e_2)=\{\langle \dot{s}_{[6,7]}, \dot{s}_{[7,8]}\rangle, \langle \ddot{s}_{[9,10]}, \ddot{s}_{[4,6]}\rangle\},$$
$$S_2(c_3,e_2)=\{\langle \dot{s}_{[7,10]}, \dot{s}_{[2,5]}\rangle, \langle \ddot{s}_{[7,8]}, \ddot{s}_{[5,7]}\rangle\},$$
$$S_3(c_1,e_3)=\{\langle \dot{s}_{[5,7]}, \dot{s}_{[6,7]}\rangle, \langle \ddot{s}_{[8,9]}, \ddot{s}_{[2,4]}\rangle\},$$
$$S_3(c_2,e_3)=\{\langle \dot{s}_{[9,10]}, \dot{s}_{[4,5]}\rangle, \langle \ddot{s}_{[9,10]}, \ddot{s}_{[4,6]}\rangle\},$$
$$S_3(c_3,e_3)=\{\langle \dot{s}_{[6,7]}, \dot{s}_{[7,8]}\rangle, \langle \ddot{s}_{[9,12]}, \ddot{s}_{[4,5]}\rangle\}.$$

Now, in order to take into account the decisions of engineers the executive committee uses the technique that the greatest similarity result obtained in favor for specific proposal between the opinions of experts on the behalf of distance measure between them.

Now here we imply the modified hamming distance

$$d_{MH}(S_1,S_2,S_3)_{\mathrm{u}_1,\mathrm{u}_2} = \sum_{i=1}^{2}\left[\sum_{k=1}^{2}\left(\frac{\sum_{j=1}^{3} S_k(c_j,u_i,e_k)}{3} - \frac{\sum_{j=1}^{3} S_{k+1}(c_j,u_i,e_{k+1})}{3}\right)\right],$$

this represents the expression for overall distance measure by taking into account the all alternatives. Here the summation sign is used because as if for instantly the distance measure between GLIVIFSESs is '2' in case of u_1 and in case of u_2 its '4' then obviously by considering both u_1 and u_2 at the same time it will be '6'. Now in case of specific alternative u_1 we have

$$d_{MH}(S_1,S_2,S_3)_{\mathrm{u}_1} = \sum_{k=1}^{2}\left(\frac{\sum_{j=1}^{3} S_k(c_j,u_1,e_k)}{3} - \frac{\sum_{j=1}^{3} S_{k+1}(c_j,u_1,e_{k+1})}{3}\right),$$

by substituting and evaluating these values we have

$$d_{MH}(S_1,S_2,S_3)_{\mathrm{u}_1} = (\langle \dot{s}_{[3.7,6.93]}, \dot{s}_{[3.22,5.03]}\rangle, \langle \ddot{s}_{[7.77,10.83]}, \ddot{s}_{[1.25,2.98]}\rangle).$$

Now by applying similarity measure

$$S_{d_{MH}}(S_1,S_2,S_3)_{u_1} = \left\{\langle \dot{s}_{\left[\left|\frac{t}{2}-\rho'\right|,\left|\frac{t}{2}-\rho\right|\right]}, \dot{s}_{\left[\left|\frac{t}{2}-\sigma'\right|,\left|\frac{t}{2}-\sigma\right|\right]}\rangle, \langle \ddot{s}_{\left[\left|\frac{t'}{2}-\varphi'\right|,\left|\frac{t'}{2}-\varphi\right|\right]}, \ddot{s}_{\left[\left|\frac{t'}{2}-\omega'\right|,\left|\frac{t'}{2}-\omega\right|\right]}\rangle\right\}$$
$$= (\langle \dot{s}_{[0.57,3.8]}, \dot{s}_{[2.47,4.28]}\rangle, \langle \ddot{s}_{[2.33,0.73]}, \ddot{s}_{[5.52,7.25]}\rangle).$$

Now in case of second proposal u_2 we have

$$d_{MH}(S_1,S_2,S_3)_{\mathrm{u}_2} = \sum_{k=1}^{2}\left(\frac{\sum_{j=1}^{3} S_k(c_j,u_2,e_k)}{3} - \frac{\sum_{j=1}^{3} S_{k+1}(c_j,u_2,e_{k+1})}{3}\right),$$

after simplifications we get

$$d_{MH}(S_1, S_2, S_3)_{\mathrm{u}_2} = \left(\langle \dot{s}_{[5.22,7.81]}, \dot{s}_{[3.42,6.34]} \rangle, \langle \ddot{s}_{[7.9,10.28]}, \ddot{s}_{[2.23,4.04]} \rangle\right).$$

Now by applying similarity measure which is based on modified hamming distance we have

$$S_{d_{MH}}(S_1, S_2, S_3)_{u_2} = \left\{ \langle \dot{\mathrm{s}}_{\left[\left|\frac{\mathrm{t}}{2}-\rho'\right|, \left|\frac{\mathrm{t}}{2}-\rho\right|\right]}, \dot{\mathrm{s}}_{\left[\left|\frac{\mathrm{t}}{2}-\sigma'\right|, \left|\frac{\mathrm{t}}{2}-\sigma\right|\right]} \rangle , \langle \ddot{\mathrm{s}}_{\left[\left|\frac{\mathrm{t}'}{2}-\varphi'\right|, \left|\frac{\mathrm{t}'}{2}-\varphi\right|\right]} , \ddot{\mathrm{s}}_{\left[\left|\frac{\mathrm{t}'}{2}-\omega'\right|, \left|\frac{\mathrm{t}'}{2}-\omega\right|\right]} \rangle \right\}$$
$$= \left(\langle \dot{s}_{[0.31,2.28]}, \dot{s}_{[1.16,4.08]} \rangle, \langle \ddot{s}_{[1.78,0.6]}, \ddot{s}_{[4.46,6.27]} \rangle\right).$$

From above calculations it's clear that the similarity between the opinions of engineers is greater in case of u_1 and also the similarity for expertise is also greater than u_2 so, the proposed model u_1 will be further proceeding.

5.2. MODIFIED EUCLIDEAN DISTANCE BASED SIMILARITY MEASURE FOR GLIVIFSESs.

According to Euclidean distance for fuzzy sets

$$d_E(A, B) = \sqrt{\sum_{i=1}^{n} |\alpha_A(x_i) - \alpha_B(x_i)|^2},$$

here "n" represents order of set of alternatives/universe of discourse where $\alpha_A(x_i)$ and $\alpha_B(x_i)$ represents membership functions of fuzzy sets A and B respectively.

Now in case of linguistic with 2-D IVIFSESs that distance measure takes the following form

$$d_{ME}(S_1, S_2) = \sqrt{\sum_{i=1}^{n} \left(\frac{\sum_{j=1}^{r} S_1(c_j, x_i)}{r} - \frac{\sum_{j=1}^{r} S_2(c_j, x_i)}{r} \right)^2} \qquad (5.2)$$

here c_j represents a specific alternative with "r" represents order of set of criteria's.

Where S_1 and S_2 are generally defined as

$$\mathrm{S}_1 = \left\{ \langle \dot{\mathrm{s}}_{[\alpha,\alpha']}, \dot{\mathrm{s}}_{[\beta,\beta']} \rangle , \langle \ddot{\mathrm{s}}_{[\gamma,\gamma']} , \ddot{\mathrm{s}}_{[\delta,\delta']} \rangle \right\},$$

$$S_2=\left\{\langle \dot{s}_{[\alpha_1,\alpha_1']}, \dot{s}_{[\beta_1,\beta_1']}\rangle\ ,\langle \ddot{s}_{[\gamma_1,\gamma_1']}\ , \ddot{s}_{[\delta_1,\delta_1']}\rangle\right\},$$

with normalized case is defined by dividing the expression of Modified Euclidean distance by order of set of alternatives

$$d_{NME}(S_1,S_2)=\frac{\sqrt{\sum_{i=1}^{n}\left(\frac{\sum_{j=1}^{r}S_1(c_j,x_i)}{r}-\frac{\sum_{j=1}^{r}S_2(c_j,x_i)}{r}\right)^2}}{n}.$$

Now we can extend that to finite number of GLIVIFSESs, such that instead of finding the distance between any two GLIVIFSESs we can find the overall distance between any finite orders of these systems

$$d_{ME}(S_1,S_2,S_3,\dots,S_m)=\sqrt{\sum_{i=1}^{n}\left[\sum_{k=1}^{m-1}\left(\frac{\sum_{j=1}^{r}S_k(c_j,x_i,e_k)}{r}-\frac{\sum_{j=1}^{r}S_{k+1}(c_j,x_i,e_{k+1})}{r}\right)^2\right]},$$

with

$$d_{NME}(S_1,S_2,S_3,\dots,S_m)=\frac{\sqrt{\sum_{i=1}^{n}\left[\sum_{k=1}^{m-1}\left(\frac{\sum_{j=1}^{r}S_k(c_j,x_i,e_k)}{r}-\frac{\sum_{j=1}^{r}S_{k+1}(c_j,x_i,e_{k+1})}{r}\right)^2\right]}}{n}.$$

Now by using the Modified Euclidean distance measure we define a distance based similarity measure.

Suppose $C=\{c_j: j=1,2,3,\dots,r\}$ represents a set of criteria's & $X=\{x_i: i=1,2,3,\dots,n\}$ represents a set of alternatives with

$$S_1=\left\{\langle \dot{s}_{[\alpha,\alpha']}, \dot{s}_{[\beta,\beta']}\rangle\ ,\langle \ddot{s}_{[\gamma,\gamma']}\ , \ddot{s}_{[\delta,\delta']}\rangle\right\},$$

$$S_2=\left\{\langle \dot{s}_{[\alpha_1,\alpha_1']}, \dot{s}_{[\beta_1,\beta_1']}\rangle\ ,\langle \ddot{s}_{[\gamma_1,\gamma_1']}\ , \ddot{s}_{[\delta_1,\delta_1']}\rangle\right\},$$

be two any general GLIVIFSESs. Now we also take $d_{ME}(S_1,S_2)$ with arbitrary values such that

$$d_{ME}(S_1,S_2)=\left\{\langle \dot{s}_{[a,a']}, \dot{s}_{[z,z']}\rangle\ ,\langle \ddot{s}_{[w,w']}\ , \ddot{s}_{[q,q']}\rangle\right\}.$$

Then the similarity measure based on Modified Euclidean distance is

$$S_{d_{ME}}(S_1,S_2)=\left\{\langle \dot{s}_{\left[\left|\frac{t}{2}-a'\right|,\left|\frac{t}{2}-a\right|\right]}, \dot{s}_{\left[\left|\frac{t}{2}-z'\right|,\left|\frac{t}{2}-z\right|\right]}\rangle\ ,\langle \ddot{s}_{\left[\left|\frac{t'}{2}-w'\right|,\left|\frac{t'}{2}-w\right|\right]}\ , \ddot{s}_{\left[\left|\frac{t'}{2}-q'\right|,\left|\frac{t'}{2}-q\right|\right]}\rangle\right\} \qquad (5.2.1)$$

now to compare that similarity measure with previous one's we take into account Example 4.1.1.

EXAMPLE 5.2.1.

By taking data from Example 4.1.1 we have following GLIVIFSESs

$$\begin{aligned}
S_1(h_1, a) &= \{\langle \dot{s}_{[2,3]}, \dot{s}_{[1,2]}\rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\rangle\},\\
S_1(h_2, a) &= \{\langle \dot{s}_{[3,4]}, \dot{s}_{[1,2]}\rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\rangle\},\\
S_2(h_1, b) &= \{\langle \dot{s}_{[1,3]}, \dot{s}_{[2,3]}\rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\rangle\},\\
S_2(h_2, b) &= \{\langle \dot{s}_{[1,2]}, \dot{s}_{[3,4]}\rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\rangle\}.
\end{aligned}$$

These sets represent the evaluation values by students 'a' and 'b' for the teacher t_1. Now for the second teacher t_2 the evaluation values are as under

$$\begin{aligned}
S_1(h_1, a) &= \{\langle \dot{s}_{[1,2]}, \dot{s}_{[2,4]}\rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\rangle\},\\
S_1(h_2, a) &= \{\langle \dot{s}_{[2,4]}, \dot{s}_{[1,2]}\rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\rangle\},\\
S_2(h_1, b) &= \{\langle \dot{s}_{[2,3]}, \dot{s}_{[2,3]}\rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\rangle\},\\
S_2(h_2, b) &= \{\langle \dot{s}_{[1,2]}, \dot{s}_{[3,4]}\rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\rangle\}.
\end{aligned}$$

Now

$$d_{ME}(S_1, S_2) = \sqrt{\sum_{i=1}^{n}\left(\frac{\sum_{j=1}^{r} S_1(c_j, x_i)}{r} - \frac{\sum_{j=1}^{r} S_2(c_j, x_i)}{r}\right)^2},$$

By substituting and simplifying these algebraic expressions we get

$$= \sqrt{\begin{matrix}\left[\{\langle \dot{s}_{[2.5359,3.5505]}, \dot{s}_{[1,2]}\rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\rangle\} - \{\langle \dot{s}_{[1,2.5359]}, \dot{s}_{[2.449,3.4641]}\rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\rangle\}\right]^2 + \\ \left[\{\langle \dot{s}_{[1.5279,3.17157]}, \dot{s}_{[1.4142,2.8284]}\rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\rangle\} - \{\langle \dot{s}_{[1.5279,2.5359]}, \dot{s}_{[2.449,3.4641]}\rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\rangle\}\right]^2\end{matrix}} =$$

$$\sqrt{\left(\langle \dot{s}_{[1.95855,2.51522]}, \dot{s}_{[0.40817,1.1547]}\rangle, \langle \ddot{s}_{[\frac{1}{4},1]}, \ddot{s}_{[\frac{1}{4},1]}\rangle\right)^2 + \left(\langle \dot{s}_{[0.3891,1.97613]}, \dot{s}_{[0.57723,1.63298]}\rangle, \langle \ddot{s}_{[\frac{1}{4},1]}, \ddot{s}_{[\frac{1}{4},1]}\rangle\right)^2}$$

$$= \sqrt{\left(\langle s_{[0.64,1.054]}, s_{[0.79,2.087]}\rangle, \langle s_{[0.016,0.25]}, s_{[0.48,1.75]}\rangle\right) + \left(\langle s_{[0.025,0.6508]}, s_{[1.099,2.82]}\rangle, \langle s_{[0.016,0.25]}, s_{[0.48,1.75]}\rangle\right)}$$

$$= \sqrt{\left(\langle \dot{s}_{[0.6623,1.5905]}, \dot{s}_{[0.1447,0.98089]}\rangle, \langle \ddot{s}_{[0.0319,0.484]}, \ddot{s}_{[0.0576,0.7656]}\rangle\right)}$$

$$= \left(\langle \dot{s}_{[1.9934,3.089]}, \dot{s}_{[0.0728,0.5123]}\rangle, \langle \ddot{s}_{[0.3572,1.3914]}, \ddot{s}_{[0.0289,0.4031]}\rangle\right).$$

The above calculated set represents the distance between GLIVIFSESs. Now to calculate distance based similarity between S_1 & S_2 we have

$$S_{d_{ME}}(S_1, S_2) = \left\{ \langle \dot{s}_{\left[\left|\frac{t}{2}-a'\right|, \left|\frac{t}{2}-a\right|\right]}, \dot{s}_{\left[\left|\frac{t}{2}-z'\right|, \left|\frac{t}{2}-z\right|\right]} \rangle, \langle \ddot{s}_{\left[\left|\frac{t'}{2}-w'\right|, \left|\frac{t'}{2}-w\right|\right]}, \ddot{s}_{\left[\left|\frac{t'}{2}-q'\right|, \left|\frac{t'}{2}-q\right|\right]} \rangle \right\},$$

by substituting these values we get

$$S_{d_{ME}}(S_1, S_2) = \{ \langle \dot{s}_{[0.089,1.0066]}, \dot{s}_{[2.4877,2.9272]} \rangle, \langle \ddot{s}_{[0.6086,1.6428]}, \ddot{s}_{[1.5969,1.9711]} \rangle \}.$$

Which represents distance based similarity between S_1 and S_2. Now to compare that similarity result with previous similarity measures results on the same data sets we use score function for GLIVIFSESs.

According to score function (here we use score function $s_{\left[\frac{t+\alpha+\beta}{2t}, \frac{t+\alpha'+\beta'}{2t}\right] \times \left[\frac{t'+\gamma+\delta}{2t'}, \frac{t'+\gamma'+\delta'}{2t'}\right]}$)

$$S^{\$}(S_{d_{ME}}) = s_{[0.71,0.83] \times [0.78,0.95]} = s_{[0.5538,0.7885]}.$$

For comparison reasons we take the arithmetic mean of extreme values of interval such that

$$S^{\$}(S_{d_{ME}}) = s_{0.67115}.$$

By comparing that similarity result with previous similarity results we can observe that this value is greater than Type-I, Type-II, Type-III, Type-IV, Type-V, Correlation, Max-Min similarity measure but a smallest differ in order of similarity result than the Hamming distance similarity measure result.

Now we take data from Example 5.1.2 to more briefly understand the difference between these similarity measures.

EXAMPLE 5.2.2.

