CHARACTERISTICS OF THERMALLY STRATIFIED CASSON FLUID FLOW WITH CONVECTIVE BOUNDARY CONDITIONS

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ISLAMABAD

July, 2023

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MS MATH, National University of Modern Languages, Islamabad, 2023

A THESIS SUBMITTED IN PARTIAL FULFILMENT OF
THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF SCIENCE

In **Mathematics**

To

FACULTY OF ENGINEERING & COMPUTER SCIENCE

NATIONAL UNIVERSITY OF MODERN LANGUAGES ISLAMABAD

NATIONAL UNIUVERSITY OF MODERN LANGUAGES FACULTY OF ENGINEERIING & COMPUTER SCIENCE

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Thesis Title: Characteristics of Thermally Stratified Casson Fluid Flow with Convective Boundary Conditions

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ABSTRACT

Title: Characteristics of Thermally Stratified Casson Fluid Flow with Convective Boundary Conditions

This dissertation focuses on the stretching flow of Casson fluid through horizontal sheet. The study is carried out to analyze the characteristics of velocity slip and hydromagnetic phenomena. Features of heat are elaborated with viscous dissipation, thermal radiation and nonlinear stratification. Sheet surface is subjected to convective boundary condition. The governing educations are made dimensionless by implementing suitable variables. The problem is analyzed analytically via homotopic technique. The upshots of several pertinent parameters upon the dimensionless profiles of velocity and temperature are scrutinized. Results found that Biot number intensifies the temperature field.

TABLE OF CONTENTS

CHAPTER	TITLE		PAGE
	AUTHOR'S DECLARATION		iii
	ABSTRACT		iv
	TABLE OF CONTENTS		v
	LIST OF FIGURES		vii
	LIST OF **SYMBOLS**		viii
	GREEK SYMBOLS		ix
	ACKNOWLEDGEMENT		x
	DEDICATION		xi
1	INTRODUCTION		1
2	LITERATURE SURVEY		19
3	Basics Concepts		32
	3.1	Fluid	32
	3.2	Fluid Mechanics	32
		3.2.1 Fluid Static	32
		3.2.2 Fluid Dynamics	32
	3.3	Flow	33
		3.3.1 Steady Flow	33
		3.3.2 Unsteady Flow	33
		3.3.3 Laminar Flow	33
		3.3.4 Turbulent Flow	33
		3.3.5 Compressible Flow	33
		3.3.6 Incompressible Flow	33
	3.4	Density	33
	3.5	Stress	34
		3.5.1 Shear Stress	34
		3.5.2 Normal Stress	34
	3.6	Strain	34
	3.7	Viscosity	34
	3.8	Kinematic Viscosity (υ)	35
	3.9	Viscosity Law (Newtonian)	35
	3.1	Newtonian Fluids	35

	3.11	Model of Power Law	35
	3.12	Non-Newtonian Liquid	36
	3.13	Model of Casson Liquid	36
	3.14	Modes of Heat Transfer	36
		3.14.1 Conduction	36
		3.14.2 Convection	37
		3.14.3 Thermal Radiation	37
	3.15	Body and Surface Forces	37
	3.16	Stratification	38
	3.17	Joule Heating Law	38
	3.18	Thermal Conductivity	38
	3.19	Thermal Diffusivity	39
	3.2	Convective Boundary Condition	39
	3.21	Dimensionless Numbers	40
		3.21.1 Reynold Number	40
		3.21.2 Prandtl Number	40
		3.21.3 Biot Number	41
		3.21.4 Nusselt Number	41
		3.21.5 Skin Friction	41
	3.22	Basic Equations	41
		3.22.1 Equation of Continuity	41
		3.22.2 Momentum Equation	42
		3.22.3 Energy Equation	43
	3.23	Homotopy Analysis Method	43
4	Magnetic Features in Stretching Flow of Fluid with Thermal Radiation		45
	4.1	Introduction	45
	4.2	Mathematical Modeling	45
		4.2.1 Conditions at Boundary	46
		4.2.2 Suitable Variable	47
	4.3	Homotopic Solution	48
		3.3.1 Problem of 0th Order	48
		3.3.2 Problem of mth Order	49
		4.3.3 Analysis of Convergence	50
	4.4	Discussion	51
5	Characteristics of Thermally Stratified Casson Fluid Flow with Convective Boundary Conditions		54
	5.1	Introduction	54
	5.2	Mathematical Formulation	54
		5.2.1 Conditions at Boundary	55
		5.2.2 Suitable Variables	56
		5.2.3 Homotopic Procedure	57
	5.3	Analysis of Convergence	57
	5.4	Discussion	58
6	CONCLUSION		63
	REFERENCES		64