By taking problem statement from Example 5.1.2 we have,
the opinions of engineers regarding the proposal u_1 are

$$S_1(c_1,e_1)=\{\langle \dot{s}_{[4,5]},\dot{s}_{[6,7]}\rangle,\langle \ddot{s}_{[10,12]},\ddot{s}_{[1,3]}\rangle\},$$
$$S_1(c_2,e_1)=\{\langle \dot{s}_{[3,4]},\dot{s}_{[5,7]}\rangle,\langle \ddot{s}_{[8,9]},\ddot{s}_{[3,5]}\rangle\},$$
$$S_1(c_3,e_1)=\{\langle \dot{s}_{[6,10]},\dot{s}_{[2,3]}\rangle,\langle \ddot{s}_{[11,14]},\ddot{s}_{[1,2]}\rangle\},$$
$$S_2(c_1,e_2)=\{\langle \dot{s}_{[7,8]},\dot{s}_{[3,4]}\rangle,\langle \ddot{s}_{[9,12]},\ddot{s}_{[1,3]}\rangle\},$$
$$S_2(c_2,e_2)=\{\langle \dot{s}_{[5,7]},\dot{s}_{[7,8]}\rangle,\langle \ddot{s}_{[9,10]},\ddot{s}_{[4,5]}\rangle\},$$
$$S_2(c_3,e_2)=\{\langle \dot{s}_{[6,10]},\dot{s}_{[2,3]}\rangle,\langle \ddot{s}_{[7,8]},\ddot{s}_{[5,6]}\rangle\},$$
$$S_3(c_1,e_3)=\{\langle \dot{s}_{[4,7]},\dot{s}_{[5,6]}\rangle,\langle \ddot{s}_{[8,9]},\ddot{s}_{[2,3]}\rangle\},$$
$$S_3(c_2,e_3)=\{\langle \dot{s}_{[7,8]},\dot{s}_{[4,6]}\rangle,\langle \ddot{s}_{[9,10]},\ddot{s}_{[4,5]}\rangle\},$$
$$S_3(c_3,e_3)=\{\langle \dot{s}_{[5,7]},\dot{s}_{[7,8]}\rangle,\langle \ddot{s}_{[8,12]},\ddot{s}_{[4,5]}\rangle\}.$$

Similarly, in case of proposed model u_2 the opinions of experts are

$$S_1(c_1,e_1)=\{\langle \dot{S}_{[2,5]},\dot{S}_{[5,7]}\rangle,\langle \ddot{S}_{[10,11]},\ddot{S}_{[1,4]}\rangle\},$$
$$S_1(c_2,e_1)=\{\langle \dot{S}_{[6,7]},\dot{S}_{[5,7]}\rangle,\langle \ddot{S}_{[8,10]},\ddot{S}_{[3,4]}\rangle\},$$
$$S_1(c_3,e_1)=\{\langle \dot{s}_{[9,10]},\dot{s}_{[3,5]}\rangle,\langle \ddot{s}_{[11,13]},\ddot{s}_{[2,4]}\rangle\},$$
$$S_2(c_1,e_2)=\{\langle \dot{s}_{[7,9]},\dot{s}_{[3,5]}\rangle,\langle \ddot{s}_{[9,11]},\ddot{s}_{[4,5]}\rangle\},$$
$$S_2(c_2,e_2)=\{\langle \dot{s}_{[6,7]},\dot{s}_{[7,8]}\rangle,\langle \ddot{s}_{[9,10]},\ddot{s}_{[4,6]}\rangle\},$$
$$S_2(c_3,e_2)=\{\langle \dot{s}_{[7,10]},\dot{s}_{[2,5]}\rangle,\langle \ddot{s}_{[7,8]},\ddot{s}_{[5,7]}\rangle\},$$
$$S_3(c_1,e_3)=\{\langle \dot{s}_{[5,7]},\dot{s}_{[6,7]}\rangle,\langle \ddot{s}_{[8,9]},\ddot{s}_{[2,4]}\rangle\},$$
$$S_3(c_2,e_3)=\{\langle \dot{s}_{[9,10]},\dot{s}_{[4,5]}\rangle,\langle \ddot{s}_{[9,10]},\ddot{s}_{[4,6]}\rangle\},$$
$$S_3(c_3,e_3)=\{\langle \dot{s}_{[6,7]},\dot{s}_{[7,8]}\rangle,\langle \ddot{s}_{[9,12]},\ddot{s}_{[4,5]}\rangle\}.$$

Now here we imply the Modified Euclidean distance

$$d_{ME}(S_1,S_2,S_3)=\sqrt{\sum_{i=1}^{2}\left[\sum_{k=1}^{2}\left(\frac{\sum_{j=1}^{3}S_k(c_j,x_i,e_k)}{r}-\frac{\sum_{j=1}^{3}S_{k+1}(c_j,x_i,e_{k+1})}{r}\right)^2\right]},$$

in case of specific alternative u_1 we have

$$d_{ME}(S_1,S_2,S_3)_{u_1}=\sqrt{\sum_{k=1}^{2}\left(\frac{\sum_{j=1}^{3}S_k(c_j,x_i,e_k)}{r}-\frac{\sum_{j=1}^{3}S_{k+1}(c_j,x_i,e_{k+1})}{r}\right)^2},$$

after simplifications on substituting theses values we get

$$d_{ME}(S_1,S_2,S_3)_{u_1}=\{\langle \dot{s}_{[2.82,5.56]},\dot{s}_{[4.46,6.47]}\rangle,\langle \ddot{s}_{[6.24,9.18]},\ddot{s}_{[1.98,4.25]}\rangle\}.$$

Now by implying modified Euclidean distance based similarity measure

$$S_{d_{ME}}(S_1, S_2, S_3)_{u_1} = \left\{ \left\langle \dot{s}_{\left[\left|\frac{t}{2}-a'\right|,\left|\frac{t}{2}-a\right|\right]}, \dot{s}_{\left[\left|\frac{t}{2}-z'\right|,\left|\frac{t}{2}-z\right|\right]} \right\rangle, \left\langle \ddot{s}_{\left[\left|\frac{t'}{2}-w'\right|,\left|\frac{t'}{2}-w\right|\right]}, \ddot{s}_{\left[\left|\frac{t'}{2}-q'\right|,\left|\frac{t'}{2}-q\right|\right]} \right\rangle \right\},$$

by substituting these values we get

$$S_{d_{ME}}(S_1, S_2, S_3)_{u_1} = \{\langle \dot{s}_{[1.94,4.68]}, \dot{s}_{[1.03,3.04]} \rangle, \langle \ddot{s}_{[0.68,2.26]}, \ddot{s}_{[4.25,6.52]} \rangle\}.$$

Now in case of specific alternative u_2 we have

$$d_{ME}(S_1, S_2, S_3)_{u_2} = \sqrt{\sum_{k=1}^{2} \left(\frac{\sum_{j=1}^{3} S_k(c_j, x_i, e_k)}{r} - \frac{\sum_{j=1}^{3} S_{k+1}(c_j, x_i, e_{k+1})}{r} \right)^2},$$

after simplifications with substituting theses values we get

$$d_{ME}(S_1, S_2, S_3)_{u_2} = \{\langle \dot{s}_{[4.04,6.38]}, \dot{s}_{[4.69,7.81]} \rangle, \langle \ddot{s}_{[6.33,8.61]}, \ddot{s}_{[3.31,5.51]} \rangle\}.$$

Now by implying modified Euclidean distance based similarity measure

$$S_{d_{ME}}(S_1, S_2, S_3)_{u_2} = \left\{ \left\langle \dot{s}_{\left[\left|\frac{t}{2}-a'\right|,\left|\frac{t}{2}-a\right|\right]}, \dot{s}_{\left[\left|\frac{t}{2}-z'\right|,\left|\frac{t}{2}-z\right|\right]} \right\rangle, \left\langle \ddot{s}_{\left[\left|\frac{t'}{2}-w'\right|,\left|\frac{t'}{2}-w\right|\right]}, \ddot{s}_{\left[\left|\frac{t'}{2}-q'\right|,\left|\frac{t'}{2}-q\right|\right]} \right\rangle \right\},$$

by substituting these values we get

$$S_{d_{ME}}(S_1, S_2, S_3)_{u_2} = \{\langle \dot{s}_{[1.12,3.46]}, \dot{s}_{[0.31,2.81]} \rangle, \langle \ddot{s}_{[0.11,2.17]}, \ddot{s}_{[2.99,5.19]} \rangle\}.$$

From above expressions of similarity measures in opinions of engineers it is clear that similarity measure in favor in case of u_2 is greater than u_1, thus u_1 will be considered as appropriate proposal. While by comparing results of similarity measures in u_1 case with Modified Hamming distance based similarity measure results we can observe that similarity measure in favor is higher in Modified Euclidean distance based similarity measure.

5.3. ENTROPY SIMILARITY MEASURE FOR GLIVIFSESs.

The entropy similarity measure for GLIVIFSESs is a generalization of entropy similarity measure for interval-valued intuitionistic fuzzy sets (IVIFSs) [4] which we modify according to over structure of GLIVIFSESs.

Definition. Let suppose two general GLIVIFSESs

$$S_1=\left\{\langle \dot{s}_{[\alpha,\alpha']}, \dot{s}_{[\beta,\beta']}\rangle\ ,\langle \ddot{s}_{[\gamma,\gamma']}\ ,\ddot{s}_{[\delta,\delta']}\rangle\right\},$$

and

$$S_2=\left\{\langle \dot{s}_{[\alpha_1,\alpha_1']}, \dot{s}_{[\beta_1,\beta_1']}\rangle\ ,\langle \ddot{s}_{[\gamma_1,\gamma_1']}\ ,\ddot{s}_{[\delta_1,\delta_1']}\rangle\right\}.$$

According to entropy measure for interval-valued intuitionistic fuzzy set $W = \{\langle u_i, [\alpha,\alpha'], [\beta,\beta']\rangle : i \in U\}$

$$E(W) = \frac{1}{n}\sum_{i=1}^{n}\frac{min\{\alpha,\beta\} + min\{\alpha',\beta'\} + \pi_i + \pi_i'}{max\{\alpha,\beta\} + max\{\alpha',\beta'\} + \pi_i + \pi_i'},$$

here π_i represents the hesitancy of maximum values for membership and non-membership and π_i' represents the hesitancy of lowest membership and non-membership values.

Now in case of generalized linguistic intuitionistic fuzzy soft expert sets, let $U = \{u_e : e = 1, \ldots, n\}$ represents a set of alternatives and $E = \{d_c : c = 1, \ldots, q\}$ represents a set of experts, also let $C = \{c_i : i = 1, \ldots, r\}$ which represents a set of criteria's for decision making, in mathematically the entropy measure for S_1 is defined as

$$E(S_1) = \frac{1}{n}\sum_{e=1}^{n}\left[\begin{gathered}
\frac{\left[\begin{gathered}\min\{\dot{S}_{\alpha}(d_c,c_i),\dot{S}_{\beta}(d_c,c_i)\}\oplus\min\left\{\dot{S}_{\alpha'}(d_c,c_i),\dot{S}_{\beta'}(d_c,c_i)\right\}\oplus\min\{\ddot{S}_{\gamma}(d_c,c_i),\ddot{S}_{\delta}(d_c,c_i)\}\oplus\dot{\pi}_e(d_c,c_i)\oplus\dot{\pi}'_e(d_c,c_i)\\ \oplus\min\left\{\ddot{S}_{\gamma'}(d_c,c_i),\ddot{S}_{\delta'}(d_c,c_i)\right\}\oplus\ddot{\pi}_e(d_c,c_i)\oplus\ddot{\pi}'_e(d_c,c_i)\end{gathered}\right]}{\left[\begin{gathered}\max\{\dot{S}_{\alpha}(d_c,c_i),\dot{S}_{\beta}(d_c,c_i)\}\oplus\max\left\{\dot{S}_{\alpha'}(d_c,c_i),\dot{S}_{\beta'}(d_c,c_i)\right\}\oplus\max\{\ddot{S}_{\gamma}(d_c,c_i),\ddot{S}_{\delta}(d_c,c_i)\}\oplus\dot{\pi}_e(d_c,c_i)\oplus\dot{\pi}'_e(d_c,c_i)\\ \oplus\max\left\{\ddot{S}_{\gamma'}(d_c,c_i),\ddot{S}_{\delta'}(d_c,c_i)\right\}\oplus\ddot{\pi}_e(d_c,c_i)\oplus\ddot{\pi}'_e(d_c,c_i)\end{gathered}\right]}\\
\oplus\frac{\left[\begin{gathered}\min\{\dot{S}_{\alpha}(d_c,c_{i+1}),\dot{S}_{\beta}(d_c,c_{i+1})\}\oplus\min\left\{\dot{S}_{\alpha'}(d_c,c_{i+1}),\dot{S}_{\beta'}(d_c,c_{i+1})\right\}\oplus\min\{\ddot{S}_{\gamma}(d_c,c_{i+1}),\ddot{S}_{\delta}(d_c,c_{i+1})\}\\ \oplus\dot{\pi}_e(d_c,c_{i+1})\oplus\dot{\pi}'_e(d_c,c_{i+1})\oplus\min\left\{\ddot{S}_{\gamma'}(d_c,c_{i+1}),\ddot{S}_{\delta'}(d_c,c_{i+1})\right\}\oplus\ddot{\pi}_e(d_c,c_{i+1})\oplus\ddot{\pi}'_e(d_c,c_{i+1})\end{gathered}\right]}{\left[\begin{gathered}\max\{\dot{S}_{\alpha}(d_c,c_{i+1}),\dot{S}_{\beta}(d_c,c_{i+1})\}\oplus\max\left\{\dot{S}_{\alpha'}(d_c,c_{i+1}),\dot{S}_{\beta'}(d_c,c_{i+1})\right\}\oplus\max\{\ddot{S}_{\gamma}(d_c,c_{i+1}),\ddot{S}_{\delta}(d_c,c_{i+1})\}\\ \oplus\dot{\pi}_e(d_c,c_{i+1})\oplus\dot{\pi}'_e(d_c,c_{i+1})\oplus\max\left\{\ddot{S}_{\gamma'}(d_c,c_{i+1}),\ddot{S}_{\delta'}(d_c,c_{i+1})\right\}\oplus\ddot{\pi}_e(d_c,c_{i+1})\oplus\ddot{\pi}'_e(d_c,c_{i+1})\end{gathered}\right]}\\
\oplus\ldots\oplus\\
\frac{\left[\begin{gathered}\min\{\dot{S}_{\alpha}(d_c,c_r),\dot{S}_{\beta}(d_c,c_r)\}\oplus\min\left\{\dot{S}_{\alpha'}(d_c,c_r),\dot{S}_{\beta'}(d_c,c_r)\right\}\oplus\min\{\ddot{S}_{\gamma}(d_c,c_r),\ddot{S}_{\delta}(d_c,c_r)\}\oplus\dot{\pi}_e(d_c,c_r)\oplus\dot{\pi}'_e(d_c,c_r)\\ \oplus\min\left\{\ddot{S}_{\gamma'}(d_c,c_r),\ddot{S}_{\delta'}(d_c,c_r)\right\}\oplus\ddot{\pi}_e(d_c,c_r)\oplus\ddot{\pi}'_e(d_c,c_r)\end{gathered}\right]}{\left[\begin{gathered}\max\{\dot{S}_{\alpha}(d_c,c_r),\dot{S}_{\beta}(d_c,c_r)\}\oplus\max\left\{\dot{S}_{\alpha'}(d_c,c_r),\dot{S}_{\beta'}(d_c,c_r)\right\}\oplus\max\{\ddot{S}_{\gamma}(d_c,c_r),\ddot{S}_{\delta}(d_c,c_r)\}\oplus\dot{\pi}_e(d_c,c_r)\oplus\dot{\pi}'_e(d_c,c_r)\\ \oplus\max\left\{\ddot{S}_{\gamma'}(d_c,c_r),\ddot{S}_{\delta'}(d_c,c_r)\right\}\oplus\ddot{\pi}_e(d_c,c_r)\oplus\ddot{\pi}'_e(d_c,c_r)\end{gathered}\right]}
\end{gathered}\right] \quad (5.3)$$

Similarly in case of second GLIVIFSES S_2 the entropy measure is defined mathematically as

$$
E(\mathrm{S}_2)=\frac{1}{\mathrm{n}}\sum_{\mathrm{e}=1}^{\mathrm{n}}\left[\begin{array}{c}
\dfrac{\left[\begin{array}{c}\min\left\{\dot{\mathrm{S}}_{\alpha_1}(d_{c+1},c_i),\dot{\mathrm{S}}_{\beta_1}(d_{c+1},c_i)\right\}\oplus\min\left\{\dot{\mathrm{S}}_{\alpha_1{}'}(d_{c+1},c_i),\dot{\mathrm{S}}_{\beta_1{}'}(d_{c+1},c_i)\right\}\oplus\min\left\{\ddot{\mathrm{S}}_{\gamma_1}(d_{c+1},c_i),\ddot{\mathrm{S}}_{\delta_1}(d_{c+1},c_i)\right\}\\ \oplus\dot{\pi}_e(d_{c+1},c_i)\oplus\dot{\pi}'_e(d_{c+1},c_i)\oplus\min\left\{\ddot{\mathrm{S}}_{\gamma_1'}(d_{c+1},c_i),\ddot{\mathrm{S}}_{\delta_1'}(d_{c+1},c_i)\right\}\oplus\ddot{\pi}_e(d_{c+1},c_i)\oplus\ddot{\pi}'_e(d_{c+1},c_i)\end{array}\right]}{\left[\begin{array}{c}\max\left\{\dot{\mathrm{S}}_{\alpha_1}(d_{c+1},c_i),\dot{\mathrm{S}}_{\beta_1}(d_{c+1},c_i)\right\}\oplus\max\left\{\dot{\mathrm{S}}_{\alpha_1{}'}(d_{c+1},c_i),\dot{\mathrm{S}}_{\beta_1{}'}(d_{c+1},c_i)\right\}\oplus\max\left\{\ddot{\mathrm{S}}_{\gamma_1}(d_{c+1},c_i),\ddot{\mathrm{S}}_{\delta_1}(d_{c+1},c_i)\right\}\\ \oplus\dot{\pi}_e(d_{c+1},c_i)\oplus\dot{\pi}'_e(d_{c+1},c_i)\oplus\max\left\{\ddot{\mathrm{S}}_{\gamma_1'}(d_{c+1},c_i),\ddot{\mathrm{S}}_{\delta_1'}(d_{c+1},c_i)\right\}\oplus\ddot{\pi}_e(d_{c+1},c_i)\oplus\ddot{\pi}'_e(d_{c+1},c_i)\end{array}\right]}\\
\oplus\dfrac{\left[\begin{array}{c}\min\left\{\dot{\mathrm{S}}_{\alpha_1}(d_{c+1},c_{i+1}),\dot{\mathrm{S}}_{\beta_1}(d_{c+1},c_{i+1})\right\}\oplus\min\left\{\dot{\mathrm{S}}_{\alpha_1'}(d_{c+1},c_{i+1}),\dot{\mathrm{S}}_{\beta_1'}(d_{c+1},c_{i+1})\right\}\oplus\min\left\{\ddot{\mathrm{S}}_{\gamma_1}(d_{c+1},c_{i+1}),\ddot{\mathrm{S}}_{\delta_1}(d_{c+1},c_{i+1})\right\}\\ \oplus\dot{\pi}_e(d_{c+1},c_{i+1})\oplus\dot{\pi}'_e(d_{c+1},c_{i+1})\oplus\min\left\{\ddot{\mathrm{S}}_{\gamma_1'}(d_{c+1},c_{i+1}),\ddot{\mathrm{S}}_{\delta_1'}(d_{c+1},c_{i+1})\right\}\oplus\ddot{\pi}_e(d_{c+1},c_{i+1})\oplus\ddot{\pi}'_e(d_{c+1},c_{i+1})\end{array}\right]}{\left[\begin{array}{c}\max\left\{\dot{\mathrm{S}}_{\alpha_1}(d_{c+1},c_{i+1}),\dot{\mathrm{S}}_{\beta_1}(d_{c+1},c_{i+1})\right\}\oplus\max\left\{\dot{\mathrm{S}}_{\alpha_1'}(d_{c+1},c_{i+1}),\dot{\mathrm{S}}_{\beta_1'}(d_{c+1},c_{i+1})\right\}\oplus\max\left\{\ddot{\mathrm{S}}_{\gamma_1}(d_{c+1},c_{i+1}),\ddot{\mathrm{S}}_{\delta_1}(d_{c+1},c_{i+1})\right\}\\ \oplus\dot{\pi}_e(d_{c+1},c_{i+1})\oplus\dot{\pi}'_e(d_{c+1},c_{i+1})\oplus\max\left\{\ddot{\mathrm{S}}_{\gamma_1'}(d_{c+1},c_{i+1}),\ddot{\mathrm{S}}_{\delta_1'}(d_{c+1},c_{i+1})\right\}\oplus\ddot{\pi}_e(d_{c+1},c_{i+1})\oplus\ddot{\pi}'_e(d_{c+1},c_{i+1})\end{array}\right]}\\
\oplus\ .\ .\ .\ \oplus\\
\dfrac{\left[\begin{array}{c}\min\left\{\dot{\mathrm{S}}_{\alpha_1}(d_{c+1},c_r),\dot{\mathrm{S}}_{\beta_1}(d_{c+1},c_r)\right\}\oplus\min\left\{\dot{\mathrm{S}}_{\alpha_1{}'}(d_{c+1},c_r),\dot{\mathrm{S}}_{\beta_1{}'}(d_{c+1},c_r)\right\}\oplus\min\left\{\ddot{\mathrm{S}}_{\gamma_1}(d_{c+1},c_r),\ddot{\mathrm{S}}_{\delta_1}(d_{c+1},c_r)\right\}\\ \oplus\dot{\pi}_e(d_{c+1},c_r)\oplus\dot{\pi}'_e(d_{c+1},c_r)\oplus\min\left\{\ddot{\mathrm{S}}_{\gamma_1'}(d_{c+1},c_r),\ddot{\mathrm{S}}_{\delta_1'}(d_{c+1},c_r)\right\}\oplus\ddot{\pi}_e(d_{c+1},c_r)\oplus\ddot{\pi}'_e(d_{c+1},c_r)\end{array}\right]}{\left[\begin{array}{c}\max\left\{\dot{\mathrm{S}}_{\alpha_1}(d_{c+1},c_r),\dot{\mathrm{S}}_{\beta_1}(d_{c+1},c_r)\right\}\oplus\max\left\{\dot{\mathrm{S}}_{\alpha_1{}'}(d_{c+1},c_r),\dot{\mathrm{S}}_{\beta_1{}'}(d_{c+1},c_r)\right\}\oplus\max\left\{\ddot{\mathrm{S}}_{\gamma_1}(d_{c+1},c_r),\ddot{\mathrm{S}}_{\delta_1}(d_{c+1},c_r)\right\}\\ \oplus\dot{\pi}_e(d_{c+1},c_r)\oplus\dot{\pi}'_e(d_{c+1},c_r)\oplus\max\left\{\ddot{\mathrm{S}}_{\gamma_1'}(d_{c+1},c_r),\ddot{\mathrm{S}}_{\delta_1'}(d_{c+1},c_r)\right\}\oplus\ddot{\pi}_e(d_{c+1},c_r)\oplus\ddot{\pi}'_e(d_{c+1},c_r)\end{array}\right]}
\end{array}\right]
$$

similarly so on.