List of Figures

4.1	Schematic of problem	46
4.2	h-curves of f and	51
4.3	Depiction of on	52
4.4	Depiction of on	52
4.5	Depiction of on	53
4.6	Depiction of on	53
5.1	h-curves of f and	58
5.2	Depiction of on	59
5.3	Depiction of on	60
5.4	Depiction of on	60
5.5	Depiction of on	61
5.6	Depiction of on	61
5.7	Depiction of & on	62
5.8	Depiction of & on	62

List of Symbols

Cartesian coordinates
Velocity components in x and y directions respectively
Magnetic field strength
Reference temperature
Electric conductivity
Reference length
Fluid temperature
Radiative heat flux
Specific heat
Ambient temperature
Magnetic parameter
Radiation parameter
Biot number
Eckert number
Pardantl number
Stratification parameter
Velocity Slip

Greek Symbols

Similarity variable
Stream function
Dimensionless temperature
Casson fluid parameter

ACKNOWLEDGMENT

I offer my humblest sense of gratitude to Almighty Allah the Beneficent, the most Merciful, who blessed me an opportunity to stay on the way of knowledge to seek my recognition in the universe. Great blessing to His Prophet Muhammad (S.A.W) who builds the height and standard of character for all over the world and guided His 'Ummah' to seek knowledge from cradle to grave.

With deep respect and genuineness, I feel much pleasure in expressing my heartiest gratitude to my supervisor, Dr. Aisha Anjum for her dynamic, affectionate and inspirational supervision. Deep sense of gratitude is paid to my worthy teachers their valuable suggestions, inspiring guidance and continuous encouragement.

I would like to thank my family, especially my parents, for the opportunities they provided to their children and for teaching us all that our abilities were only limited by our imaginations. For pushing us, understanding us, and helping us to achieve our dreams. Their motivations and prayers have always saved me from drifting.

Faiza Niaz

DEDICATION

This work is dedicated

To

My Beloved Parents
& Respected
Teachers

Chapter 1

Introduction

When an external force is applied, a category of substance known as a fluid deforms or flows. Fluids might be in the form of liquids, gases, or plasma. It is a substance that lacks shear modulus or, more precisely, a substance that cannot tolerate applied shear stress. Due to fluid's importance in many common processes and the fact that it is a basic necessity for daily life, researchers throughout the world are working to uncover many insider facts about fluid movement. Fluids can be further categorized as Newtonian or non-Newtonian fluids based on the relationship between two physical parameters, particularly the relationship between stress and strain. Non-Newtonian liquids are those in which the relationship between shear stress and rate of deformation is not linear. Non-Newtonian liquids are defined as substances that do not follow Newton's viscosity law. Liquids such as blood, ketchup, paint, shampoo and muck display non-Newtonian behaviour. Numerous sectors,comprising petroleum production, filters, polymer technology and porcelain manufacture, employ non-Newtonian fluid. We explore liquid movement and address its causes in the branch of fluid dynamics that is known as fluid mechanics. Moreover, in fluid mechanics we also study how the liquid motion is influenced by forces. It provides explanations for how the ocean, the stream, the geological plate, and the blood's spinning all move. A few notable applications for fluid flows include pipeline cleaning, jet engines, wind farms, and heating and cooling systems. The first scientist was Archimedes, who carefully outlined his theories about fluid changelessness and consciously anticipated the fundamentals of fluid mechanics. Beginning in the early fourteenth century, the proper research of liquid mechanical features has been conducted. A fluid can be further classified as a Newtonian or non-Newtonian liquid depending on the connection between two body parameters, namely stress and strain. The substance's deformation considering the shearing force effect is addressed in the mechanics of fluids. Because a little shearing force may distort a fluid body, the deformation's velocity will be similarly low. The study of fluids, whether they are moving or at rest, is referred to as fluid mechanics. It has been used in numerous technical developments, including ducting and piping used in commercial and residential water and air conditioning systems, piping systems regarding plants of chemical, the canal designa as well as dam systems, system of lubrication, and supersonic and automotive aerodynamics. The hydrodynamic relates the analysis of the fluid motion under incompressibility, for example, and is one of the categories under which fluid mechanics is classified; Hydraulics basically the investigation of fluid motions through open channel and cannal; gas dynamics relates liquid flows with density variation ; and aerodynamics relates liquid flowsthrough bodies. Hydrology, oceanography, and meteorology are only a few divisions because of naturally occurring flows. However, the study of fluid mechanics is fundamental to the advancement of technology as well as in science. It is very helpful in the wind turbines and design, crude oil, jet engines, boats, aeroplanes, submarines, natural gas, rockets, water transportation, cooling of electronic components, and biomedical devices, among other things. Additionally, it aids in construction and bridge design. Fluid mechanics concepts also govern a number of other naturally occurring phenomena, including the rain cycle, the ground liquid ascent to the tops of trees, weather patterns, winds, ocean waves. Magnetohydrodynamics is the study of how magnetic fields interact with conducting fluids. In other terms, magnetohydrodynamics is the study of flows in which the fluid conducts electricity. Fluids are referred to as hydro, forces are referred to as forces, and rules of motion are referred to as dynamics. Liquid metals (such as antimony, mercury, molten magnesium, gallium, sodium, etc.) and plasmas are a couple of well-known examples (electrically conducting gases or ionised gases). Fundamentally, an electromotive force is produced by the relative movement of a magneto fluid and a magnetic field, and that relative motion produces electrical currents. The order of density affects the fundamental principle of magnetohydrodynamics. Due to the fact that current causes an additional induced magnetic field, the fluid produces flow with magnetic field lines as a result. In summary, it can be shown that fluids may drag magnetic field lines whereas magnetic fields can pull on conducting fluids.

1.1 Boundary Layer Flow

A fluid's viscosity is crucial at a thin layer close to the surface. Shear layer or boundary layer are the terms used in English. The flow field is divided into two regions: one within the boundary layer, where viscosity dominates and is primarily responsible for the drag experienced by the boundary body; and one outside the boundary layer, where viscosity may be discarded without having a substantial impact on the solution. The term "boundary layer" is used frequently in physics and fluid mechanics to describe the layer of fluid immediately adjacent to a bounding surface where viscosity has a substantial impact. An essential link between the flow of ideal fluid and that of real fluid is provided by the boundary layer. Only a tiny region immediately surrounding the fluid boundaries exhibits the effects of internal friction in fluids with relatively low viscosities. There is a sharp velocity gradient from the barrier into the flow because the fluid has zero velocity at the boundaries. In a real fluid, this velocity differential creates shear forces close to the border that lower the flow speed to that of the barrier. The boundary layer is the fluid layer whose velocity has been impacted by boundary shear. By combining the boundary layer with the inviscid outer zone, the total flow field is discovered. There will also be a lot of focus on the coupling process (both physically and mathematically). We can examine one or a mixture of the boundary layers outlined below, depending on the kind of fluid flow under investigation.

Velocity boundary layer

The tangential velocity of fluid particles in contact with a stationary surface is zero. The same holds true for fluid particles in touch with a moving surface; they will move at the surface's speed. The no slip condition is the name given to this occurrence in fluid dynamics. When a fluid moves, there is a net momentum transfer from high- to low-velocity regions, which produces a viscous shear stress in the flow direction. The purpose of the velocity boundary layer is to estimate the fluid's surface (or skin) friction.

Thermal boundary layer

Temperature differential between the surface and the free stream zone cause the thermal boundary layer to form. The rate of convection heat transmission is significantly influenced by the thermal boundary layer.

Concentration boundary layer

In a fluid area with concentration gradients between the surface and the free stream, a concentration boundary layer forms. This boundary layer is important for figuring out the convection mass transfer rate.