AXIOMS FOR ENTROPY.

According to axioms for function defined on interval-valued intuitionistic fuzzy sets [5] to become entropy

a. E(W)=0 if and only if 'W' is a crisp set
b. E(W)=1 if and only if $[\alpha,\alpha']=[\beta,\beta']$ for each $x\in X$
c. E(W)= E(W^C)
d. If $W_1\subseteq W_2$ With $\alpha_{W_2}\leq\beta_{W_2}$ and $\alpha'_{W_2}\leq\beta'_{W_2}$ for all alternatives then $E(W_1)\leq E(W_2)$.

Now in case of generalized linguistic intuitionistic fuzzy soft expert sets

a. $E(\mathrm{S}_1)=\mathrm{s}_0$ if and only if S_1 is a crisp set
b. $E(\mathrm{S}_1)=\left[\mathrm{s}_{\frac{1}{\mathrm{t}}}\oplus\mathrm{s}_{\frac{1}{\mathrm{t}}}\oplus\ .\ .\ .\ \oplus\mathrm{s}_{\frac{1}{\mathrm{t}}}\right]$ (n-times) if and only if $\dot{\mathrm{s}}_{[\alpha,\alpha']}=\dot{\mathrm{s}}_{[\beta,\beta']}$ and $\ddot{\mathrm{s}}_{[\gamma,\gamma']}=\ddot{\mathrm{s}}_{[\delta,\delta']}$
c. $E(\mathrm{S}_1)=E\left({\mathrm{S}_1}^{\mathrm{C}}\right)$

d. If $S_1 \subseteq S_2$ and $\dot{S}_{[\alpha_2,\alpha_2']} \leq \dot{S}_{[\beta_2,\beta_2']}$ and $\ddot{S}_{[\gamma_2,\gamma_2']} \leq \ddot{S}_{[\delta_2,\delta_2']}$ for all alternatives then $E(S_1) \leq E(S_2)$.

PROOF (a). Suppose that S_1 is a crisp set then

$$E(S_1) = \frac{1}{n}\sum_{e=1}^{n}\begin{bmatrix} \frac{[\dot{s}_0\oplus\dot{s}_0\oplus\dot{s}_0 \oplus \ddot{s}_0\oplus\dot{s}_0\oplus\ddot{s}_0\oplus\ddot{s}_0\oplus\ddot{s}_0]}{[\dot{s}_t\oplus\dot{s}_t\oplus\ddot{s}_{t'}\oplus\dot{s}_0\oplus\dot{s}_0\oplus\ddot{s}_{t'}\oplus\ddot{s}_0\oplus\ddot{s}_0]} \\ \oplus \frac{[\dot{s}_0\oplus\dot{s}_0\oplus\dot{s}_0 \oplus \ddot{s}_0\oplus\dot{s}_0\oplus\ddot{s}_0\oplus\ddot{s}_0\oplus\ddot{s}_0]}{[\dot{s}_t\oplus\dot{s}_t\oplus\ddot{s}_{t'}\oplus\dot{s}_0\oplus\dot{s}_0\oplus\ddot{s}_{t'}\oplus\ddot{s}_0\oplus\ddot{s}_0]} \\ \oplus\, .\,.\,.\, \oplus \\ \frac{[\dot{s}_0\oplus\dot{s}_0\oplus\dot{s}_0 \oplus \ddot{s}_0\oplus\dot{s}_0\oplus\ddot{s}_0\oplus\ddot{s}_0\oplus\ddot{s}_0]}{[\dot{s}_t\oplus\dot{s}_t\oplus\ddot{s}_{t'}\oplus\dot{s}_0\oplus\dot{s}_0\oplus\ddot{s}_{t'}\oplus\ddot{s}_0\oplus\ddot{s}_0]} \end{bmatrix} = \begin{bmatrix} \frac{[\dot{s}_0\oplus\dot{s}_0\oplus\dot{s}_0 \oplus \ddot{s}_0\oplus\dot{s}_0\oplus\ddot{s}_0\oplus\ddot{s}_0\oplus\ddot{s}_0]}{[\dot{s}_t\oplus\dot{s}_t\oplus\ddot{s}_{t'}\oplus\dot{s}_0\oplus\dot{s}_0\oplus\ddot{s}_{t'}\oplus\ddot{s}_0\oplus\ddot{s}_0]} \\ \oplus \frac{[\dot{s}_0\oplus\dot{s}_0\oplus\dot{s}_0 \oplus \ddot{s}_0\oplus\dot{s}_0\oplus\ddot{s}_0\oplus\ddot{s}_0\oplus\ddot{s}_0]}{[\dot{s}_t\oplus\dot{s}_t\oplus\ddot{s}_{t'}\oplus\dot{s}_0\oplus\dot{s}_0\oplus\ddot{s}_{t'}\oplus\ddot{s}_0\oplus\ddot{s}_0]} \\ \oplus\, .\,.\,.\, \oplus \\ \frac{[\dot{s}_0\oplus\dot{s}_0\oplus\dot{s}_0 \oplus \ddot{s}_0\oplus\dot{s}_0\oplus\ddot{s}_0\oplus\ddot{s}_0\oplus\ddot{s}_0]}{[\dot{s}_t\oplus\dot{s}_t\oplus\ddot{s}_{t'}\oplus\dot{s}_0\oplus\dot{s}_0\oplus\ddot{s}_{t'}\oplus\ddot{s}_0\oplus\ddot{s}_0]} \end{bmatrix}.$$

As we know that

$$s_0\oplus s_0 = s_0 \;\&\; s_t\oplus s_t = s_t \;\&\; s_t\oplus s_0 = s_0\oplus s_t = s_t,$$

Thus

$$E(S_1) = \left[\frac{[\dot{s}_0\oplus\ddot{s}_0]}{[\dot{s}_t\oplus\ddot{s}_{t'}]} \oplus \frac{[\dot{s}_0\oplus\ddot{s}_0]}{[\dot{s}_t\oplus\ddot{s}_{t'}]} \oplus\, .\,.\,.\, \oplus \frac{[\dot{s}_0\oplus\ddot{s}_0]}{[\dot{s}_t\oplus\ddot{s}_{t'}]}\right],$$

now from here we have two choices

$\dot{s}_t\oplus\ddot{s}_{t'} = s_{t+t'-\frac{tt'}{t}}$ or $\dot{s}_t\oplus\ddot{s}_{t'} = s_{t+t'-\frac{tt'}{t'}}$,

here we take

$\dot{s}_t\oplus\ddot{s}_{t'} = s_{t+t'-\frac{tt'}{t}} = s_t$,

this implies that

$$E(S_1) = \left[\frac{s_0}{s_t} \oplus \frac{s_0}{s_t} \oplus\, .\,.\,.\, \oplus \frac{s_0}{s_t}\right],$$

here $\frac{s_0}{s_t} = s_0 \otimes s_{\frac{1}{t}} = s_{\frac{0}{t^2}} = s_0$,

thus

$$E(S_1) = [s_0 \oplus s_0 \oplus\, .\,.\,.\, \oplus s_0] = s_0,$$

this states that if S_1 is a crisp set then $E(S_1) = s_0$.

Now conversely suppose that $E(S_1) = s_0$ and we wanted to show that S_1 is crisp set, to do this we follow the steps done when S_1 was crisp in a reverse order and at the end we get

$$E(\mathrm{S}_1)=\frac{1}{\mathrm{n}}\sum_{\mathrm{e}=1}^{\mathrm{n}}\left[\begin{array}{c}\frac{[\dot{\mathrm{s}}_0\oplus\dot{\mathrm{s}}_0\oplus\dot{\mathrm{s}}_0\oplus\ddot{\mathrm{s}}_0\oplus\dot{\mathrm{s}}_0\oplus\ddot{\mathrm{s}}_0\oplus\ddot{\mathrm{s}}_0\oplus\ddot{\mathrm{s}}_0]}{[\dot{\mathrm{s}}_\mathrm{t}\oplus\dot{\mathrm{s}}_\mathrm{t}\oplus\ddot{\mathrm{s}}_{\mathrm{t}'}\oplus\dot{\mathrm{s}}_0\oplus\dot{\mathrm{s}}_0\oplus\ddot{\mathrm{s}}_{\mathrm{t}'}\oplus\ddot{\mathrm{s}}_0\oplus\ddot{\mathrm{s}}_0]}\\ \oplus\frac{[\dot{\mathrm{s}}_0\oplus\dot{\mathrm{s}}_0\oplus\dot{\mathrm{s}}_0\oplus\ddot{\mathrm{s}}_0\oplus\dot{\mathrm{s}}_0\oplus\ddot{\mathrm{s}}_0\oplus\ddot{\mathrm{s}}_0\oplus\ddot{\mathrm{s}}_0]}{[\dot{\mathrm{s}}_\mathrm{t}\oplus\dot{\mathrm{s}}_\mathrm{t}\oplus\ddot{\mathrm{s}}_{\mathrm{t}'}\oplus\dot{\mathrm{s}}_0\oplus\dot{\mathrm{s}}_0\oplus\ddot{\mathrm{s}}_{\mathrm{t}'}\oplus\ddot{\mathrm{s}}_0\oplus\ddot{\mathrm{s}}_0]}\\ \oplus\ .\ .\ .\ \oplus\\ \frac{[\dot{\mathrm{s}}_0\oplus\dot{\mathrm{s}}_0\oplus\dot{\mathrm{s}}_0\oplus\ddot{\mathrm{s}}_0\oplus\dot{\mathrm{s}}_0\oplus\ddot{\mathrm{s}}_0\oplus\ddot{\mathrm{s}}_0\oplus\ddot{\mathrm{s}}_0]}{[\dot{\mathrm{s}}_\mathrm{t}\oplus\dot{\mathrm{s}}_\mathrm{t}\oplus\ddot{\mathrm{s}}_{\mathrm{t}'}\oplus\dot{\mathrm{s}}_0\oplus\dot{\mathrm{s}}_0\oplus\ddot{\mathrm{s}}_{\mathrm{t}'}\oplus\ddot{\mathrm{s}}_0\oplus\ddot{\mathrm{s}}_0]}\end{array}\right],$$

which shows that S_1 is a crisp set. Thus for GLIVIFSES S_1 $E(\mathrm{S}_1)=\mathrm{s}_0$ if and only if S_1 is crisp.

PROOF (b). Firstly suppose that $\dot{\mathrm{s}}_{[\alpha,\alpha']}=\dot{\mathrm{s}}_{[\beta,\beta']}$and $\ddot{\mathrm{s}}_{[\gamma,\gamma']}=\ddot{\mathrm{s}}_{[\delta,\delta']}$

then

$$E(\mathrm{S}_1)=\frac{1}{\mathrm{n}}\sum_{\mathrm{e}=1}^{\mathrm{n}}\left[\begin{array}{c}\frac{\left[\dot{\mathrm{s}}_\alpha(d_c,c_i)\oplus\dot{\mathrm{s}}_{\alpha'}(d_c,c_i)\oplus\ddot{\mathrm{s}}_\gamma(d_c,c_i)\oplus\dot{\mathrm{s}}_{\mathrm{t}-2\alpha}\oplus\dot{\mathrm{s}}_{\mathrm{t}-2\alpha'}\oplus\ddot{\mathrm{s}}_{\gamma'}(d_c,c_i)\oplus\ddot{\mathrm{s}}_{\mathrm{t}-2\gamma}\oplus\ddot{\mathrm{s}}_{\mathrm{t}-2\gamma'}\right]}{\left[\dot{\mathrm{s}}_\alpha(d_c,c_i)\oplus\dot{\mathrm{s}}_{\alpha'}(d_c,c_i)\oplus\ddot{\mathrm{s}}_\gamma(d_c,c_i)\oplus\dot{\mathrm{s}}_{\mathrm{t}-2\alpha}\oplus\dot{\mathrm{s}}_{\mathrm{t}-2\alpha'}\oplus\ddot{\mathrm{s}}_{\gamma'}(d_c,c_i)\oplus\ddot{\mathrm{s}}_{\mathrm{t}-2\gamma}\oplus\ddot{\mathrm{s}}_{\mathrm{t}-2\gamma'}\right]}\\ \oplus\frac{\left[\dot{\mathrm{s}}_\alpha(d_c,c_{i+1})\oplus\dot{\mathrm{s}}_{\alpha'}(d_c,c_{i+1})\oplus\ddot{\mathrm{s}}_\gamma(d_c,c_{i+1})\oplus\dot{\mathrm{s}}_{\mathrm{t}-2\alpha}\oplus\dot{\mathrm{s}}_{\mathrm{t}-2\alpha'}\oplus\ddot{\mathrm{s}}_{\gamma'}(d_c,c_{i+1})\oplus\ddot{\mathrm{s}}_{\mathrm{t}-2\gamma}\oplus\ddot{\mathrm{s}}_{\mathrm{t}-2\gamma'}\right]}{\left[\dot{\mathrm{s}}_\alpha(d_c,c_{i+1})\oplus\dot{\mathrm{s}}_{\alpha'}(d_c,c_{i+1})\oplus\ddot{\mathrm{s}}_\gamma(d_c,c_{i+1})\oplus\dot{\mathrm{s}}_{\mathrm{t}-2\alpha}\oplus\dot{\mathrm{s}}_{\mathrm{t}-2\alpha'}\oplus\ddot{\mathrm{s}}_{\gamma'}(d_c,c_{i+1})\oplus\ddot{\mathrm{s}}_{\mathrm{t}-2\gamma}\oplus\ddot{\mathrm{s}}_{\mathrm{t}-2\gamma'}\right]}\\ \oplus\ .\ .\ .\ \oplus\\ \frac{\left[\dot{\mathrm{s}}_\alpha(d_c,c_r)\oplus\dot{\mathrm{s}}_{\alpha'}(d_c,c_r)\oplus\ddot{\mathrm{s}}_\gamma(d_c,c_r)\oplus\dot{\mathrm{s}}_{\mathrm{t}-2\alpha}\oplus\dot{\mathrm{s}}_{\mathrm{t}-2\alpha'}\oplus\ddot{\mathrm{s}}_{\gamma'}(d_c,c_r)\oplus\ddot{\mathrm{s}}_{\mathrm{t}-2\gamma}\oplus\ddot{\mathrm{s}}_{\mathrm{t}-2\gamma'}\right]}{\left[\dot{\mathrm{s}}_\alpha(d_c,c_r)\oplus\dot{\mathrm{s}}_{\alpha'}(d_c,c_r)\oplus\ddot{\mathrm{s}}_\gamma(d_c,c_r)\oplus\dot{\mathrm{s}}_{\mathrm{t}-2\alpha}\oplus\dot{\mathrm{s}}_{\mathrm{t}-2\alpha'}\oplus\ddot{\mathrm{s}}_{\gamma'}(d_c,c_r)\oplus\ddot{\mathrm{s}}_{\mathrm{t}-2\gamma}\oplus\ddot{\mathrm{s}}_{\mathrm{t}-2\gamma'}\right]}\end{array}\right]$$

$$=\frac{1}{\mathrm{n}}\sum_{\mathrm{e}=1}^{\mathrm{n}}\left[\frac{\mathrm{S_a}}{\mathrm{S_a}}\oplus\frac{\mathrm{S_x}}{\mathrm{S_x}}\oplus\ .\ .\ .\ \oplus\frac{\mathrm{S_m}}{\mathrm{S_m}}\right]=\frac{1}{\mathrm{n}}\sum_{\mathrm{e}=1}^{\mathrm{n}}\left[\mathrm{s}_{\frac{1}{\mathrm{t}}}\oplus\mathrm{s}_{\frac{1}{\mathrm{t}}}\oplus\ .\ .\ .\ \oplus\mathrm{s}_{\frac{1}{\mathrm{t}}}\right]=\left[\mathrm{s}_{\frac{1}{\mathrm{t}}}\oplus\mathrm{s}_{\frac{1}{\mathrm{t}}}\oplus\ .\ .\ .\ \oplus\mathrm{s}_{\frac{1}{\mathrm{t}}}\right].$$

Conversely suppose that

$E(\mathrm{S}_1)=\left[\mathrm{s}_{\frac{1}{\mathrm{t}}}\oplus\mathrm{s}_{\frac{1}{\mathrm{t}}}\oplus\ .\ .\ .\ \oplus\mathrm{s}_{\frac{1}{\mathrm{t}}}\right]$ (n-times),

by applying the steps done previously in converse order we get $\dot{\mathrm{s}}_{[\alpha,\alpha']}=\dot{\mathrm{s}}_{[\beta,\beta']}$and $\ddot{\mathrm{s}}_{[\gamma,\gamma']}=\ddot{\mathrm{s}}_{[\delta,\delta']}$.