When fluid is moving over the surface, or when fluid and body are in direct touch, boundary layer flow is extremely important. Due to drag and friction forces, a thin boundary barrier formed in which the fluid adjusted the body's velocity. The effect of these forces is less than in the turbulent layer (boundary) in the laminar (boundary). Recent research has focused on finding ways to reduce drag and friction forces in order to increase the efficiency of various devices. Thus, numerous attempts have been made to reduce forces of drag for motion over a wing's surface, a wind turbine rotor's surface, a tail plane's surface, etc. However, that type of forces can be mitigated by preventing layer (boundary) separation and delaying the changeover from laminar towards flow with turbulency. This task can be carried out using a variety of physical techniques, such as moving the surface, injecting and suctioning fluid, using body forces, and chilling or heating the surface. As a result, a variety of industrial production processes make extensive use of the heat transpory features in layer (boundary) flow caused by a movable surface. A few examples of these procedures are the manufacturing of glass fibre, hot rolling, paper, drawing of plastic films, continuous casting, metal spinning, metal and polymer extrusion, and wire drawing. The heat transfer rate at the stretching surface has a significant impact on the final result when annealing and thinnin copper wires. In the metallurgical process, such flow consideration in the presence of a magnetic field is crucial. Using MHD liquid to create stripes and ensuring that the finished result is of the impuls quality, the cooling rate may be controlled. The quality of the finished product is significantly influenced by the mechanics of stretching and any accompanying heating or cooling. The important problem of boundary layer flow across a stretched sheet occurs in a number of industrial production processes. Making paper and drawing sheets out of plastic, glass, fibre, and rubber is one example of this. Crystal growth while extending polymers for manufacture, chilling metallic sheets in a cooling solution, etc. Because it has applications in polymer processing technologies, this flow issue with heat transmission has attracted the attention of researchers. Due to its broad applicability to industrial issues, the study of laminar flow over a stretched plate in porous media and heat transfer has attracted a lot of attention. From the perspective of existence, boundary layer fluxes in convergent channels and in divergent channels are fundamentally different. The former one can often exist, while the later one is less common, and depending on the Reynolds number, boundary layer separation happens in the backflow zone. If anything else can prevent the backflow construction, i.e., the separation cause, then the structure of the boundary layer in the immediate vicinity of both channel walls will be a possibility. Additionally, there is a substantial body of work on similarity solutions in the field of fluid mechanics for the problems of boundary layer flow, heat transfer, and mass transfer. Therefore, one of the important methods to lower the number of independent variables is the similarity solution. It is said that similarity solutions have been carried out for various parameter values. The convective boundary-layer context may be solved using the similarity approach distant from the downstream or close to the leading edge.

1.2 Non-Newtonian Fluid

Non-Newtonian fluids are widely used in engineering and industrial applications and have received significant inspection. It is not feasible to use a single model to represent the fluid characteristics of the vast class of non-Newtonian fluids due to the limitations of fluid models. The literature presents a number of non-Newtonian fluid models. Power-law fluid is the simplest fluid model in this context, however it has limits since it cannot precisely predict the flow parameters in lower and higher Newtonian areas. Furthermore, a slight variation in viscosity can have a significant impact on a variety of phenomena, including lubrication issues and the processing of polymers. Non-Newtonian fluids typically have complex constitutive interactions. Both the relationship between heat current and temperature gradient in thermodynamics and the relationship between stress and strain in rheology are nonlinear in these fluids. A single mathematical model cannot account for all the rheological fluid characteristics due to the diversity of flows. It is all around noticed reality that the fluids which go against the viscosity law of Newtonian described as non-Newtonian liquids. The thickness of these type of liquids depend on the shear type stress executed to the liquid. The employed shear type stress transmutted the liquids into extra strong state or more liquid state which relies on the idea of non-Newtonian liquids. Broad models connected with these liquids found in fluid items utilized in day to day existence, for example, blood, cleanser, toothpaste, custards, honey, paint, softened spread, corn starch, and a few more. Different liquid models that