PROOF(c). As we know that

$$E(\mathrm{S}_1)=\frac{1}{\mathrm{n}}\sum_{\mathrm{e}=1}^{\mathrm{n}}\left[\begin{array}{c}
\dfrac{\left[\begin{array}{c}\min\{\dot{\mathrm{S}}_{\alpha}(d_c,c_i),\dot{\mathrm{S}}_{\beta}(d_c,c_i)\}\oplus\min\left\{\dot{\mathrm{S}}_{\alpha'}(d_c,c_i),\dot{\mathrm{S}}_{\beta'}(d_c,c_i)\right\}\oplus\min\{\ddot{\mathrm{S}}_{\gamma}(d_c,c_i),\ddot{\mathrm{S}}_{\delta}(d_c,c_i)\}\oplus\dot{\pi}_e(d_c,c_i)\oplus\dot{\pi}'_e(d_c,c_i)\\ \oplus\min\left\{\ddot{\mathrm{S}}_{\gamma'}(d_c,c_i),\ddot{\mathrm{S}}_{\delta'}(d_c,c_i)\right\}\oplus\ddot{\pi}_e(d_c,c_i)\oplus\ddot{\pi}'_e(d_c,c_i)\end{array}\right]}{\left[\begin{array}{c}\max\{\dot{\mathrm{S}}_{\alpha}(d_c,c_i),\dot{\mathrm{S}}_{\beta}(d_c,c_i)\}\oplus\max\left\{\dot{\mathrm{S}}_{\alpha'}(d_c,c_i),\dot{\mathrm{S}}_{\beta'}(d_c,c_i)\right\}\oplus\max\{\ddot{\mathrm{S}}_{\gamma}(d_c,c_i),\ddot{\mathrm{S}}_{\delta}(d_c,c_i)\}\oplus\dot{\pi}_e(d_c,c_i)\oplus\dot{\pi}'_e(d_c,c_i)\\ \oplus\max\left\{\ddot{\mathrm{S}}_{\gamma'}(d_c,c_i),\ddot{\mathrm{S}}_{\delta'}(d_c,c_i)\right\}\oplus\ddot{\pi}_e(d_c,c_i)\oplus\ddot{\pi}'_e(d_c,c_i)\end{array}\right]}\\
\oplus\dfrac{\left[\begin{array}{c}\min\{\dot{\mathrm{S}}_{\alpha}(d_c,c_{i+1}),\dot{\mathrm{S}}_{\beta}(d_c,c_{i+1})\}\oplus\min\left\{\dot{\mathrm{S}}_{\alpha'}(d_c,c_{i+1}),\dot{\mathrm{S}}_{\beta'}(d_c,c_{i+1})\right\}\oplus\min\{\ddot{\mathrm{S}}_{\gamma}(d_c,c_{i+1}),\ddot{\mathrm{S}}_{\delta}(d_c,c_{i+1})\}\\ \oplus\dot{\pi}_e(d_c,c_{i+1})\oplus\dot{\pi}'_e(d_c,c_{i+1})\oplus\min\left\{\ddot{\mathrm{S}}_{\gamma'}(d_c,c_{i+1}),\ddot{\mathrm{S}}_{\delta'}(d_c,c_{i+1})\right\}\oplus\ddot{\pi}_e(d_c,c_{i+1})\oplus\ddot{\pi}'_e(d_c,c_{i+1})\end{array}\right]}{\left[\begin{array}{c}\max\{\dot{\mathrm{S}}_{\alpha}(d_c,c_{i+1}),\dot{\mathrm{S}}_{\beta}(d_c,c_{i+1})\}\oplus\max\left\{\dot{\mathrm{S}}_{\alpha'}(d_c,c_{i+1}),\dot{\mathrm{S}}_{\beta'}(d_c,c_{i+1})\right\}\oplus\max\{\ddot{\mathrm{S}}_{\gamma}(d_c,c_{i+1}),\ddot{\mathrm{S}}_{\delta}(d_c,c_{i+1})\}\\ \oplus\dot{\pi}_e(d_c,c_{i+1})\oplus\dot{\pi}'_e(d_c,c_{i+1})\oplus\max\left\{\ddot{\mathrm{S}}_{\gamma'}(d_c,c_{i+1}),\ddot{\mathrm{S}}_{\delta'}(d_c,c_{i+1})\right\}\oplus\ddot{\pi}_e(d_c,c_{i+1})\oplus\ddot{\pi}'_e(d_c,c_{i+1})\end{array}\right]}\\
\oplus\ .\ .\ .\ \oplus\\
\dfrac{\left[\begin{array}{c}\min\{\dot{\mathrm{S}}_{\alpha}(d_c,c_r),\dot{\mathrm{S}}_{\beta}(d_c,c_r)\}\oplus\min\left\{\dot{\mathrm{S}}_{\alpha'}(d_c,c_r),\dot{\mathrm{S}}_{\beta'}(d_c,c_r)\right\}\oplus\min\{\ddot{\mathrm{S}}_{\gamma}(d_c,c_r),\ddot{\mathrm{S}}_{\delta}(d_c,c_r)\}\oplus\dot{\pi}_e(d_c,c_r)\oplus\dot{\pi}'_e(d_c,c_r)\\ \oplus\min\left\{\ddot{\mathrm{S}}_{\gamma'}(d_c,c_r),\ddot{\mathrm{S}}_{\delta'}(d_c,c_r)\right\}\oplus\ddot{\pi}_e(d_c,c_r)\oplus\ddot{\pi}'_e(d_c,c_r)\end{array}\right]}{\left[\begin{array}{c}\max\{\dot{\mathrm{S}}_{\alpha}(d_c,c_r),\dot{\mathrm{S}}_{\beta}(d_c,c_r)\}\oplus\max\left\{\dot{\mathrm{S}}_{\alpha'}(d_c,c_r),\dot{\mathrm{S}}_{\beta'}(d_c,c_r)\right\}\oplus\max\{\ddot{\mathrm{S}}_{\gamma}(d_c,c_r),\ddot{\mathrm{S}}_{\delta}(d_c,c_r)\}\oplus\dot{\pi}_e(d_c,c_r)\oplus\dot{\pi}'_e(d_c,c_r)\\ \oplus\max\left\{\ddot{\mathrm{S}}_{\gamma'}(d_c,c_r),\ddot{\mathrm{S}}_{\delta'}(d_c,c_r)\right\}\oplus\ddot{\pi}_e(d_c,c_r)\oplus\ddot{\pi}'_e(d_c,c_r)\end{array}\right]}
\end{array}\right],$$

now

$$E\left({\mathrm{S}_1}^{\mathrm{C}}\right)=\frac{1}{\mathrm{n}}\sum_{\mathrm{e}=1}^{\mathrm{n}}\left[\begin{array}{c}
\dfrac{\left[\begin{array}{c}\min\{\dot{\mathrm{S}}_{\beta}(d_c,c_i),\dot{\mathrm{S}}_{\alpha}(d_c,c_i)\}\oplus\min\left\{\dot{\mathrm{S}}_{\beta'}(d_c,c_i),\dot{\mathrm{S}}_{\alpha'}(d_c,c_i)\right\}\oplus\min\{\ddot{\mathrm{S}}_{\delta}(d_c,c_i),\ddot{\mathrm{S}}_{\gamma}(d_c,c_i)\}\oplus\dot{\pi}_e(d_c,c_i)\oplus\dot{\pi}'_e(d_c,c_i)\\ \oplus\min\left\{\ddot{\mathrm{S}}_{\delta'}(d_c,c_i),\ddot{\mathrm{S}}_{\gamma'}(d_c,c_i)\right\}\oplus\ddot{\pi}_e(d_c,c_i)\oplus\ddot{\pi}'_e(d_c,c_i)\end{array}\right]}{\left[\begin{array}{c}\max\{\dot{\mathrm{S}}_{\beta}(d_c,c_i),\dot{\mathrm{S}}_{\alpha}(d_c,c_i)\}\oplus\max\left\{\dot{\mathrm{S}}_{\beta'}(d_c,c_i),\dot{\mathrm{S}}_{\alpha'}(d_c,c_i)\right\}\oplus\max\{\ddot{\mathrm{S}}_{\delta}(d_c,c_i),\ddot{\mathrm{S}}_{\gamma}(d_c,c_i)\}\oplus\dot{\pi}_e(d_c,c_i)\oplus\dot{\pi}'_e(d_c,c_i)\\ \oplus\max\left\{\ddot{\mathrm{S}}_{\delta'}(d_c,c_i),\ddot{\mathrm{S}}_{\gamma'}(d_c,c_i)\right\}\oplus\ddot{\pi}_e(d_c,c_i)\oplus\ddot{\pi}'_e(d_c,c_i)\end{array}\right]}\\
\oplus\dfrac{\left[\begin{array}{c}\min\{\dot{\mathrm{S}}_{\beta}(d_c,c_{i+1}),\dot{\mathrm{S}}_{\alpha}(d_c,c_{i+1})\}\oplus\min\left\{\dot{\mathrm{S}}_{\beta'}(d_c,c_{i+1}),\dot{\mathrm{S}}_{\alpha'}(d_c,c_{i+1})\right\}\oplus\min\{\ddot{\mathrm{S}}_{\delta}(d_c,c_{i+1}),\ddot{\mathrm{S}}_{\gamma}(d_c,c_{i+1})\}\\ \oplus\dot{\pi}_e(d_c,c_{i+1})\oplus\dot{\pi}'_e(d_c,c_{i+1})\oplus\min\left\{\ddot{\mathrm{S}}_{\delta'}(d_c,c_{i+1}),\ddot{\mathrm{S}}_{\gamma'}(d_c,c_{i+1})\right\}\oplus\ddot{\pi}_e(d_c,c_{i+1})\oplus\ddot{\pi}'_e(d_c,c_{i+1})\end{array}\right]}{\left[\begin{array}{c}\max\{\dot{\mathrm{S}}_{\beta}(d_c,c_{i+1}),\dot{\mathrm{S}}_{\alpha}(d_c,c_{i+1})\}\oplus\max\left\{\dot{\mathrm{S}}_{\beta'}(d_c,c_{i+1}),\dot{\mathrm{S}}_{\alpha'}(d_c,c_{i+1})\right\}\oplus\max\{\ddot{\mathrm{S}}_{\delta}(d_c,c_{i+1}),\ddot{\mathrm{S}}_{\gamma}(d_c,c_{i+1})\}\\ \oplus\dot{\pi}_e(d_c,c_{i+1})\oplus\dot{\pi}'_e(d_c,c_{i+1})\oplus\max\left\{\ddot{\mathrm{S}}_{\delta'}(d_c,c_{i+1}),\ddot{\mathrm{S}}_{\gamma'}(d_c,c_{i+1})\right\}\oplus\ddot{\pi}_e(d_c,c_{i+1})\oplus\ddot{\pi}'_e(d_c,c_{i+1})\end{array}\right]}\\
\oplus\ .\ .\ .\ \oplus\\
\dfrac{\left[\begin{array}{c}\min\{\dot{\mathrm{S}}_{\beta}(d_c,c_r),\dot{\mathrm{S}}_{\alpha}(d_c,c_r)\}\oplus\min\left\{\dot{\mathrm{S}}_{\beta'}(d_c,c_r),\dot{\mathrm{S}}_{\alpha'}(d_c,c_r)\right\}\oplus\min\{\ddot{\mathrm{S}}_{\delta}(d_c,c_r),\ddot{\mathrm{S}}_{\gamma}(d_c,c_r)\}\oplus\dot{\pi}_e(d_c,c_i)\oplus\dot{\pi}'_e(d_c,c_i)\\ \oplus\min\left\{\ddot{\mathrm{S}}_{\delta'}(d_c,c_r),\ddot{\mathrm{S}}_{\gamma'}(d_c,c_r)\right\}\oplus\ddot{\pi}_e(d_c,c_i)\oplus\ddot{\pi}'_e(d_c,c_i)\end{array}\right]}{\left[\begin{array}{c}\max\{\dot{\mathrm{S}}_{\beta}(d_c,c_r),\dot{\mathrm{S}}_{\alpha}(d_c,c_r)\}\oplus\max\left\{\dot{\mathrm{S}}_{\beta'}(d_c,c_r),\dot{\mathrm{S}}_{\alpha'}(d_c,c_r)\right\}\oplus\max\{\ddot{\mathrm{S}}_{\delta}(d_c,c_r),\ddot{\mathrm{S}}_{\gamma}(d_c,c_r)\}\oplus\dot{\pi}_e(d_c,c_i)\oplus\dot{\pi}'_e(d_c,c_i)\\ \oplus\max\left\{\ddot{\mathrm{S}}_{\delta'}(d_c,c_r),\ddot{\mathrm{S}}_{\gamma'}(d_c,c_r)\right\}\oplus\ddot{\pi}_e(d_c,c_i)\oplus\ddot{\pi}'_e(d_c,c_i)\end{array}\right]}
\end{array}\right],$$

by observation we can see that the equations for $E(\mathrm{S}_1)$ and $E\left({\mathrm{S}_1}^{\mathrm{C}}\right)$ are same with the alteration in intervals of linguistic terms from membership to non-membership but at the end we obtain same result since

$$\min\left\{\dot{\mathrm{s}}_{\alpha}(d_c,c_i),\dot{\mathrm{s}}_{\beta}(d_c,c_i)\right\}=\min\left\{\dot{\mathrm{s}}_{\beta}(d_c,c_i),\dot{\mathrm{s}}_{\alpha}(d_c,c_i)\right\},$$

and

$$\max\left\{\dot{\mathrm{s}}_{\alpha}(d_c,c_i),\dot{\mathrm{s}}_{\beta}(d_c,c_i))\right\}=\max\left\{\dot{\mathrm{s}}_{\beta}(d_c,c_i),\dot{\mathrm{s}}_{\alpha}(d_c,c_i)\right\},$$

with no variation in linguistic term for hesitancy degree. Thus

$$E(\mathrm{S}_1) = E\left(\mathrm{S}_1{}^{\mathrm{C}}\right).$$

PROOF (d). As we know that

$$\mathrm{S}_1 = \left\{\langle \dot{\mathrm{s}}_{[\alpha,\alpha']}, \dot{\mathrm{s}}_{[\beta,\beta']}\rangle\ , \langle \ddot{\mathrm{s}}_{[\gamma,\gamma']}\ , \ddot{\mathrm{s}}_{[\delta,\delta']}\rangle\right\},$$
$$\mathrm{S}_2 = \left\{\langle \dot{\mathrm{s}}_{[\alpha_1,\alpha_1']}, \dot{\mathrm{s}}_{[\beta_1,\beta_1']}\rangle\ , \langle \ddot{\mathrm{s}}_{[\gamma_1,\gamma_1']}\ , \ddot{\mathrm{s}}_{[\delta_1,\delta_1']}\rangle\right\}.$$

Suppose that $S_1 \subseteq S_2$ and $\dot{\mathrm{s}}_{[\alpha_2,\alpha_2']} \leq \dot{\mathrm{s}}_{[\beta_2,\beta_2']}, \ddot{\mathrm{s}}_{[\gamma_2,\gamma_2']} \leq \ddot{\mathrm{s}}_{[\delta_1,\delta_1']}$,
here $S_1 \subseteq S_2$ implies $\dot{\mathrm{s}}_{[\alpha,\alpha']} \leq \dot{\mathrm{s}}_{[\alpha_1,\alpha_1']}$ & $\dot{\mathrm{s}}_{[\beta_1,\beta_1']} \leq \dot{\mathrm{s}}_{[\beta,\beta']}$ & $\ddot{\mathrm{s}}_{[\gamma,\gamma']} \leq \ddot{\mathrm{s}}_{[\gamma_1,\gamma_1']}$ & $\ddot{\mathrm{s}}_{[\delta_1,\delta_1']} \leq \ddot{\mathrm{s}}_{[\delta,\delta']}$.
Where Entropy measure,

$$E(\mathrm{S}_1) = \frac{1}{\mathrm{n}}\sum_{\mathrm{e}=1}^{\mathrm{n}}\left[\begin{array}{c}
\dfrac{\begin{bmatrix}\min\{\dot{\mathrm{S}}_{\alpha}(d_c,c_i),\dot{\mathrm{S}}_{\beta}(d_c,c_i)\}\oplus\min\left\{\dot{\mathrm{S}}_{\alpha'}(d_c,c_i),\dot{\mathrm{S}}_{\beta'}(d_c,c_i)\right\}\oplus\min\{\ddot{\mathrm{S}}_{\gamma}(d_c,c_i),\ddot{\mathrm{S}}_{\delta}(d_c,c_i)\}\oplus\dot{\pi}_e(d_c,c_i)\oplus\dot{\pi}_e'(d_c,c_i)\\ \oplus\min\left\{\ddot{\mathrm{S}}_{\gamma'}(d_c,c_i),\ddot{\mathrm{S}}_{\delta'}(d_c,c_i)\right\}\oplus\ddot{\pi}_e(d_c,c_i)\oplus\ddot{\pi}_e'(d_c,c_i)\end{bmatrix}}{\begin{bmatrix}\max\{\dot{\mathrm{S}}_{\alpha}(d_c,c_i),\dot{\mathrm{S}}_{\beta}(d_c,c_i)\}\oplus\max\left\{\dot{\mathrm{S}}_{\alpha'}(d_c,c_i),\dot{\mathrm{S}}_{\beta'}(d_c,c_i)\right\}\oplus\max\{\ddot{\mathrm{S}}_{\gamma}(d_c,c_i),\ddot{\mathrm{S}}_{\delta}(d_c,c_i)\}\oplus\dot{\pi}_e(d_c,c_i)\oplus\dot{\pi}_e'(d_c,c_i)\\ \oplus\max\left\{\ddot{\mathrm{S}}_{\gamma'}(d_c,c_i),\ddot{\mathrm{S}}_{\delta'}(d_c,c_i)\right\}\oplus\ddot{\pi}_e(d_c,c_i)\oplus\ddot{\pi}_e'(d_c,c_i)\end{bmatrix}}\\
\oplus\dfrac{\begin{bmatrix}\min\{\dot{\mathrm{S}}_{\alpha}(d_c,c_{i+1}),\dot{\mathrm{S}}_{\beta}(d_c,c_{i+1})\}\oplus\min\left\{\dot{\mathrm{S}}_{\alpha'}(d_c,c_{i+1}),\dot{\mathrm{S}}_{\beta'}(d_c,c_{i+1})\right\}\oplus\min\{\ddot{\mathrm{S}}_{\gamma}(d_c,c_{i+1}),\ddot{\mathrm{S}}_{\delta}(d_c,c_{i+1})\}\\ \oplus\dot{\pi}_e(d_c,c_{i+1})\oplus\dot{\pi}_e'(d_c,c_{i+1})\oplus\min\left\{\ddot{\mathrm{S}}_{\gamma'}(d_c,c_{i+1}),\ddot{\mathrm{S}}_{\delta'}(d_c,c_{i+1})\right\}\oplus\ddot{\pi}_e(d_c,c_{i+1})\oplus\ddot{\pi}_e'(d_c,c_{i+1})\end{bmatrix}}{\begin{bmatrix}\max\{\dot{\mathrm{S}}_{\alpha}(d_c,c_{i+1}),\dot{\mathrm{S}}_{\beta}(d_c,c_{i+1})\}\oplus\max\left\{\dot{\mathrm{S}}_{\alpha'}(d_c,c_{i+1}),\dot{\mathrm{S}}_{\beta'}(d_c,c_{i+1})\right\}\oplus\max\{\ddot{\mathrm{S}}_{\gamma}(d_c,c_{i+1}),\ddot{\mathrm{S}}_{\delta}(d_c,c_{i+1})\}\\ \oplus\dot{\pi}_e(d_c,c_{i+1})\oplus\dot{\pi}_e'(d_c,c_{i+1})\oplus\max\left\{\ddot{\mathrm{S}}_{\gamma'}(d_c,c_{i+1}),\ddot{\mathrm{S}}_{\delta'}(d_c,c_{i+1})\right\}\oplus\ddot{\pi}_e(d_c,c_{i+1})\oplus\ddot{\pi}_e'(d_c,c_{i+1})\end{bmatrix}}\\
\oplus\ .\ .\ .\ \oplus\\
\dfrac{\begin{bmatrix}\min\{\dot{\mathrm{S}}_{\alpha}(d_c,c_r),\dot{\mathrm{S}}_{\beta}(d_c,c_r)\}\oplus\min\left\{\dot{\mathrm{S}}_{\alpha'}(d_c,c_r),\dot{\mathrm{S}}_{\beta'}(d_c,c_r)\right\}\oplus\min\{\ddot{\mathrm{S}}_{\gamma}(d_c,c_r),\ddot{\mathrm{S}}_{\delta}(d_c,c_r)\}\oplus\dot{\pi}_e(d_c,c_r)\oplus\dot{\pi}_e'(d_c,c_r)\\ \oplus\min\left\{\ddot{\mathrm{S}}_{\gamma'}(d_c,c_r),\ddot{\mathrm{S}}_{\delta'}(d_c,c_r)\right\}\oplus\ddot{\pi}_e(d_c,c_r)\oplus\ddot{\pi}_e'(d_c,c_r)\end{bmatrix}}{\begin{bmatrix}\max\{\dot{\mathrm{S}}_{\alpha}(d_c,c_r),\dot{\mathrm{S}}_{\beta}(d_c,c_r)\}\oplus\max\left\{\dot{\mathrm{S}}_{\alpha'}(d_c,c_r),\dot{\mathrm{S}}_{\beta'}(d_c,c_r)\right\}\oplus\max\{\ddot{\mathrm{S}}_{\gamma}(d_c,c_r),\ddot{\mathrm{S}}_{\delta}(d_c,c_r)\}\oplus\dot{\pi}_e(d_c,c_r)\oplus\dot{\pi}_e'(d_c,c_r)\\ \oplus\max\left\{\ddot{\mathrm{S}}_{\gamma'}(d_c,c_r),\ddot{\mathrm{S}}_{\delta'}(d_c,c_r)\right\}\oplus\ddot{\pi}_e(d_c,c_r)\oplus\ddot{\pi}_e'(d_c,c_r)\end{bmatrix}}
\end{array}\right],$$

and

$$E(\mathrm{S}_2)=\frac{1}{\mathrm{n}}\sum_{\mathrm{e}=1}^{\mathrm{n}}\left[\begin{array}{c}
\dfrac{\left[\begin{array}{c}\min\left\{\dot{\mathrm{S}}_{\alpha_1}(d_{c+1},c_i),\dot{\mathrm{S}}_{\beta_1}(d_{c+1},c_i)\right\}\oplus\min\left\{\dot{\mathrm{S}}_{\alpha_1{}'}(d_{c+1},c_i),\dot{\mathrm{S}}_{\beta_1{}'}(d_{c+1},c_i)\right\}\oplus\min\left\{\ddot{\mathrm{S}}_{\gamma_1}(d_{c+1},c_i),\ddot{\mathrm{S}}_{\delta_1}(d_{c+1},c_i)\right\}\\ \oplus\dot{\pi}_e(d_{c+1},c_i)\oplus\dot{\pi}'_e(d_{c+1},c_i)\oplus\min\left\{\ddot{\mathrm{S}}_{\gamma_1{}'}(d_{c+1},c_i),\ddot{\mathrm{S}}_{\delta_1{}'}(d_{c+1},c_i)\right\}\oplus\ddot{\pi}_e(d_{c+1},c_i)\oplus\ddot{\pi}'_e(d_{c+1},c_i)\end{array}\right]}{\left[\begin{array}{c}\max\left\{\dot{\mathrm{S}}_{\alpha_1}(d_{c+1},c_i),\dot{\mathrm{S}}_{\beta_1}(d_{c+1},c_i)\right\}\oplus\max\left\{\dot{\mathrm{S}}_{\alpha_1{}'}(d_{c+1},c_i),\dot{\mathrm{S}}_{\beta_1{}'}(d_{c+1},c_i)\right\}\oplus\max\left\{\ddot{\mathrm{S}}_{\gamma_1}(d_{c+1},c_i),\ddot{\mathrm{S}}_{\delta_1}(d_{c+1},c_i)\right\}\\ \oplus\dot{\pi}_e(d_{c+1},c_i)\oplus\dot{\pi}'_e(d_{c+1},c_i)\oplus\max\left\{\ddot{\mathrm{S}}_{\gamma_1{}'}(d_{c+1},c_i),\ddot{\mathrm{S}}_{\delta_1{}'}(d_{c+1},c_i)\right\}\oplus\ddot{\pi}_e(d_{c+1},c_i)\oplus\ddot{\pi}'_e(d_{c+1},c_i)\end{array}\right]}\\
\oplus\dfrac{\left[\begin{array}{c}\min\left\{\dot{\mathrm{S}}_{\alpha_1}(d_{c+1},c_{i+1}),\dot{\mathrm{S}}_{\beta_1}(d_{c+1},c_{i+1})\right\}\oplus\min\left\{\dot{\mathrm{S}}_{\alpha'_1}(d_{c+1},c_{i+1}),\dot{\mathrm{S}}_{\beta'_1}(d_{c+1},c_{i+1})\right\}\oplus\min\left\{\ddot{\mathrm{S}}_{\gamma_1}(d_{c+1},c_{i+1}),\ddot{\mathrm{S}}_{\delta_1}(d_{c+1},c_{i+1})\right\}\\ \oplus\dot{\pi}_e(d_{c+1},c_{i+1})\oplus\dot{\pi}'_e(d_{c+1},c_{i+1})\oplus\min\left\{\ddot{\mathrm{S}}_{\gamma_1{}'}(d_{c+1},c_{i+1}),\ddot{\mathrm{S}}_{\delta_1{}'}(d_{c+1},c_{i+1})\right\}\oplus\ddot{\pi}_e(d_{c+1},c_{i+1})\oplus\ddot{\pi}'_e(d_{c+1},c_{i+1})\end{array}\right]}{\left[\begin{array}{c}\max\left\{\dot{\mathrm{S}}_{\alpha_1}(d_{c+1},c_{i+1}),\dot{\mathrm{S}}_{\beta_1}(d_{c+1},c_{i+1})\right\}\oplus\max\left\{\dot{\mathrm{S}}_{\alpha'_1}(d_{c+1},c_{i+1}),\dot{\mathrm{S}}_{\beta'_1}(d_{c+1},c_{i+1})\right\}\oplus\max\left\{\ddot{\mathrm{S}}_{\gamma_1}(d_{c+1},c_{i+1}),\ddot{\mathrm{S}}_{\delta_1}(d_{c+1},c_{i+1})\right\}\\ \oplus\dot{\pi}_e(d_{c+1},c_{i+1})\oplus\dot{\pi}'_e(d_{c+1},c_{i+1})\oplus\max\left\{\ddot{\mathrm{S}}_{\gamma_1{}'}(d_{c+1},c_{i+1}),\ddot{\mathrm{S}}_{\delta_1{}'}(d_{c+1},c_{i+1})\right\}\oplus\ddot{\pi}_e(d_{c+1},c_{i+1})\oplus\ddot{\pi}'_e(d_{c+1},c_{i+1})\end{array}\right]}\\
\oplus\,.\,.\,.\,\oplus\\
\dfrac{\left[\begin{array}{c}\min\left\{\dot{\mathrm{S}}_{\alpha_1}(d_{c+1},c_r),\dot{\mathrm{S}}_{\beta_1}(d_{c+1},c_r)\right\}\oplus\min\left\{\dot{\mathrm{S}}_{\alpha_1{}'}(d_{c+1},c_r),\dot{\mathrm{S}}_{\beta_1{}'}(d_{c+1},c_r)\right\}\oplus\min\left\{\ddot{\mathrm{S}}_{\gamma_1}(d_{c+1},c_r),\ddot{\mathrm{S}}_{\delta_1}(d_{c+1},c_r)\right\}\\ \oplus\dot{\pi}_e(d_{c+1},c_r)\oplus\dot{\pi}'_e(d_{c+1},c_r)\oplus\min\left\{\ddot{\mathrm{S}}_{\gamma_1{}'}(d_{c+1},c_r),\ddot{\mathrm{S}}_{\delta_1{}'}(d_{c+1},c_r)\right\}\oplus\ddot{\pi}_e(d_{c+1},c_r)\oplus\ddot{\pi}'_e(d_{c+1},c_r)\end{array}\right]}{\left[\begin{array}{c}\max\left\{\dot{\mathrm{S}}_{\alpha_1}(d_{c+1},c_r),\dot{\mathrm{S}}_{\beta_1}(d_{c+1},c_r)\right\}\oplus\max\left\{\dot{\mathrm{S}}_{\alpha_1{}'}(d_{c+1},c_r),\dot{\mathrm{S}}_{\beta_1{}'}(d_{c+1},c_r)\right\}\oplus\max\left\{\ddot{\mathrm{S}}_{\gamma_1}(d_{c+1},c_r),\ddot{\mathrm{S}}_{\delta_1}(d_{c+1},c_r)\right\}\\ \oplus\dot{\pi}_e(d_{c+1},c_r)\oplus\dot{\pi}'_e(d_{c+1},c_r)\oplus\max\left\{\ddot{\mathrm{S}}_{\gamma_1{}'}(d_{c+1},c_r),\ddot{\mathrm{S}}_{\delta_1{}'}(d_{c+1},c_r)\right\}\oplus\ddot{\pi}_e(d_{c+1},c_r)\oplus\ddot{\pi}'_e(d_{c+1},c_r)\end{array}\right]}
\end{array}\right]$$

After substituting the supposed conditions we get $E(S_1)\leq E(S_2)$.

Now we make a rule to find the similarity between two GLIVIFSESs.

Let

$$M_{S_1S_2 1}(u)=\frac{\sum_{i=1}^{r}\left[min\{\dot{S}_{|\alpha}-\dot{S}_{\alpha_1|},\dot{S}_{|\beta}-\dot{S}_{\beta_1|}\}\right]}{2},$$

$$M_{S_1S_2 2}(u)=\frac{\sum_{i=1}^{r}\left[min\left\{\dot{S}_{|\alpha'}-\dot{S}_{\alpha_1{}'|},\dot{S}_{|\beta'}-\dot{S}_{\beta_1{}'|}\right\}\right]}{2},$$

$$M_{S_1S_2 3}(u)=\frac{\sum_{i=1}^{r}\left[max\{\dot{S}_{|\alpha}-\dot{S}_{\alpha_1|},\dot{S}_{|\beta}-\dot{S}_{\beta_1|}\}\right]}{2},$$

$$M_{S_1S_2 4}(u)=\frac{\sum_{i=1}^{r}\left[max\left\{\dot{S}_{|\alpha'}-\dot{S}_{\alpha_1{}'|},\dot{S}_{|\beta'}-\dot{S}_{\beta_1{}'|}\right\}\right]}{2},$$

$$M_{S_1S_2 5}(u)=\frac{\sum_{i=1}^{r}\left[min\{\ddot{S}_{|\gamma}-\ddot{S}_{\gamma_1|},\ddot{S}_{|\delta}-\ddot{S}_{\delta_1|}\}\right]}{2},$$

$$M_{S_1S_2 6}(u)=\frac{\sum_{i=1}^{r}\left[min\left\{\ddot{S}_{|\gamma'}-\ddot{S}_{\gamma_1{}'|},\ddot{S}_{|\delta'}-\ddot{S}_{\delta_1{}'|}\right\}\right]}{2},$$

$$M_{S_1S_2 7}(u)=\frac{\sum_{i=1}^{r}\left[max\{\ddot{S}_{|\gamma}-\ddot{S}_{\gamma_1|},\ddot{S}_{|\delta}-\ddot{S}_{\delta_1|}\}\right]}{2},$$

$$M_{S_1S_28}(u) = \frac{\sum_{i=1}^{r}\left[max\left\{\ddot{S}_{|\gamma'} - \ddot{S}_{{\gamma_1}'|}, \ddot{S}_{|\delta'} - \ddot{S}_{{\delta_1}'|}\right\}\right]}{2},$$

here

$$\dot{S}_{|\alpha} - \dot{S}_{\alpha_1|} = \dot{S}_{|\alpha-\alpha_1+\frac{\alpha\alpha_1}{t}|},$$

and 'r' represents the number of criteria's, 'u' represents specific alternative such that $u \in U$ where 'U' represents set of alternatives.

Now take

$\left[min\left(M_{S_1S_21}(u), M_{S_1S_22}(u)\right), max(M_{S_1S_21}(u), M_{S_1S_22}(u))\right] = [\alpha_{s_1s_2}, \alpha'_{s_1s_2}]$, $[\min(M_{S_1S_23}(u), M_{S_1S_24}(u)), \max(M_{S_1S_23}(u), M_{S_1S_24}(u))] = [\beta_{s_1s_2}, \beta'_{s_1s_2}]$, $\left[min\left(M_{S_1S_25}(u), M_{S_1S_26}(u)\right), max\left(M_{S_1S_25}(u), M_{S_1S_26}(u)\right)\right] = [\gamma_{s_1s_2}, \gamma'_{s_1s_2}]$, $\left[min\left(M_{S_1S_27}(u), M_{S_1S_28}(u)\right), max\left(M_{S_1S_27}(u), M_{S_1S_28}(u)\right)\right] = [\delta_{s_1s_2}, \delta'_{s_1s_2}]$,

such that

$$S_u = \left\{\langle \dot{s}_{[\alpha_{s_1s_2}, \alpha'_{s_1s_2}]}, \dot{s}_{[\beta_{s_1s_2}, \beta'_{s_1s_2}]}\rangle, \langle \ddot{s}_{[\gamma_{s_1s_2}, \gamma'_{s_1s_2}]}, \ddot{s}_{[\delta_{s_1s_2}, \delta'_{s_1s_2}]}\rangle\right\},$$

represents GLIVIFSES, where 'u' represents specific criteria. Similarly we find the GLIVIFSESs for all alternatives and then at the end we again apply the above technique and GLIVIFSES 'S', such that $E(S)$ represents similarity measure.

Now to illustrate that similarity measure we consider Example 4.1.1 and find entropy similarity measure and then find the score function to compare that similarity measure with previous similarity measures.

EXAMPLE 5.3.1.

By taking data from Example 4.1.1 we have the following generalized linguistic interval-valued intuitionistic fuzzy soft expert sets

$$S_1(h_1, a) = \left\{\langle \dot{s}_{[2,3]}, \dot{s}_{[1,2]}\rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\rangle\right\},$$
$$S_1(h_2, a) = \left\{\langle \dot{s}_{[3,4]}, \dot{s}_{[1,2]}\rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\rangle\right\},$$
$$S_2(h_1, b) = \left\{\langle \dot{s}_{[1,3]}, \dot{s}_{[2,3]}\rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\rangle\right\},$$
$$S_2(h_2, b) = \left\{\langle \dot{s}_{[1,2]}, \dot{s}_{[3,4]}\rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\rangle\right\}.$$

These sets represent the evaluation values by students 'a' and 'b' for the teacher t_1. Now for the second teacher t_2 the evaluation values are as under

$$\mathrm{S}_1(\mathrm{h}_1, \mathrm{a})=\{\langle \dot{\mathrm{s}}_{[1,2]}, \dot{\mathrm{s}}_{[2,4]}\rangle, \langle \ddot{\mathrm{s}}_{[1,2]}, \ddot{\mathrm{s}}_{[1,2]}\rangle\},$$
$$\mathrm{S}_1(\mathrm{h}_2, \mathrm{a})=\{\langle \dot{\mathrm{s}}_{[2,4]}, \dot{\mathrm{s}}_{[1,2]}\rangle, \langle \ddot{\mathrm{s}}_{[1,2]}, \ddot{\mathrm{s}}_{[1,2]}\rangle\},$$
$$\mathrm{S}_2(\mathrm{h}_1, \mathrm{b})=\{\langle \dot{\mathrm{s}}_{[2,3]}, \dot{\mathrm{s}}_{[2,3]}\rangle, \langle \ddot{\mathrm{s}}_{[1,2]}, \ddot{\mathrm{s}}_{[1,2]}\rangle\},$$
$$\mathrm{S}_2(\mathrm{h}_2, \mathrm{b})=\{\langle \dot{\mathrm{s}}_{[1,2]}, \dot{\mathrm{s}}_{[3,4]}\rangle, \langle \ddot{\mathrm{s}}_{[1,2]}, \ddot{\mathrm{s}}_{[1,2]}\rangle\}.$$

Now

$$M_{S_1S_2 1}(t_1) = \frac{[min\{\dot{S}_{|2} - \dot{S}_{1|}, \dot{S}_{|1} - \dot{S}_{2|}\}] \oplus [min\{\dot{S}_{|3} - \dot{S}_{1|}, \dot{S}_{|1} - \dot{S}_{3|}\}]}{2}$$
$$= \dot{\mathrm{s}}_{1.101021}$$
$$M_{S_1S_2 2}(t_1) = \frac{[min\{\dot{S}_{|3} - \dot{S}_{3|}, \dot{S}_{|2} - \dot{S}_{3|}\}] \oplus [min\{\dot{S}_{|4} - \dot{S}_{2|}, \dot{S}_{|2} - \dot{S}_{4|}\}]}{2}$$
$$= \dot{\mathrm{s}}_{0.343146}$$
$$M_{S_1S_2 3}(t_1) = \frac{[max\{\dot{S}_{|2} - \dot{S}_{1|}, \dot{S}_{|1} - \dot{S}_{2|}\}] \oplus [max\{\dot{S}_{|3} - \dot{S}_{1|}, \dot{S}_{|1} - \dot{S}_{3|}\}]}{2}$$
$$= \dot{\mathrm{s}}_{1.9585}$$
$$M_{S_1S_2 4}(t_1) = \frac{[max\{\dot{S}_{|3} - \dot{S}_{3|}, \dot{S}_{|2} - \dot{S}_{3|}\}] \oplus [max\{\dot{S}_{|4} - \dot{S}_{2|}, \dot{S}_{|2} - \dot{S}_{4|}\}]}{2}$$
$$= \dot{\mathrm{s}}_{2.535898}$$
$$M_{S_1S_2 5}(t_1) = \frac{[min\{\ddot{S}_{|1} - \ddot{S}_{1|}, \ddot{S}_{|1} - \ddot{S}_{1|}\}] \oplus [min\{\ddot{S}_{|1} - \ddot{S}_{1|}, \ddot{S}_{|1} - \ddot{S}_{1|}\}]}{2}$$
$$= \ddot{\mathrm{s}}_{\frac{1}{4}}$$
$$M_{S_1S_2 6}(t_1) = \frac{[min\{\ddot{S}_{|2} - \ddot{S}_{2|}, \ddot{S}_{|2} - \ddot{S}_{2|}\}] \oplus [min\{\ddot{S}_{|2} - \ddot{S}_{2|}, \ddot{S}_{|2} - \ddot{S}_{2|}\}]}{2}$$
$$= \ddot{\mathrm{s}}_{1}$$
$$M_{S_1S_2 7}(t_1) = \frac{[max\{\ddot{S}_{|1} - \ddot{S}_{1|}, \ddot{S}_{|1} - \ddot{S}_{1|}\}] \oplus [max\{\ddot{S}_{|1} - \ddot{S}_{1|}, \ddot{S}_{|1} - \ddot{S}_{1|}\}]}{2}$$
$$= \ddot{\mathrm{s}}_{\frac{1}{4}}$$
$$M_{S_1S_2 8}(t_1) = \frac{[min\{\ddot{S}_{|2} - \ddot{S}_{2|}, \ddot{S}_{|2} - \ddot{S}_{2|}\}] \oplus [min\{\ddot{S}_{|2} - \ddot{S}_{2|}, \ddot{S}_{|2} - \ddot{S}_{2|}\}]}{2}$$
$$= \ddot{\mathrm{s}}_{1}$$

thus

$$S_{t_1} = \left\{\langle \dot{s}_{[0.343146, 1.101021]}, \dot{s}_{[1.9585, 2.535898]}\rangle, \langle \ddot{s}_{\left[\frac{1}{4}, \frac{1}{4}\right]}, \ddot{s}_{[1,1]}\rangle\right\}.$$

Now to find the GLIVIFSES S_{t_2} we have

$$M_{S_1S_21}(t_2)=\frac{\left[min\{\dot{S}_{|1}-\dot{S}_{2|},\dot{S}_{|2}-\dot{S}_{2|}\}\right]\oplus\left[min\{\dot{S}_{|2}-\dot{S}_{1|},\dot{S}_{|1}-\dot{S}_{3|}\}\right]}{2}$$

$$=\dot{s}_{0.78251}$$

$$M_{S_1S_22}(t_2)=\frac{\left[min\{\dot{S}_{|2}-\dot{S}_{3|},\dot{S}_{|4}-\dot{S}_{3|}\}\right]\oplus\left[min\{\dot{S}_{|4}-\dot{S}_{2|},\dot{S}_{|2}-\dot{S}_{4|}\}\right]}{2}$$

$$=\dot{s}_{0.343146}$$

$$M_{S_1S_23}(t_2)=\frac{\left[max\{\dot{S}_{|1}-\dot{S}_{2|},\dot{S}_{|2}-\dot{S}_{2|}\}\right]\oplus\left[max\{\dot{S}_{|2}-\dot{S}_{1|},\dot{S}_{|1}-\dot{S}_{3|}\}\right]}{2}$$

$$=\dot{s}_{1.58412}$$

$$M_{S_1S_24}(t_2)=\frac{\left[max\{\dot{S}_{|2}-\dot{S}_{3|},\dot{S}_{|4}-\dot{S}_{3|}\}\right]\oplus\left[max\{\dot{S}_{|4}-\dot{S}_{2|},\dot{S}_{|2}-\dot{S}_{4|}\}\right]}{2}$$

$$=\dot{s}_{3.17157}$$

$$M_{S_1S_25}(t_2)=\frac{\left[min\{\ddot{S}_{|1}-\ddot{S}_{1|},\ddot{S}_{|1}-\ddot{S}_{1|}\}\right]\oplus\left[min\{\ddot{S}_{|1}-\ddot{S}_{1|},\ddot{S}_{|1}-\ddot{S}_{1|}\}\right]}{2}$$

$$=\ddot{s}_{\frac{1}{4}}$$

$$M_{S_1S_26}(t_2)=\frac{\left[min\{\ddot{S}_{|2}-\ddot{S}_{2|},\ddot{S}_{|2}-\ddot{S}_{2|}\}\right]\oplus\left[min\{\ddot{S}_{|2}-\ddot{S}_{2|},\ddot{S}_{|2}-\ddot{S}_{2|}\}\right]}{2}$$

$$=\ddot{s}_1$$

$$M_{S_1S_27}(t_2)=\frac{\left[max\{\ddot{S}_{|1}-\ddot{S}_{1|},\ddot{S}_{|1}-\ddot{S}_{1|}\}\right]\oplus\left[max\{\ddot{S}_{|1}-\ddot{S}_{1|},\ddot{S}_{|1}-\ddot{S}_{1|}\}\right]}{2}$$

$$=\ddot{s}_{\frac{1}{4}}$$

$$M_{S_1S_28}(t_2)=\frac{\left[min\{\ddot{S}_{|2}-\ddot{S}_{2|},\ddot{S}_{|2}-\ddot{S}_{2|}\}\right]\oplus\left[min\{\ddot{S}_{|2}-\ddot{S}_{2|},\ddot{S}_{|2}-\ddot{S}_{2|}\}\right]}{2}$$

$$=\ddot{s}_1$$

thus

$$S_{t_2}=\left\{\langle\dot{s}_{[0.343146,0.78251]},\dot{s}_{[1.58412,3.17157]}\rangle,\langle\ddot{s}_{\left[\frac{1}{4},\frac{1}{4}\right]},\ddot{s}_{[1,1]}\rangle\right\}.$$