represent non-Newtonian liquids are available. These liquids show thickning (shear) and diminishing (shear) qualities. Between these, liquid of Casson is a particularly model which portrays the diminishing (shear) capacity, and after employing some more pressure, it turns incredibly. The Casson model related to liquid can generally be utilized to separate the liquids' non-Newtonian way of behaving. Casson fluid, a thinning (shear) liquid, is predicted to have no viscosity at undefined shear rate, an immeasurable viscosity with zero shear rate, and a yield stress reduce which no movement occurs. The most widely used non-Newtonian fluid, Casson fluid has several uses in the fields of bioengineering, drilling, metallurgy, and food processing. Casson developed the Casson fluid model to predict how pigment-oil suspensions will flow. Jelly, tomato sauce, honey, soup, concentrated fruit liquids, and human blood are a few examples of casson fluid. Due to various engineering and industrial applications, such as those in the fields of pharmaceuticals, chemicals, oil, cosmetics, gas, polymer solutions, filtration, polymeric liquids, and ceramic processes of ceramic and enhanced oil recovery, biomechanics. The heat transport mechanism has a substantial significance regarding industrial view point. Several scientists and explorer focus on this area. In technically advanced technologies and numerous industrial related fields, the theory of non-Newtonian liquid has great influence because model of Newtonian liquid unable to express diverse flow features. The Newtonian as well as non-Newtonian material motion analysis is very significant and important. Because of their complicated flow properties, rate of deformation, and structures, industrial related fluids are frequently referred to as non-Newtonian materials. Scientists, mathematicians, engineers, and numerical analyzers now have the chance to demonstrate the behaviour of common non-Newtonian fluid properties across a variety of scientific and technological disciplines. Thinning-thickening (Shear), retardation-relaxation, normal-yield (stress) variations in shear motion, asphalts in geomechanics and concrete, many more, are common non-Newtonian fluids characteristic. Furthermore, it is undeniable that non-Newtonian fluids are more difficult to manage than Newtonian fluids are, and that it is difficult to explain all of their features. There is no one governing equation with the capacity to predict all types of rheological properties of non-Newtonian fluids due to the non-linear relationship between the shear stress and shear rate. As a result, a number of models have been put out by researchers in order to predict all of the traits and non-Newtonian material propertie. It is common knowledge that non-Newtonian fluids may be divided into the following categories: (i) differential types, (ii) rate types, and (iii) integral kinds of models. The models of Maxwell, Burger, and Oldroyd-B, all fall under the rate type fluids group since they allow for memory and elastic effects as well as explain a very tiny memory. Characterizing viscoelastic fluids is the primary benefit of rate type fluid (i-e viscous and elastic memory effects). From the perspective of mechanical engineering, the complex rheological behaviour of a shear-thinning fluid like blood cannot be represented by Newton's relatively simple, one parameter, and linearized viscosity formula. Only higher-order constitutive equations, such the power-law model, which take into account the fluid's non-Newtonian traits like shear thinning and yield stress characteristics, can adequately describe the properties of this type of fluid. One of the most crucial problems for academics in recent years has been the exploration of fluid dynamics and heat transmission in the non-Newtonian fluid flow. A better understanding of scientific phenomena that occur in the real world can be achieved by understanding the non-Newtonian fluid movement property and the thermal-mechanical behaviour of the fluid stream, which has led many researchers in various engineering branches to focus on simulating non-Newtonian fluid flow using various methodologies. Numerous research have been done up to this point on the behaviour and properties of non-Newtonian fluid flow under various boundary conditions. The formulation and production of complicated liquid goods across a variety of sectors frequently involves liquid mixing in mechanically agitated containers. Paints and inks, meals, home and personal care items, catalyst intermediates, and battery materials are a few examples of such goods. With flow measurement techniques like Laser Doppler Anemometry (LDA), Particle Image Velocimetry (PIV), and Planar Laser Induced Fluorescence (PLIF), which enable quantitative analysis of mixing time, flow patterns, turnover rates, and turbulent fluctuations, this unit operation has received significant research attention. Even though mixing occurs often under transitional flow regimes in industry, particularly for goods whose rheology changes throughout production, the majority of experimental research in the literature concentrate on entirely laminar or totally turbulent flows.