Now to find the GLIVIFSES ‘S’ we have

$$M_{S_1S_21}(t_2)=\frac{\left[min\{\dot{s}_{|0.343146}-\dot{s}_{0.343146|},\dot{s}_{|1.9585}-\dot{s}_{1.58412|}\}\right]}{2}=\dot{s}_{0.0098205},$$

$$M_{S_1S_22}(t_1)=\frac{\left[min\{\dot{s}_{|1.101021}-\dot{s}_{0.78251|},\dot{s}_{|2.535898}-\dot{s}_{3.17157|}\}\right]}{2}=\dot{s}_{0.23568},$$

$$M_{S_1S_23}(t_2)=\frac{\left[max\{\dot{s}_{|0.343146}-\dot{s}_{0.343146|},\dot{s}_{|1.9585}-\dot{s}_{1.58412|}\}\right]}{2}=\dot{s}_{0.07178},$$

$$M_{S_1S_24}(t_1)=\frac{\left[max\{\dot{s}_{|1.101021}-\dot{s}_{0.78251|},\dot{s}_{|2.535898}-\dot{s}_{3.17157|}\}\right]}{2}=\dot{s}_{0.3634},$$

$$M_{S_1S_25}(t_2)=\frac{\left[min\left\{\ddot{s}_{|\frac{1}{4}}-\ddot{s}_{\frac{1}{4}|},\ddot{s}_{|1}-\ddot{s}_{1|}\right\}\right]}{2}=\ddot{s}_{0.00782},$$

$$M_{S_1S_26}(t_2) = \frac{\left[min\left\{\ddot{s}_{|\frac{1}{4}}-\ddot{s}_{\frac{1}{4}|}, \ddot{s}_{|1}-\ddot{s}_{1|}\right\}\right]}{2} = \ddot{s}_{0.00782},$$

$$M_{S_1S_27}(t_2) = \frac{\left[max\left\{\ddot{s}_{|\frac{1}{4}}-\ddot{s}_{\frac{1}{4}|}, \ddot{s}_{|1}-\ddot{s}_{1|}\right\}\right]}{2} = \ddot{s}_{0.127017},$$

$$M_{S_1S_28}(t_2) = \frac{\left[max\left\{\ddot{s}_{|\frac{1}{4}} - \ddot{s}_{\frac{1}{4}|}, \ddot{s}_{|1} - \ddot{s}_{1|}\right\}\right]}{2} = \ddot{s}_{0.127017},$$

so,

$$S = \left\{\langle \dot{s}_{[0.0098205,0.23568]}, \dot{s}_{[0.07178,0.3634]}\rangle, \langle \ddot{s}_{[0.00782,0.00782]}, \ddot{s}_{[0.127017,0.127017]}\rangle\right\}.$$

Now by applying entropy measure on ‘S’ we get

$$E(S) = \left[\frac{\begin{bmatrix}\min\{\dot{s}_{0.0098205}, \dot{s}_{0.07178}\}\oplus\min\{\dot{s}_{0.23568}, \dot{s}_{0.3634})\}\oplus\min\{\ddot{s}_{0.00782}, \ddot{s}_{0.127017}\} \oplus \dot{s}_{5.9183995}\oplus\dot{s}_{5.40092} \\ \oplus\min\{\ddot{s}_{0.00782}, \ddot{s}_{0.127017}\}\oplus\ddot{s}_{3.865163}\oplus\ddot{s}_{3.865163}\end{bmatrix}}{\begin{bmatrix}\max\{\dot{s}_{0.0098205}, \dot{s}_{0.07178}\}\oplus\max\{\dot{s}_{0.23568}, \dot{s}_{0.3634})\}\oplus\max\{\ddot{s}_{0.00782}, \ddot{s}_{0.127017}\} \oplus \dot{s}_{5.9183995}\oplus\dot{s}_{5.40092} \\ \oplus\max\{\ddot{s}_{0.00782}, \ddot{s}_{0.127017}\}\oplus\ddot{s}_{3.865163}\oplus\ddot{s}_{3.865163}\end{bmatrix}}\right],$$

which after simplifications we obtain

$$= \frac{\dot{s}_{5.99739}\oplus\ddot{s}_{3.99547}}{\dot{s}_{5.992437}\oplus\ddot{s}_{3.995739}} = \frac{s_{5.99739}}{s_{5.99747}} = s_{5.99746}.$$

Now by comparing that similarity measure we can observe that the result obtained by entropy similarity measure is highest in order then the previous similarity measures techniques, also we can observe from theorem that $S_1$and S_2 are significantly similar.

5.4. DISSIMILARITY MEASURE FOR GLIVIFSESs.

We now present the dissimilarity measure for generalized linguistic interval valued intuitionistic fuzzy soft expert sets to compare these sets, for that purpose we utilized the idea of Li 2004 [6] up to some sense because it measures the dissimilarity in case of intuitionistic fuzzy sets which are some numerical numbers lies between the range of 0 and 1 and our sets contains linguistic terms which are totally different from numerical numbers with differences in their operators for aggregation.

Definition. Let $U = \{u_i : i = 1, \ldots, n\}$ represents the set of alternatives and $C = \{c_j : j = 1, \ldots, r\}$ represents the set of criteria's fixed for decision making process and $E = \{e_k : k = 1, \ldots, m\}$ represents the set of decision maker's.

According to general form of GLIVIFSESs

$$S=\left\{\langle \dot{s}_{[\alpha,\alpha']}, \dot{s}_{[\beta,\beta']}\rangle, \langle \ddot{s}_{[\gamma,\gamma']}, \ddot{s}_{[\delta,\delta']}\rangle\right\}.$$

For each $u_i \in U$ the dissimilarity measure between GLIVIFSESs is defined as

$$DSM(S_a, S_b) = \sum_{j=1}^{r} \left\{ \langle \dot{s}(c_j)_{\left[\left|\alpha'-\alpha_1'+\frac{\alpha'\alpha_1'}{t}\right|, \left|\alpha-\alpha_1+\frac{\alpha\alpha_1}{t}\right|\right]}, \dot{s}(c_j)_{\left[\frac{\beta\beta_1}{t}, \frac{\beta'\beta_1'}{t}\right]}\rangle, \langle \ddot{s}(c_j)_{\left[\left|\gamma-\gamma_1+\frac{\gamma\gamma_1}{t'}\right|, \left|\gamma'-\gamma_1'+\frac{\gamma'\gamma_1'}{t'}\right|\right]}, \ddot{s}(c_j)_{\left[\frac{\delta\delta_1}{t'}, \frac{\delta'\delta_1'}{t'}\right]}\rangle \right\},$$

now to demonstrate the above listed methodology we solve a practical problem related to dissimilarity.

EXAMPLE 5.4.1.

X government wants to improve the educational facilities in rural areas of X state which are almost approaches to negligence due to many features such as corruption (which is the major problem in facilitating the educational facilities to the child of rural areas), lack of recruitments on merits (which is linked with corruption because by taking illegal money from candidates members of X state authorities bring out the testing paper's before the examination dates through which ineligible persons get recruited), fake or bugs degrees (these degrees are awarded by authorities in examination centers by illegal contents with peoples who wanted to avail these degrees).

To achieve these objectives X government education minister conduct a survey with one person e_1 from civilian and one e_2 person from education department such that $\{e_1, e_2\}$ represents a set of experts and allows them to make judgments about SED (school education department) on the behalf of criteria's

e. Teaching capability of teaching staff

f. Check and balance system

the second restriction on experts that along with judgments about SED also provide the information regarding their linkage with that department.

Now the secret society of X government makes a strategy based on the dissimilarity between the opinions of experts because a person nominated from SED will give its judgments in the favor of fellow colleagues but the civilian who one is not availing any benefit from department but its relatives or either he/she is student. They fixed the dissimilarity level 20% such that if it is greater than 20% then the actions will be taken against the SED authorities.

The judgments of experts will in the form of generalized linguistic interval valued intuitionistic fuzzy soft expert sets because they use linguistic terms to express their opinions with generalization on 2-D which represents their linkage information. Here we take predefined linguistic term set S_t with variation of $'t'$ as $0 \leq t \leq 5$ and predefined linguistic term set $S_{t'}$ for 2-D linguistic information with variation of t' as $0 \leq t' \leq 6$.

The opinions of experts are as under

$$S_1(c_1, e_1)=\{\langle \dot{s}_{[1,2]}, \dot{s}_{[2,3]} \rangle, \langle \ddot{s}_{[3,4]}, \ddot{s}_{[1,2]} \rangle\},$$
$$S_1(c_2, e_1)=\{\langle \dot{s}_{[0,1]}, \dot{s}_{[2,3]} \rangle, \langle \ddot{s}_{[2,3]}, \ddot{s}_{[1,2]} \rangle\},$$
$$S_2(c_1, e_2)=\{\langle \dot{s}_{[3,4]}, \dot{s}_{[0,1]} \rangle, \langle \ddot{s}_{[3,4]}, \ddot{s}_{[0,1]} \rangle\},$$
$$S_2(c_2, e_2)=\{\langle \dot{s}_{[3.5,4]}, \dot{s}_{[0,1]} \rangle, \langle \ddot{s}_{[3,4.5]}, \ddot{s}_{[0,1]} \rangle\},$$

By substituting these values in dissimilarity expression, we have

$$DSM(S_1, S_2) = \left[\begin{array}{c} \left\{\langle \dot{s}(c_1)_{\left[\left|2-4+\frac{8}{5}\right|, \left|1-3+\frac{3}{5}\right|\right]}, \dot{s}(c_1)_{\left[0, \frac{3}{5}\right]} \rangle' \langle \ddot{s}(c_1)_{\left[\left|3-3+\frac{9}{6}\right|, \left|4-4+\frac{16}{6}\right|\right]}, \ddot{s}(c_1)_{\left[0, \frac{4}{6}\right]} \rangle\right\} \oplus \\ \left\{\langle \dot{s}(c_2)_{\left[\left|1-4+\frac{4}{5}\right|, \left|0-3.5+0\right|\right]}, \dot{s}(c_2)_{\left[0, \frac{3}{5}\right]} \rangle' \langle \ddot{s}(c_2)_{\left[\left|2-3+\frac{6}{6}\right|, \left|3-4.5+\frac{13.5}{6}\right|\right]}, \ddot{s}(c_2)_{\left[0, \frac{2}{6}\right]} \rangle\right\} \end{array}\right],$$

after evaluating the operation of addition we get

$$DSM(S_1, S_2) = \{\langle \dot{s}_{[2.424,3.92]}, \dot{s}_{[0,0.072]} \rangle, \langle \ddot{s}_{[1.5,3.0833]}, \ddot{s}_{[0,0.037037]} \rangle\}.$$

From above expression it is clear that dissimilarity between opinions of experts is greater than 20% or in other words it is greater than s_1 because s_t the pick value in linguistic term set is s_5. So the decisions will be taken out against the authorities of X state school education department.

5.5. CORRELATION OF GLIVIFSESs.

Correlation of generalized linguistic IVIFSESs is basically measure of similarity between them, for that we used the sense of correlation of intuitionistic fuzzy sets [7] which represents the amount of correlation between these sets.

Definition. Let $U = \{u_i : i = 1, \ldots, n\}$ represents a set of alternatives and $C = \{c_j : j = 1, \ldots, m\}$ represents a set of different criteria's such that

$$S_1 = \left\{\langle \dot{s}_{[\alpha,\alpha']}, \dot{s}_{[\beta,\beta']}\rangle, \langle \ddot{s}_{[\gamma,\gamma']}, \ddot{s}_{[\delta,\delta']}\rangle\right\},$$
$$S_2 = \left\{\langle \dot{s}_{[\alpha_1,\alpha_1']}, \dot{s}_{[\beta_1,\beta_1']}\rangle, \langle \ddot{s}_{[\gamma_1,\gamma_1']}, \ddot{s}_{[\delta_1,\delta_1']}\rangle\right\},$$

represents two different GLIVIFSESs. Now for a specific alternative u_i the correlation of S_1 and S_2 denoted as $C_{u_i}(S_1, S_2)$ defined as

$$C_{u_i}(S_1, S_2) = \sum_{j=1}^{m} \begin{bmatrix} \langle \left(\dot{s}(c_j)_{[\alpha,\alpha']} \otimes \dot{s}(c_j)_{[\alpha_1,\alpha_1']}\right), \left(\dot{s}(c_j)_{[\beta,\beta']} \otimes \dot{s}(c_j)_{[\beta_1,\beta_1']}\right)\rangle, \\ \langle \left(\ddot{s}(c_j)_{[\gamma,\gamma']} \otimes \ddot{s}(c_j)_{[\gamma_1,\gamma_1']}\right), \left(\ddot{s}(c_j)_{[\delta,\delta']} \otimes \ddot{s}(c_j)_{[\delta_1,\delta_1']}\right)\rangle \end{bmatrix} \quad (5.5)$$

similarly for each alternative u_i we calculate $C_{u_i}(S_1, S_2)$ where $i = 1,2,3, \ldots, m$.

Now for correlation of (S_1, S_2) for all alternatives we use

$$C(S_1, S_2) = \sum_{i=1}^{n} C_{u_i}(S_1, S_2) \rightarrow (B)$$

To demonstrate the above listed methodology we consider Example 4.1.1 for comparison reasons and to illustrate the above methodology.

EXAMPLE 5.5.1.

Taking data from Example 4.1.1 we have

$$S_1(h_1, a) = \left\{\langle \dot{s}_{[2,3]}, \dot{s}_{[1,2]}\rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\rangle\right\},$$
$$S_1(h_2, a) = \left\{\langle \dot{s}_{[3,4]}, \dot{s}_{[1,2]}\rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\rangle\right\},$$
$$S_2(h_1, b) = \left\{\langle \dot{s}_{[1,3]}, \dot{s}_{[2,3]}\rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\rangle\right\},$$

$$\mathrm{S}_2(\mathrm{h}_2,\mathrm{b})=\left\{\langle\dot{\mathrm{s}}_{[1,2]},\dot{\mathrm{s}}_{[3,4]}\rangle,\langle\ddot{\mathrm{s}}_{[1,2]},\ddot{\mathrm{s}}_{[1,2]}\rangle\right\}.$$

These sets represent the evaluation values by students 'a' and 'b' for the teacher t_1. Now for the second teacher t_2 the evaluation values are as under

$$\begin{aligned}
\mathrm{S}_1(\mathrm{h}_1,\mathrm{a})&=\left\{\langle\dot{\mathrm{s}}_{[1,2]},\dot{\mathrm{s}}_{[2,4]}\rangle,\langle\ddot{\mathrm{s}}_{[1,2]},\ddot{\mathrm{s}}_{[1,2]}\rangle\right\},\\
\mathrm{S}_1(\mathrm{h}_2,\mathrm{a})&=\left\{\langle\dot{\mathrm{s}}_{[2,4]},\dot{\mathrm{s}}_{[1,2]}\rangle,\langle\ddot{\mathrm{s}}_{[1,2]},\ddot{\mathrm{s}}_{[1,2]}\rangle\right\},\\
\mathrm{S}_2(\mathrm{h}_1,\mathrm{b})&=\left\{\langle\dot{\mathrm{s}}_{[2,3]},\dot{\mathrm{s}}_{[2,3]}\rangle,\langle\ddot{\mathrm{s}}_{[1,2]},\ddot{\mathrm{s}}_{[1,2]}\rangle\right\},\\
\mathrm{S}_2(\mathrm{h}_2,\mathrm{b})&=\left\{\langle\dot{\mathrm{s}}_{[1,2]},\dot{\mathrm{s}}_{[3,4]}\rangle,\langle\ddot{\mathrm{s}}_{[1,2]},\ddot{\mathrm{s}}_{[1,2]}\rangle\right\}.
\end{aligned}$$

Now for alternative t_1

$$\mathrm{C}_{t_1}(\mathrm{S}_1,\mathrm{S}_2)=\left[\begin{array}{c}\left\{\begin{array}{l}\langle\left(\dot{s}(h_1)_{[2,3]}\otimes\dot{s}(h_1)_{[1,3]}\right),\left(\dot{s}(h_1)_{[1,2]}\otimes\dot{s}(h_1)_{[2,3]}\right)\rangle,\\ \langle\left(\ddot{s}(h_1)_{[1,2]}\otimes\ddot{s}(h_1)_{[1,2]}\right),\left(\ddot{s}(h_1)_{[1,2]}\otimes\ddot{s}(h_1)_{[1,2]}\right)\rangle\end{array}\right\}\oplus\\ \left\{\begin{array}{l}\langle\left(\dot{s}(h_2)_{[3,4]}\otimes\dot{s}(h_2)_{[1,2]}\right),\left(\dot{s}(h_2)_{[1,2]}\otimes\dot{s}(h_2)_{[3,4]}\right)\rangle,\\ \langle\left(\ddot{s}(h_2)_{[1,2]}\otimes\ddot{s}(h_2)_{[1,2]}\right),\left(\ddot{s}(h_2)_{[1,2]}\otimes\ddot{s}(h_2)_{[1,2]}\right)\rangle\end{array}\right\}\end{array}\right]$$

$$=\left\{\langle\dot{\mathrm{s}}_{\left[\frac{47}{72},\frac{5}{2}\right]},\dot{\mathrm{s}}_{\left[\frac{14}{9},\frac{28}{9}\right]}\rangle,\langle\ddot{\mathrm{s}}_{\left[\frac{31}{64},\frac{7}{4}\right]},\ddot{\mathrm{s}}_{\left[\frac{49}{64},\frac{9}{4}\right]}\rangle\right\}.$$

Similarly for alternative t_2

$$\mathrm{C}_{t_2}(\mathrm{S}_1,\mathrm{S}_2)=\left[\begin{array}{c}\left\{\begin{array}{l}\langle\left(\dot{s}(h_1)_{[1,2]}\otimes\dot{s}(h_1)_{[1,3]}\right),\left(\dot{s}(h_1)_{[2,4]}\otimes\dot{s}(h_1)_{[2,3]}\right)\rangle,\\ \langle\left(\ddot{s}(h_1)_{[1,2]}\otimes\ddot{s}(h_1)_{[1,2]}\right),\left(\ddot{s}(h_1)_{[1,2]}\otimes\ddot{s}(h_1)_{[1,2]}\right)\rangle\end{array}\right\}\oplus\\ \left\{\begin{array}{l}\langle\left(\dot{s}(h_2)_{[2,4]}\otimes\dot{s}(h_2)_{[1,2]}\right),\left(\dot{s}(h_2)_{[1,2]}\otimes\dot{s}(h_2)_{[3,4]}\right)\rangle,\\ \langle\left(\ddot{s}(h_2)_{[1,2]}\otimes\ddot{s}(h_2)_{[1,2]}\right),\left(\ddot{s}(h_2)_{[1,2]}\otimes\ddot{s}(h_2)_{[1,2]}\right)\rangle\end{array}\right\}\end{array}\right]$$

$$=\left\{\langle\dot{\mathrm{s}}_{\left[\frac{53}{108},\frac{19}{9}\right]},\dot{\mathrm{s}}_{\left[\frac{35}{18},\frac{35}{9}\right]}\rangle,\langle\ddot{\mathrm{s}}_{\left[\frac{31}{64},\frac{7}{4}\right]},\ddot{\mathrm{s}}_{\left[\frac{49}{64},\frac{9}{4}\right]}\rangle\right\}.$$

Now for correlation of $(\mathrm{S}_1,\mathrm{S}_2)$ for all alternatives we use Equation (B) so, that

$$\mathrm{C}(\mathrm{S}_1,\mathrm{S}_2)=\left\{\langle\dot{\mathrm{s}}_{\left[\frac{50861}{46656},\frac{403}{108}\right]},\dot{\mathrm{s}}_{\left[\frac{245}{486},\frac{490}{243}\right]}\rangle,\langle\ddot{\mathrm{s}}_{\left[\frac{14911}{16384},\frac{175}{64}\right]},\ddot{\mathrm{s}}_{\left[\frac{2401}{16384},\frac{81}{64}\right]}\rangle\right\}.$$

Now to compare that with the previous similarity types we use the score and accuracy function. According to score function we have

$$\mathrm{S}\left(\mathrm{C}(\mathrm{S}_1,\mathrm{S}_2)\right)=\mathrm{s}_{[0.5488,0.64292]\times[0.5954,0.6836]}=\mathrm{s}_{[0.36268,0.4395]}.$$

To compare it with previous score function results we take the average value from the interval such that

$$\mathrm{S}\big(\mathrm{C}(\mathrm{S}_1, \mathrm{S}_2)\big) = \mathrm{s}_{0.40109},$$

now by comparing that score function result with previous results of score functions we can observe easily that its order is greater than Type-I, Type-II, Type-III and Type-IV measures.