1.3 Heat Transfer

A heat transportation process is depicted as the transmute of thermal energy from a hotter object to a colder one. Only when temperature variation occurs between the physical body's surroundings or another body can heat transfer occur. Heat is regularly transmitted from a hotter body to a colder one, according to the second rule of thermodynamics.Traditionally, only radiation, conduction, and convection, or any combination of these, are used to transfer heat energy. Heat transfer using the material that conducts phase-variation heat (such as boiling heat is transported by steam). The word "mass transfer" is typically used in the engineering field to refer to related physical processes, such as the molecular and convection transport of atoms (or molecules) inside physically constructed systems. The processes of mass, heat, and molecular transport are found to have many parallels. The well-known Newton's law for momentum transfer, Fourier's law for heat transfer, and Fick's law for mass transport balancing all share many similarities. As a result, there are several similarities in these molecular transfer mechanisms. The driving force for mass transport is predicted by the concentration difference; specifically, mobility of random molecules contributes to a net movement of mass from a region of greater concentration to one of small concentration. By computing and using the mass transit co-efficient, the amount of mass transfer can be estimated. Effects of mass transfer have drawn considerable attention in a variety of processes, including the purification of alcohol, the evaporation of water from a vessel towards the atmosphere, and the distillation of blood in the liver and kidneys. Additionally, characteristics of heat and mass transmission have a significant impact on processes related to engineering as well as industry. Such mechanisms include dispersion, the destruction of crops due to freezing, moisture, fog formation, and temperature dispensation over fruit tree groves, agricultural fields, permeable solids drying, environmental pollution, the design of various chemical processing equipment, insulation, geothermal reservoirs, packed bed catalytic reactors, improved subterranean energy transport, and recovery of oil. Heat and mass transport analyses are quite interested in the stratification process in fluids. It happens as a result of changes in temperature, concentration, and densities of various fluids. As a result, dual stratification is thoroughly examined in flows where mass and heat transport are occurring simultaneously. Double stratification is a phenomenon that occurs in many natural as well as industrial domains. It is caused by changes in temperature, concentration discrepancies, or fluids with different densities. Numerous practical applications, including oceans, rivers, mixes of different compositions, thermal stratification of reservoirs, manufacturing procedures, ground-water reservoirs, and many more, make considerable use of the double stratification process. The presence of biological mechanisms in reservoirs causes the bottom water to become anoxic.The stratification method could also be used in solar engineering and energy storage. The movement of thermal energy from a hot to a cold body is the mechanism of heat transfer. The transmission of thermal energy between two physical objects is sometimes referred to as heat transfer when the temperatures of the objects are different. The process is such that thermal equilibrium is achieved between the body and its environment. The second rule of thermodynamics states that heat transfer from a hot body to a cold one always takes place. The only traditional methods for transferring thermal energy are conduction, convection, radiation, or any combination of these. It is sometimes considered a type of convective heat transfer when a material (like steam, which carries the heat of boiling) transports the heat of a phase shift. There are two types of convection: forced and free (natural). Free convection occurs when fluid near a heat source absorbs heat, lowers density, and rises. The fluid in the vicinity of the cooler then moves to replace it. Convection current is created when the process of heating this colder fluid proceeds. Buoyancy, which results from variations in fluid density when gravity or any other kind of acceleration is present in the system, is what drives natural convection. In contrast, forced convection happens when a fluid is propelled by pumps, fans, or other devices in order to generate a convection current. Both spontaneous and induced convection have a significant role in the rate of heat transfer in several heat transfer systems. The need for energy increases rapidly as technology and human population both develop. A recent chain of events has resulted in a worldwide energy crisis, underlining the necessity to investigate unconventional energy sources and increase the effectiveness of energy conversion systems, which presently mostly use thermal systems (steam power plants, nuclear power plants, and internal combustion engines). Thus, improving the performance of the associated thermal system is one obvious way to increase the efficiency of the energy conversion system. Enhancing heat transfer performance has, unsurprisingly, been the primary focus in the discipline of thermal engineering. It has also been suggested and studied to combine two or more passive heat transfer improvement techniques into a single heat transfer system. Numerous technical, physiological, and industrial applications depend heavily on the effects of heat and mass transmission. These consequences of chemical reactions are particularly important in chemical and hydrometallurgical processes. The formation and dispersion design of chemical processing equipment, food processing and cooling towers, temperature distribution and moisture over agriculture fields and fruit tree groves, damage to crops due to freezing, oxygenation and dialysis processes, cryosurgery, hyperthermia, etc. are some interesting fields where heat and mass transfer phenomena subject to chemical reaction are significant. Numerous technical and industrial activities considerably worsen the issue of heat transfer. Researchers have employed convectional transport ideas for heat in literature. But because waves propagate at an infinitely fast rate, this convectional theory is inadequate. Thermal and relaxation times are taken into account to get around this problem. Because of the addition of the thermal relaxation time, the traditional Fourier's law has been modified in this way, meaning that heat is transmitted normally.