5.6. LINGUISTIC FUZZY IMPLICATION FOR DISTANCE MEASURE BETWEEN GLIVIFSESs.

Fuzzy implication [10] which is basically a simple extension of classical implication from the domain set $\{0,1\}$ to the domain $[0,1]$. Operator of classical implication is a map

$$q: \{0,1\} \times \{0,1\} \to \{0,1\},$$

this satisfies the following conditions

- $q(0,0) = q(0,1) = q(1,1) = 1$
- $q(1,0) = 0.$

Similarly fuzzy implication is a map

$$t_{\Rightarrow}: [0,1] \times [0,1] \to [0,1],$$

with conditions

- $t_{\Rightarrow}(0,1) = t_{\Rightarrow}(1,1) = t_{\Rightarrow}(0,0) = 1$
- $t_{\Rightarrow}(1,0) = 0.$

We now extend the idea of classical implication and fuzzy implication into "classical linguistic implication" and "linguistic fuzzy implication" respectively with term set $\{s_0, s_t\}$ and $[s_0, s_t]$ respectively, where $t \in \mathbb{Z}^+$ with odd cardinality.

The classical linguistic implication is simply a map with following properties

$$w: \{s_0, s_t\} \times \{s_0, s_t\} \to \{s_0, s_t\},$$

- $w(s_0, s_0) = (s_0, s_t) = (s_t, s_t) = s_t$
- $w(s_1, s_0) = s_0.$

Similarly fuzzy linguistic implication is a map

$$x_{\Rightarrow}: [s_0, s_t] \times [s_0, s_t] \to [s_0, s_t],$$

with following properties

- $x_{\Rightarrow}(s_0, s_t) = x_{\Rightarrow}(s_t, s_t) = x_{\Rightarrow}(s_0, s_0) = s_t$
- $x_{\Rightarrow}(s_t, s_0) = s_0$

from these above boundary conditions the following properties must hold.

If $s_{i,}s_j \in S_t$ with the property $s_0 \leq s_i \leq s_j \neq s_t$ then for any arbitrary $s_x \in S_t$

- $x_{\Rightarrow}(s_i, s_x) \geq x_{\Rightarrow}(s_j, s_x)$
- $x_{\Rightarrow}(s_x, s_i) \leq x_{\Rightarrow}(s_x, s_j)$
- $x_{\Rightarrow}(s_t, s_x) = s_x$ & $x_{\Rightarrow}(s_0, s_x) = s_t$
- $x_{\Rightarrow}(s_i, s_j) = s_t$.

The extension of Mamdani rule [11] and Larsen rule [12] as $x_{\Rightarrow}(a, b) = min\{a, b\}$ *into* $x_{\Rightarrow}(s_i, s_j) = min\{s_i, s_j\}$ & $x_{\Rightarrow}(a, b) = a.b$ *into* $x_{\Rightarrow}(s_i, s_j) = s_i.s_j$ respectively which contradicts with fuzzy implication and linguistic fuzzy implication, we call these as "engineering implications with linguistic approach".

Now we use the concept of linguistic fuzzy implication to define a distance measure between GLIVIFSESs, the idea of distance measure using fuzzy implications and norms was proposed by Hatzimichailidis et al. for fuzzy sets [13] and intuitionistic fuzzy sets [14] which is defined as under

$$d(A, B, t_{\Rightarrow}) = \left\| \mathbb{I}_{\alpha_A} - \mathbb{I}_{\alpha_B} \right\| + \left\| \mathbb{I}_{\beta_A} - \mathbb{I}_{\beta_B} \right\|,$$

where α_A represents a membership function of a set A, α_B represents a membership function of B. similarly β_A and β_B represents non-membership function of A and B respectively with the property that $\alpha_A(i) + \beta_A(i) \leq 1$ also $\alpha_B(i) + \beta_B(i) \leq 1$ where $\alpha_A(i), \beta_A(i), \alpha_B(i), \beta_B(i) \in [0,1]$.

Where

$$\mathbb{I}_{\alpha_A} = \left[t \underset{i=1,2,\dots,n}{\Rightarrow} \big(\alpha_A(x_i), \alpha_A(x_i)\big) \right] = t_{\Rightarrow}\left(\begin{bmatrix} \alpha_A(x_1) \\ \alpha_A(x_2) \\ \alpha_A(x_3) \\ . \\ . \\ . \\ \alpha_A(x_n) \end{bmatrix}, [\alpha_A(x_1), \alpha_A(x_2), \alpha_A(x_3), \dots, \alpha_A(x_n)] \right)$$

$$=\begin{bmatrix} t_{\Rightarrow}\big(\alpha_A(x_1),\alpha_A(x_1)\big) & t_{\Rightarrow}\big(\alpha_A(x_1),\alpha_A(x_2)\big) & \cdots & t_{\Rightarrow}\big(\alpha_A(x_1),\alpha_A(x_n)\big) \\ t_{\Rightarrow}\big(\alpha_A(x_2),\alpha_A(x_1)\big) & t_{\Rightarrow}\big(\alpha_A(x_2),\alpha_A(x_2)\big) & \cdots & t_{\Rightarrow}\big(\alpha_A(x_2),\alpha_A(x_n)\big) \\ t_{\Rightarrow}\big(\alpha_A(x_3),\alpha_A(x_1)\big) & t_{\Rightarrow}\big(\alpha_A(x_3),\alpha_A(x_2)\big) & \cdots & t_{\Rightarrow}\big(\alpha_A(x_3),\alpha_A(x_n)\big) \\ \vdots & \vdots & \vdots & \vdots \\ t_{\Rightarrow}\big(\alpha_A(x_n),\alpha_A(x_1)\big) & t_{\Rightarrow}\big(\alpha_A(x_n),\alpha_A(x_2)\big) & \cdots & t_{\Rightarrow}\big(\alpha_A(x_n),\alpha_A(x_n)\big) \end{bmatrix},$$

here "n" represents the number of alternatives present in universal set or in other words it represents order of universal set. Similarly for a membership function of fuzzy set B.

Now

$$\mathbb{III}_{\beta_A}=\left[t\underset{i=1,2,\dots,n}{\Rightarrow}\Big(\beta_A(x_i),\beta_A(x_i)\Big)\right]=t_{\Rightarrow}\left(\begin{bmatrix}\beta_A(x_1)\\ \beta_A(x_2)\\ \beta_A(x_3)\\ \vdots\\ \beta_A(x_n)\end{bmatrix},\left[\beta_A(x_1),\beta_A(x_2),\beta_A(x_3),\dots,\beta_A(x_n)\right]\right)$$

$$=\begin{bmatrix} t_{\Rightarrow}\Big(\beta_A(x_1),\beta_A(x_1)\Big) & t_{\Rightarrow}\Big(\beta_A(x_1),\beta_A(x_2)\Big) & \cdots & t_{\Rightarrow}\Big(\beta_A(x_1),\beta_A(x_n)\Big) \\ t_{\Rightarrow}\Big(\beta_A(x_2),\beta_A(x_1)\Big) & t_{\Rightarrow}\Big(\beta_A(x_2),\beta_A(x_2)\Big) & \cdots & t_{\Rightarrow}\Big(\beta_A(x_2),\beta_A(x_n)\Big) \\ t_{\Rightarrow}\Big(\beta_A(x_3),\beta_A(x_1)\Big) & t_{\Rightarrow}\Big(\beta_A(x_3),\beta_A(x_2)\Big) & \cdots & t_{\Rightarrow}\Big(\beta_A(x_3),\beta_A(x_n)\Big) \\ \vdots & \vdots & \vdots & \vdots \\ t_{\Rightarrow}\Big(\beta_A(x_n),\beta_A(x_1)\Big) & t_{\Rightarrow}\Big(\beta_A(x_n),\beta_A(x_2)\Big) & \cdots & t_{\Rightarrow}\Big(\beta_A(x_n),\beta_A(x_n)\Big) \end{bmatrix},$$

similarly for a non-membership function of fuzzy set B.

Now we extend that idea in case of linguistic approach, suppose S_1 and S_2 are two GLIVIFSESs defined as

$$\mathrm{S}_1=\Big\{\langle \dot{s}_{[\alpha,\alpha']},\dot{s}_{[\beta,\beta']}\rangle\ ,\langle \ddot{s}_{[\gamma,\gamma']}\ ,\ddot{s}_{[\delta,\delta']}\rangle\Big\},$$
$$\mathrm{S}_2=\Big\{\langle \dot{s}_{[\alpha_1,\alpha_1']},\dot{s}_{[\beta_1,\beta_1']}\rangle\ ,\langle \ddot{s}_{[\gamma_1,\gamma_1']}\ ,\ddot{s}_{[\delta_1,\delta_1']}\rangle\Big\},$$

with $C=\{c_j\colon j=1,\dots,r\}$ represents a set of criteria's and $X=\{x_i\colon i=1,\dots,n\}$ represents a set of alternatives.

Here instead of $\alpha_A(x_i)$ and $\beta_A(x_i)$ we have

$$\mathrm{S}_1(\mathrm{c}_1,\mathrm{x}_\mathrm{i})=\Big\{\langle \dot{s}_{[\alpha_2,\alpha'_2]},\dot{s}_{[\beta_2,\beta'_2]}\rangle\ ,\langle \ddot{s}_{[\gamma_2,\gamma'_2]}\ ,\ddot{s}_{[\delta_2,\delta'_2]}\rangle\Big\},$$

$$S_1(c_2, x_i)=\left\{\langle \dot{s}_{[\alpha_3,\alpha'_3]}, \dot{s}_{[\beta_3,\beta'_3]}\rangle\ ,\langle \ddot{s}_{[\gamma_3,\gamma'_3]}\ ,\ddot{s}_{[\delta_3,\delta'_3]}\rangle\right\},$$
$$S_1(c_3, x_i)=\left\{\langle \dot{s}_{[\alpha_4,\alpha'_4]}, \dot{s}_{[\beta_4,\beta'_4]}\rangle\ ,\langle \ddot{s}_{[\gamma_4,\gamma'_4]}\ ,\ddot{s}_{[\delta_4,\delta'_4]}\rangle\right\},$$

•

•

•

$$S_1(c_r, x_i)=\left\{\langle \dot{s}_{[\alpha_{r+1},\alpha'_{r+1}]}, \dot{s}_{[\beta_{r+1},\beta'_{r+1}]}\rangle\ ,\langle \ddot{s}_{[\gamma_{r+1},\gamma'_{r+1}]}\ ,\ddot{s}_{[\delta_{r+1},\delta'_{r+1}]}\rangle\right\}.$$

Now we take the average for all the values obtained by different criteria's such that for x_i the membership and non-membership function in the form of GLIVIFSES is defined as

$$= \frac{\sum_{j=1}^{r} S_1(c_j, x_i)}{r}.$$

Now the distance between GLIVIFSESs using linguistic fuzzy implication is defined as

$$d(S_1, S_2, x_{\Rightarrow}) = \left\|\mathbb{M}_{S_1} - \mathbb{M}_{S_2}\right\| \qquad (5.6)$$

with

$$\mathbb{M}_{S_1} = \left[\underset{i=1,2,\dots,n}{x_{\Rightarrow}} \left(\frac{\sum_{j=1}^{r} S_1(c_j, x_i)}{r}, \frac{\sum_{j=1}^{r} S_1(c_j, x_i)}{r} \right) \right] = x_{\Rightarrow} \left(\begin{bmatrix} \frac{\sum_{j=1}^{r} S_1(c_j, x_1)}{r} \\ \frac{\sum_{j=1}^{r} S_1(c_j, x_2)}{r} \\ \frac{\sum_{j=1}^{r} S_1(c_j, x_3)}{r} \\ \vdots \\ \frac{\sum_{j=1}^{r} S_1(c_j, x_n)}{r} \end{bmatrix}, \left[\frac{\sum_{j=1}^{r} S_1(c_j, x_1)}{r}, \frac{\sum_{j=1}^{r} S_1(c_j, x_2)}{r}, \frac{\sum_{j=1}^{r} S_1(c_j, x_3)}{r}, \dots, \frac{\sum_{j=1}^{r} S_1(c_j, x_n)}{r} \right] \right)$$

$$= \begin{bmatrix} x_{\Rightarrow}\left(\frac{\sum_{j=1}^{r} S_1(c_j,x_1)}{r},\frac{\sum_{j=1}^{r} S_1(c_j,x_1)}{r}\right) & x_{\Rightarrow}\left(\frac{\sum_{j=1}^{r} S_1(c_j,x_1)}{r},\frac{\sum_{j=1}^{r} S_1(c_j,x_2)}{r}\right) & \cdots & x_{\Rightarrow}\left(\frac{\sum_{j=1}^{r} S_1(c_j,x_1)}{r},\frac{\sum_{j=1}^{r} S_1(c_j,x_n)}{r}\right) \\ x_{\Rightarrow}\left(\frac{\sum_{j=1}^{r} S_1(c_j,x_2)}{r},\frac{\sum_{j=1}^{r} S_1(c_j,x_1)}{r}\right) & x_{\Rightarrow}\left(\frac{\sum_{j=1}^{r} S_1(c_j,x_2)}{r},\frac{\sum_{j=1}^{r} S_1(c_j,x_2)}{r}\right) & \cdots & x_{\Rightarrow}\left(\frac{\sum_{j=1}^{r} S_1(c_j,x_2)}{r},\frac{\sum_{j=1}^{r} S_1(c_j,x_n)}{r}\right) \\ x_{\Rightarrow}\left(\frac{\sum_{j=1}^{r} S_1(c_j,x_3)}{r},\frac{\sum_{j=1}^{r} S_1(c_j,x_1)}{r}\right) & x_{\Rightarrow}\left(\frac{\sum_{j=1}^{r} S_1(c_j,x_3)}{r},\frac{\sum_{j=1}^{r} S_1(c_j,x_2)}{r}\right) & \cdots & x_{\Rightarrow}\left(\frac{\sum_{j=1}^{r} S_1(c_j,x_3)}{r},\frac{\sum_{j=1}^{r} S_1(c_j,x_n)}{r}\right) \\ \vdots & \vdots & \vdots & \vdots \\ x_{\Rightarrow}\left(\frac{\sum_{j=1}^{r} S_1(c_j,x_n)}{r},\frac{\sum_{j=1}^{r} S_1(c_j,x_1)}{r}\right) & x_{\Rightarrow}\left(\frac{\sum_{j=1}^{r} S_1(c_j,x_n)}{r},\frac{\sum_{j=1}^{r} S_1(c_j,x_2)}{r}\right) & \cdots & x_{\Rightarrow}\left(\frac{\sum_{j=1}^{r} S_1(c_j,x_n)}{r},\frac{\sum_{j=1}^{r} S_1(c_j,x_n)}{r}\right) \end{bmatrix} \quad (5.6.1)$$

similarly in the case of $\mathbb{III}_{S_2}$ such that

$$\mathbb{III}_{S_2} = \left[x_{\underset{i=1,2,\ldots,n}{\Rightarrow}}\left(\frac{\sum_{j=1}^{r} S_2(c_j,x_i)}{r},\frac{\sum_{j=1}^{r} S_2(c_j,x_i)}{r}\right)\right] = x_{\Rightarrow}\left(\begin{bmatrix} \frac{\sum_{j=1}^{r} S_2(c_j,x_1)}{r} \\ \frac{\sum_{j=1}^{r} S_2(c_j,x_2)}{r} \\ \frac{\sum_{j=1}^{r} S_2(c_j,x_3)}{r} \\ \vdots \\ \frac{\sum_{j=1}^{r} S_2(c_j,x_n)}{r} \end{bmatrix}, \left[\frac{\sum_{j=1}^{r} S_2(c_j,x_1)}{r},\frac{\sum_{j=1}^{r} S_2(c_j,x_2)}{r},\frac{\sum_{j=1}^{r} S_2(c_j,x_3)}{r},\ldots,\frac{\sum_{j=1}^{r} S_2(c_j,x_n)}{r}\right]\right)$$

$$= \begin{bmatrix} x_{\Rightarrow}\left(\frac{\sum_{j=1}^{r} S_2(c_j,x_1)}{r},\frac{\sum_{j=1}^{r} S_2(c_j,x_1)}{r}\right) & x_{\Rightarrow}\left(\frac{\sum_{j=1}^{r} S_2(c_j,x_1)}{r},\frac{\sum_{j=1}^{r} S_2(c_j,x_2)}{r}\right) & \cdots & x_{\Rightarrow}\left(\frac{\sum_{j=1}^{r} S_2(c_j,x_1)}{r},\frac{\sum_{j=1}^{r} S_2(c_j,x_n)}{r}\right) \\ x_{\Rightarrow}\left(\frac{\sum_{j=1}^{r} S_2(c_j,x_2)}{r},\frac{\sum_{j=1}^{r} S_2(c_j,x_1)}{r}\right) & x_{\Rightarrow}\left(\frac{\sum_{j=1}^{r} S_2(c_j,x_2)}{r},\frac{\sum_{j=1}^{r} S_2(c_j,x_2)}{r}\right) & \cdots & x_{\Rightarrow}\left(\frac{\sum_{j=1}^{r} S_2(c_j,x_2)}{r},\frac{\sum_{j=1}^{r} S_2(c_j,x_n)}{r}\right) \\ x_{\Rightarrow}\left(\frac{\sum_{j=1}^{r} S_2(c_j,x_3)}{r},\frac{\sum_{j=1}^{r} S_2(c_j,x_1)}{r}\right) & x_{\Rightarrow}\left(\frac{\sum_{j=1}^{r} S_2(c_j,x_3)}{r},\frac{\sum_{j=1}^{r} S_2(c_j,x_2)}{r}\right) & \cdots & x_{\Rightarrow}\left(\frac{\sum_{j=1}^{r} S_2(c_j,x_3)}{r},\frac{\sum_{j=1}^{r} S_2(c_j,x_n)}{r}\right) \\ \vdots & \vdots & \vdots & \vdots \\ x_{\Rightarrow}\left(\frac{\sum_{j=1}^{r} S_2(c_j,x_n)}{r},\frac{\sum_{j=1}^{r} S_2(c_j,x_1)}{r}\right) & x_{\Rightarrow}\left(\frac{\sum_{j=1}^{r} S_2(c_j,x_n)}{r},\frac{\sum_{j=1}^{r} S_2(c_j,x_2)}{r}\right) & \cdots & x_{\Rightarrow}\left(\frac{\sum_{j=1}^{r} S_2(c_j,x_n)}{r},\frac{\sum_{j=1}^{r} S_2(c_j,x_n)}{r}\right) \end{bmatrix} \quad (5.6.2).$$

To demonstrate the above mentioned methodology we simply apply it to Example 4.1.1 to find the distance between GLIVIFSESs given by different experts.

EXAMPLE 5.6.1.

By taking data from Example 4.1.1 the opinions of experts are given as

$$S_1(h_1,a)=\{\langle \dot{s}_{[2,3]},\dot{s}_{[1,2]}\rangle,\langle \ddot{s}_{[1,2]},\ddot{s}_{[1,2]}\rangle\},$$
$$S_1(h_2,a)=\{\langle \dot{s}_{[3,4]},\dot{s}_{[1,2]}\rangle,\langle \ddot{s}_{[1,2]},\ddot{s}_{[1,2]}\rangle\},$$
$$S_2(h_1,b)=\{\langle \dot{s}_{[1,3]},\dot{s}_{[2,3]}\rangle,\langle \ddot{s}_{[1,2]},\ddot{s}_{[1,2]}\rangle\},$$

$$S_2(h_2, b)=\{\langle \dot{s}_{[1,2]}, \dot{s}_{[3,4]}\rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\rangle\}.$$

These sets represent the evaluation values by students 'a' and 'b' for the teacher t_1. Now for the second teacher t_2 the evaluation values are as under

$$S_1(h_1, a)=\{\langle \dot{s}_{[1,2]}, \dot{s}_{[2,4]}\rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\rangle\},$$
$$S_1(h_2, a)=\{\langle \dot{s}_{[2,4]}, \dot{s}_{[1,2]}\rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\rangle\},$$
$$S_2(h_1, b)=\{\langle \dot{s}_{[2,3]}, \dot{s}_{[2,3]}\rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\rangle\},$$
$$S_2(h_2, b)=\{\langle \dot{s}_{[1,2]}, \dot{s}_{[3,4]}\rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\rangle\}.$$

Here we use the Mamdani rule in fuzzy implication, thus by using linguistic fuzzy implication based distance measure in case of S_1 we have

$$\mathbb{III}_{S_1} = \begin{bmatrix} x_{\Rightarrow}\left(\frac{\sum_{j=1}^{2} S_1(h_j, t_1)}{2}, \frac{\sum_{j=1}^{2} S_1(h_j, t_1)}{2}\right) & x_{\Rightarrow}\left(\frac{\sum_{j=1}^{2} S_1(h_j, t_1)}{2}, \frac{\sum_{j=1}^{2} S_1(h_j, t_2)}{2}\right) \\ x_{\Rightarrow}\left(\frac{\sum_{j=1}^{2} S_1(h_j, t_2)}{2}, \frac{\sum_{j=1}^{2} S_1(h_j, t_1)}{2}\right) & x_{\Rightarrow}\left(\frac{\sum_{j=1}^{2} S_1(h_j, t_2)}{2}, \frac{\sum_{j=1}^{2} S_1(h_j, t_2)}{2}\right) \end{bmatrix},$$

here firstly by calculating the required values

$$\frac{\sum_{j=1}^{2} S_1(h_j, t_1)}{2} = \frac{\{\langle \dot{s}_{[2,3]}, \dot{s}_{[1,2]}\rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\rangle\} \oplus \{\langle \dot{s}_{[3,4]}, \dot{s}_{[1,2]}\rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\rangle\}}{2}$$
$$= \{\langle \dot{s}_{[2.5359,3.5505]}, \dot{s}_{[1,2]}\rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\rangle\},$$

similarly

$$\frac{\sum_{j=1}^{2} S_1(h_j, t_2)}{2} = \frac{\{\langle \dot{s}_{[1,2]}, \dot{s}_{[2,4]}\rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\rangle\} \oplus \{\langle \dot{s}_{[2,4]}, \dot{s}_{[1,2]}\rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\rangle\}}{2}$$
$$= \{\langle \dot{s}_{[1.5279,3.172]}, \dot{s}_{[1.4142,2.828]}\rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\rangle\}.$$

Now by substituting these calculated values in expression of $\mathbb{III}_{S_1}$ we obtain

$$\mathbb{III}_{S_1} = \begin{bmatrix} \{\langle \dot{s}_{[2.5359,3.5505]}, \dot{s}_{[1,2]}\rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\rangle\} & \{\langle \dot{s}_{[1.5279,3.172]}, \dot{s}_{[1.4142,2.828]}\rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\rangle\} \\ \{\langle \dot{s}_{[1.5279,3.172]}, \dot{s}_{[1.4142,2.828]}\rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\rangle\} & \{\langle \dot{s}_{[1.5279,3.172]}, \dot{s}_{[1.4142,2.828]}\rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\rangle\} \end{bmatrix}.$$

Similarly in case of opinions from second expert "b"

$$\mathbb{III}_{S_2} = \begin{bmatrix} x_{\Rightarrow}\left(\frac{\sum_{j=1}^{2} S_2(h_j, t_1)}{2}, \frac{\sum_{j=1}^{2} S_2(h_j, t_1)}{2}\right) & x_{\Rightarrow}\left(\frac{\sum_{j=1}^{2} S_2(h_j, t_1)}{2}, \frac{\sum_{j=1}^{2} S_2(h_j, t_2)}{2}\right) \\ x_{\Rightarrow}\left(\frac{\sum_{j=1}^{2} S_2(h_j, t_2)}{2}, \frac{\sum_{j=1}^{2} S_2(h_j, t_1)}{2}\right) & x_{\Rightarrow}\left(\frac{\sum_{j=1}^{2} S_2(h_j, t_2)}{2}, \frac{\sum_{j=1}^{2} S_2(h_j, t_2)}{2}\right) \end{bmatrix}.$$

Firstly by calculating the required values

$$\frac{\sum_{j=1}^{2} S_2(h_j, t_1)}{2} = \frac{\{\langle \dot{s}_{[1,3]}, \dot{s}_{[2,3]}\rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\rangle\} \oplus \{\langle \dot{s}_{[1,2]}, \dot{s}_{[3,4]}\rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\rangle\}}{2}$$
$$= \{\langle \dot{s}_{[1,2.536]}, \dot{s}_{[2.449,3.464]}\rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\rangle\}$$
$$\frac{\sum_{j=1}^{2} S_2(h_j, t_2)}{2} = \frac{\{\langle \dot{s}_{[2,3]}, \dot{s}_{[2,3]}\rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\rangle\} \oplus \{\langle \dot{s}_{[1,2]}, \dot{s}_{[3,4]}\rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\rangle\}}{2}$$
$$= \{\langle \dot{s}_{[1.5279,2.536]}, \dot{s}_{[2.449,3.464]}\rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\rangle\}.$$

Now by substituting these values in expression of $\mathbb{M}_{S_2}$ we obtain

$$\mathbb{M}_{S_2} = \begin{bmatrix} \{\langle \dot{s}_{[1,2.536]}, \dot{s}_{[2.449,3.464]}\rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\rangle\} & \{\langle \dot{s}_{[1,2.536]}, \dot{s}_{[2.449,3.464]}\rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\rangle\} \\ \{\langle \dot{s}_{[1,2.536]}, \dot{s}_{[2.449,3.464]}\rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\rangle\} & \{\langle \dot{s}_{[1.5279,2.536]}, \dot{s}_{[2.449,3.464]}\rangle, \langle \ddot{s}_{[1,2]}, \ddot{s}_{[1,2]}\rangle\} \end{bmatrix}.$$

Now by substituting these values in $\mathbb{M}_{S_1} - \mathbb{M}_{S_2}$ expression we get

$$= \begin{bmatrix} \left\{\langle \dot{s}_{[1.9575,2.515]}, \dot{s}_{[0.40817,1.15467]}\rangle, \langle \ddot{s}_{[\frac{1}{4},1]}, \ddot{s}_{[\frac{1}{4},1]}\rangle\right\} & \left\{\langle \dot{s}_{[0.78255,1.977]}, \dot{s}_{[0.577,1.6327]}\rangle, \langle \ddot{s}_{[\frac{1}{4},1]}, \ddot{s}_{[\frac{1}{4},1]}\rangle\right\} \\ \left\{\langle \dot{s}_{[0.78255,1.977]}, \dot{s}_{[0.577,1.6327]}\rangle, \langle \ddot{s}_{[\frac{1}{4},1]}, \ddot{s}_{[\frac{1}{4},1]}\rangle\right\} & \left\{\langle \dot{s}_{[0.3891,1.977]}, \dot{s}_{[0.577,1.6327]}\rangle, \langle \ddot{s}_{[\frac{1}{4},1]}, \ddot{s}_{[\frac{1}{4},1]}\rangle\right\} \end{bmatrix},$$

Since the above matrix is a square matrix then the for norm we take the determinant of that matrix such that

$$d(S_1, S_2, x_{\Rightarrow}) = |\mathbb{M}_{S_1} - \mathbb{M}_{S_2}|$$
$$= \begin{vmatrix} \left\{\langle \dot{s}_{[1.9575,2.515]}, \dot{s}_{[0.40817,1.15467]}\rangle, \langle \ddot{s}_{[\frac{1}{4},1]}, \ddot{s}_{[\frac{1}{4},1]}\rangle\right\} & \left\{\langle \dot{s}_{[0.78255,1.977]}, \dot{s}_{[0.577,1.6327]}\rangle, \langle \ddot{s}_{[\frac{1}{4},1]}, \ddot{s}_{[\frac{1}{4},1]}\rangle\right\} \\ \left\{\langle \dot{s}_{[0.78255,1.977]}, \dot{s}_{[0.577,1.6327]}\rangle, \langle \ddot{s}_{[\frac{1}{4},1]}, \ddot{s}_{[\frac{1}{4},1]}\rangle\right\} & \left\{\langle \dot{s}_{[0.3891,1.977]}, \dot{s}_{[0.577,1.6327]}\rangle, \langle \ddot{s}_{[\frac{1}{4},1]}, \ddot{s}_{[\frac{1}{4},1]}\rangle\right\} \end{vmatrix}$$
$$= \left\{\langle \dot{s}_{[0.1269,0.8287]}, \dot{s}_{[1.0124,2.47317]}\rangle, \langle \ddot{s}_{[\frac{1}{64},\frac{1}{4}]}, \ddot{s}_{[\frac{31}{64},\frac{7}{4}]}\rangle\right\} \ominus \left\{\langle \dot{s}_{[0.1021,0.651]}, \dot{s}_{[1.10229,2.8211]}\rangle, \langle \ddot{s}_{[\frac{1}{64},\frac{1}{4}]}, \ddot{s}_{[\frac{31}{64},\frac{7}{4}]}\rangle\right\}$$

here by evaluating subtraction operator we get

$$d(S_1, S_2, x_{\Rightarrow}) = \{\langle s_{[0.02696,0.2676]}, s_{[0.186,1.1628]}\rangle, \langle s_{[0.000061,0.0156]}, s_{[0.05865,0.765625]}\rangle\}.$$

From above expression we can observe that the result obtained by linguistic fuzzy implication to measure distance between GLIVIFSESs give appropriate results because we can observe that the values of sets S_1 and S_2 slightly differ from each other.

CHAPTER 6

CONCLUSIONS

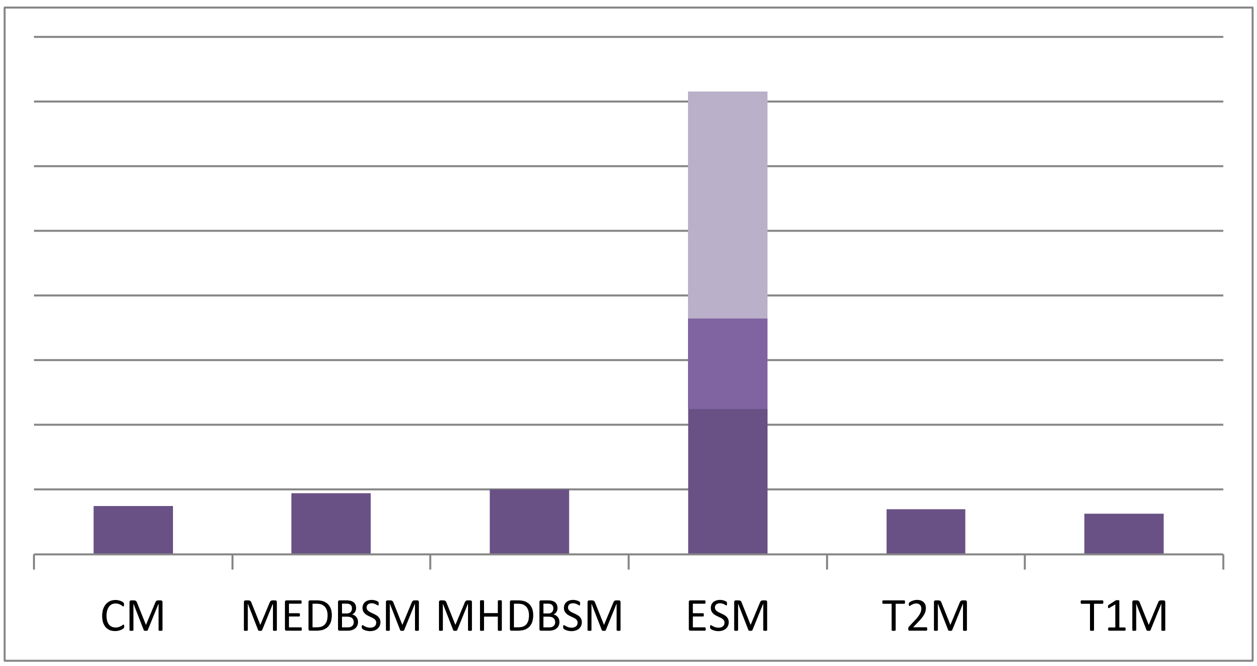
Since the concept of GLIVIFSESs introduced by Tasaduq Mahmood and Afshan Qayyum [23] is the comprehensive model in the field of linguistic approach, as it covers all the necessary possibilities which the fuzzy structures may have, so we used that structure for further study on it and applied a lot of new similarity measures on that structure, due to their importance for many decision making problems and a lot of other applications in real world situations we compared the results by taking out common example throughout these measures. Also separately for each similarity measure we constructed practical problems from real world data examples and checked-out the accuracy level of these measures. We have the following observations for GLIVIFSESs,

Modified Hamming Distance Based Similarity Measure	$\left\{\begin{matrix}\langle \dot{s}_{[0.66295,\ 0.779362]}, \dot{s}_{[2.68573,\ 2.960732]} \rangle \\ , \langle \ddot{s}_{[0.25,1.515625]}, \ddot{s}_{[1.75,1.984375]} \rangle\end{matrix}\right\}$	$s_{0.672635}$
Modified Euclidean Distance Based Similarity Measure	$\left\{\begin{matrix}\langle \dot{s}_{[0.089,\ 1.0066]}, \dot{s}_{[2.4877,\ 2.9272]} \rangle, \\ \langle \ddot{s}_{[0.6086,1.6428]}, \ddot{s}_{[1.5969,1.9711]} \rangle\end{matrix}\right\}$	$s_{0.67115}$
Entropy Similarity Measure	$\dfrac{\dot{s}_{5.99739} \bigoplus \ddot{s}_{3.99547}}{\dot{s}_{5.992437} \bigoplus \ddot{s}_{3.995739}}$	$s_{5.99746}$
Correlation Measure	$\left\{\begin{matrix}\langle \dot{s}_{\left[\frac{50861}{46656}, \frac{403}{108}\right]}, \dot{s}_{\left[\frac{245}{486}, \frac{490}{243}\right]} \rangle, \\ \langle \ddot{s}_{\left[\frac{14911}{16384}, \frac{175}{64}\right]}, \ddot{s}_{\left[\frac{2401}{16384}, \frac{81}{64}\right]} \rangle\end{matrix}\right\}$	$s_{0.40109}$

Type-I Similarity Measure	$\left\{\begin{matrix}\langle \dot{s}_{\frac{8}{9}}, \dot{s}_{1.01835015}\rangle, \\ \langle \ddot{s}_{\frac{5}{6}}, \ddot{s}_{\frac{1}{6}}\rangle\end{matrix}\right\}$	$s_{0.2853734}$
Type-II Similarity Measure	$\left(\begin{matrix}\langle \dot{s}_{\frac{18-\sqrt{203}}{3}}, \dot{s}_{\frac{1}{3}}\rangle, \\ \langle \ddot{s}_{\frac{5}{6}}, \ddot{s}_{\frac{1}{6}}\rangle\end{matrix}\right)$	$s_{0.33626239}$
Type-III Similarity Measure	$\left\{\begin{matrix}\langle \dot{s}_{0.3236134}, \dot{s}_{1.4515819}\rangle, \\ \langle \ddot{s}_{\frac{199}{256}}, \ddot{s}_{\frac{299}{256}}\rangle\end{matrix}\right\}$	$s_{0.1831769}$
Type-IV Similarity Measure	$\left\{\begin{matrix}\langle \dot{s}_{\left[\frac{251}{2592}, 0.3214993\right]}, \dot{s}_{[1.26505565, 2.75803167]}\rangle, \\ \langle \ddot{s}_{\left[\frac{63}{256}, \frac{15}{16}\right]}, \ddot{s}_{\left[\frac{121}{256}, \frac{25}{16}\right]}\rangle\end{matrix}\right\}$	$s_{0.1575996157}$
Type-V Similarity Measure	$\left\{\begin{matrix}\langle \dot{s}_{0.49965}, \dot{s}_{1.59285}\rangle, \\ \langle \ddot{s}_{\frac{31}{64}}, \ddot{s}_{\frac{529}{576}}\rangle\end{matrix}\right\}$	$s_{0.1822658}$
Max-Min Similarity Measure	$\left(\langle \dot{s}_{\left[\frac{671}{7560}, \frac{49}{285}\right]}, \dot{s}_{\left[\frac{683}{3888}, \frac{167}{162}\right]}\rangle, \langle \ddot{s}_{\left[\frac{35}{192}, \frac{12}{35}\right]}, \ddot{s}_{\left[\frac{511}{16384}, \frac{31}{64}\right]}\rangle\right)$	$s_{0.26997}$
Modified Max-Min Similarity Measure	$\left\{\langle \dot{s}_{\left[\frac{25135}{11664}, \frac{370}{81}\right]}, \dot{s}_{\left[\frac{943}{3888}, \frac{319}{243}\right]}\rangle, \langle \ddot{s}_{\left[\frac{22687}{16384}, \frac{207}{64}\right]}, \ddot{s}_{\left[\frac{961}{16384}, \frac{49}{64}\right]}\rangle\right\}$	$s_{0.5313}$

- Entropy similarity measure produces more accurate result's than Modified Hamming distance based similarity measure
- Modified Hamming distance based similarity measure produces more accurate results then Modified Euclidean distance based similarity measure
- Modified Euclidean distance based similarity measure produces more accurate results than Modified Max-Min similarity measure
- Modified Max-Min similarity measure produces more accurate results than correlation similarity measure

- Correlation measure produces more accurate results than Type-II similarity measure
- Type-II similarity measure produces more accurate results than Type-I similarity measure
- Type-I similarity measure produces more accurate results than Max-Min similarity measure
- Max-Min similarity measure produces more accurate results than Type-III similarity measure
- Type-III similarity measure produce ore accurate results than Type-V similarity measure
- Type-V similarity measure produces more accurate results than Type-IV similarity measure. In graphically representation we have



Here in Type-V similarity measure we made some amendments and call the new one as Modified Type-V similarity measure and further us extended the practical example related to mining license under Modified Type-V similarity measure and got the most appropriate results. Here one can also employ example.1 under Modified Type-V similarity measure.

Behind similarity measures we attempted to apply dissimilarity measure, which plays an essential role in decision making problems. In which we firstly introduced the mathematical expression to measure dissimilarity for GLIVIFSESs and then tested the validity of that dissimilarity measure by considering the practical example related to judgments regarding the authorities of X state education department, and we obtained mostly accurate result.

Later on we introduced the concept of linguistic fuzzy implication for distance measure between GLIVIFSESs and then employed the exports opinions from Example 4.1.1 to measure distance between them under linguistic fuzzy implication environment and obtained considerable accurate results.

Future Plan

We intend to investigate these measures using programming tools with complexed real life situations for infinitely large data's to apply these practically.

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