1.4 Dissipation Effects

Fluids have a physical characteristic called viscosity that describes how fluids resist flowing. Its transport feature, which expresses the movement of momentum over a velocity gradient, is more precise. The effects of viscosity are seen in straightforward flow scenarios like forced convection in a pipe and flow over a flat plate; the form of the velocity gradient is determined by the requirement for no-slip at the surface as well as the fluid's viscosity properties. The viscous dissipation of energy is the process by which mechanical energy is transformed into thermal energy by shearing action. It is sometimes referred to as "heat production" and is a common word in the domains of fluid mechanics and heat transport. This terminology is inadequate in two ways. It appears to be a breach of the first rule of thermodynamics, and it also suggests heat movement over a boundary. The fluid medium and the flow patterns determine how significant the viscous dissipation of energy is. Its quantitative contribution is often negligible. Temperature data often make it difficult to identify viscous dissipation in the fluid, but in some circumstances, it is possible to confirm its existence by calculating the reduction in mechanical energy, also known as the pressure drop. The shearing action is discovered to have two impacts on the fluid element during the construction of the energy equation for the fluid element. The fluid element is compressed or expanded as a result of the viscous forces, which also result in an increase in internal energy. The general energy equation has four distinct sets of components for the energy that is held within the element, the energy that is transmitted through convection, the energy that is transferred through conduction, and the energy that is dissipated through viscous motion. The viscous dissipation function has gotten little attention and is typically ignored entirely since the quantitative contribution of the last set of components is typically low compared to the previous variables. The energy equation is substantially more difficult to solve mathematically as a result of its inclusion. The change in mechanical energy must be large for the viscous dissipation function to produce a significant quantity in comparison to the conduction and convection terms. Since the size of the dissipated energy depends on the velocity gradients, flow conditions with a big velocity gradient as a defining feature should be researched. The flow of fluids via constraints is a two-dimensional flow scenario with significant mechanical energy changes. Equations are created using an analytical model to represent the viscous energy dissipation in the situation of unconstrained boundary expansions downstream of limitations. These equations provide a way to figure out how the energy wasted is distributed in both the axial and radial directions. By altering the input parameters in the expressions, the consequences of the various physical circumstances may be taken into account. The outcomes are contrasted with the values discovered using a thermodynamic system technique. Consider the continuous flow of a fluid through a constriction to demonstrate the crucial role that viscous energy dissipation plays in meeting the tenets of classical thermodynamics. The viscous energy dissipation increases the fluid's energy level; nevertheless, the expansion work in the case of a compressible fluid significantly reduces the fluid's thermal energy level. As a result, the viscous dissipation does not account for the expansion work in a compressible fluid, and the reduction in temperature with a matching fall in pressure for a compressible fluid is the outcome of an expansion process. The heat transfer phenomena heavily relies on dissipation effects as an energy source. When plates are heating or cooling, Ohmic heating and dissipation (viscous) become more important. The heat transfer phenomena can be witnessed in the cooling of nuclear reactors, electronic chip cooling, cooling of metallic sheets, and cooling of power generation systems. Additionally, the resistance that develops as electric current passes through the material during the Joule heating process causes heat to be produced. The Joule heating has both positive and negative effects on the system. PCR reactors, micro-valves for fluid management, di-electrophoretic traps Hot plates, thermometers and soldering irons„ thermometers, and soldering irons, manipulating bio-particles in diluted medium, electric fuses, etc. are a few examples of systems that utilise the Joule heating effects. On the other hand, excessive heat can result in various processes that melt or degrade the machinery parts, denature biological materials (such as proteins and DNA), develop bubbles, or cause chip systems to malfunction, among other things. The mechanism of viscous dissipation, in which the fluid's kinetic energy is converted to thermal energy, is important to note in this context. During ohmic dissipation, heat is generated by the uninterrupted fluid flow through the nearby boundary layer as a result of the interactions of the shear forces. Mixed convective flow has an irregular fluid flow through gravitational pull or viscous dissipation, that is clearly defined in regards of Eckert number. Viscous dissipation is the mechanism by which a fluid's viscosity absorbs energy from fluid motion and converts it into internal fluid energy that acts as an energy source. Most analyses of high velocity systems have overlooked viscous dissipation, despite the fact that it is crucial, particularly in high velocity systems. Many techniques that operate at high velocities or are subject to considerable variations in gravitational energy depend heavily on the effects of viscous dissipation. To illustrate the important function that viscous energy dissipation performs in fulfilling the requirements of classical thermodynamics, imagine a fluid flowing continuously through a constriction. Although the viscous energy dissipation raises the fluid's energy level, compressible fluids' thermal energy level is greatly decreased by the expansion effort. As a result, the expansion work in a compressible fluid is not accounted for by viscous dissipation, and the decrease in temperature and corresponding fall in pressure for a compressible fluid are the results of an expansion process. The energy source for the heat transfer phenomenon mainly depends on dissipation effects.

1.5 Slip Phenomenon

Because of its importance in several liquid dynamics systems, slip effects across stretching sheets have captured the interest of numerous researchers. It has been found in the literature that the majority of analyses made the assumption that surfaces have no slip conditions. However, it is imperative to have slip conditions rather than no slip requirements in a variety of physical situations where the aforementioned conditions are no longer valid. Hydrophobic surfaces that have been chemically treated or lubricated, wire nettings and perforated sheets, shear skin, porous or rough surfaces, hysteresis and spurts effects, and super-hydrophobic nano-surfaces have all demonstrated considerable slip situations. The flow of fluid across many interfaces, the polishing of prosthetic heart valves, and issues with rarefied fluid are some other industrial slip examples. The condition at a solid wall boundary for a viscous fluid is of a no-slip type, i.e., the fluid's velocity corresponds with the velocity of the solid boundary, which is a well-known fact at the macroscopic level in fluid dynamics. However, the fluid may slide across the surface of a solid in many real-world circumstances.

1.6 Thermal Radiation

The study of heat transfer phenomena has gained popularity across several industries. However, due to its widespread use in nuclear power plants, missiles, various aircraft thrusters, gas turbines, spacecraft, and satellites, radiative heat transfer has attracted some interest from engineers and scientists. The electromagnetic waves act as a conduit for thermal radiation. Due to the two surfaces' different temperatures, energy is transmitted between them. In this process, the energy is dispersed in all directions as waves. The surface temperature, surface type, and radiation frequency all affect how these radiations are emitted. While linear thermal radiation is relevant when the temperature difference is small, it is not inapplicable when the temperature difference is great. Due to the diverse uses of magnetohydrodynamics, many researchers have recently concentrated on using it in conjunction with the thermal radiation phenomena to explore heat and mass transfer analyses. The scientific community has given substantial consideration to the important role that thermal radiation phenomena plays in heat transmission. The thermal radiation phenomenon has a significant influence on fluid temperature and motion in the context of dynamics and industrial advancement at high temperatures. The phenomena of thermal radiation becomes more significant if there is a difference between the surface temperature and the surrounding air temperature. While nonlinear thermal radiation is true for all differences, linear thermal radiation assumes that the temperature difference is sufficiently modest. In this phenomena, energy expands outward in the form of electromagnetic waves from a bright surface to the point of absorption over the whole area. The mechanism of radiation is distinct from the two transport processes we've already covered, namely: (1) energy transport by conduction, which is proportional to the temperature gradient's negative value; and (2) energy transport by convection, which is proportional to the temperature difference between the body and the moving fluid in contact with it. Thermal radiation is an electromagnetic process that enables energy to go over regions of space devoid of any matter at the speed of light. The Stefan-Boltzmann law, which says that the radiant energy communicated is proportional to the difference in the fourth power of the temperatures of the surfaces, describes the exchange of radiant energy between surfaces or between an area and its surroundings. The proportionality parameter is known as the Stefan-Boltzmann parameter." Similar to conduction and convection, radiation is a method of heat transfer that doesn't need direct contact between the heat source and the object being warmed. Warm radiation, often known as infrared radiation, has the ability to transmit warmth through empty space. This electromagnetic radiation is one type. During the radiation time, neither a medium nor a mass are exchanged. The sun's heat and the warmth emitted by a light fibre are examples of radiation. Because of the unpredictable temperature fluctuations of charges inside the material, all things generate electromagnetic radiation. An innate characteristic of nature is the emission of electromagnetic radiation as a function of a body's temperature. Thermal emission-based electromagnetic radiation sources are essential for energy harvesting, illumination, spectroscopy, and sensing applications. Many of these sources, which are frequently made of bulk materials that are several hundred microns thick, are inefficient and emit a lot less power than a perfect blackbody. Because of its applications in engineering fields like solar ponds, solar collectors, gas turbines, furnace design, photochemical reactors, turbid water bodies, nuclear power plants, and the various propulsion systems for missiles, satellites, aircraft, and spacecraft, for example, researchers have discussed the effect of non-linear thermal radiation on fluid flows. Recently, a lot of researchers have looked into how thermal radiation affects heat transfer processes on surfaces that have been stretched by conventional and nanofluids. A radiating surface produces electromagnetic waves transporting heat energy in thermal radiation. Thermal radiation must be used to produce high temperatures. Thermal radiation has a significant influence on a variety of technical processes that take place at high temperatures, such as the reentry of missiles and spacecraft. Many scientists are interested in investigating the effects of radiation on the convective flow of hydromagnetic nanofluids in various geometries and combinations. The parameters of thermal radiation (T-R), which is electromagnetic radiation given off by a substance as a function of its heat, depend on the material's temperature. Thermal diffusivity rises as a result of thermal radiation heating. Uses for thermal radiation's results include electrical energy, food, solar energy panels, projectiles, nuclear power plants, gas turbines, and aerospace engineering. These are all high-temperature, commercial applications. In the polymer processing sectors, where the end product's quality is somewhat affected by heat-controlling factors, thermal radiation effects may be critical for sustaining heat transmission. Thermal cycling, high-energy protons, high vacuum, ultraviolet (UV) radiation, atomic oxygen (AO) radiation, and other factors make up the space environment. Materials that are left unprotected in the space environment must have their performance under space radiation assessed since this can cause material deterioration, component or structural damage, decreased system dependability, and even a reduction in the amount of time a spacecraft can operate. Only such materials could be chosen for aerospace usage if they could endure the extreme space radiations for the duration of their intended service life. Materials are often subjected to terrestrial radiation testing prior to the space flight review. Due to their constant exposure to shifting solar light, spacecraft frequently experience alternate temperature fluctuations. Materials used in space applications must be able to tolerate temperature variations between extreme high and low temperatures, or "thermal cycling," as it is known.

1.7 MHD

Given its wide range of practical uses, researchers have taken a keen interest in studying electrically conducting non-Newtonian liquid. Numerous domains, including material processing, geophysics, astrophysics, liquid metal systems for fusion reactors, plasma research, MHD power generation, the petroleum industry, materials purification, and boundary layer control in aerodynamics, are where these applications are used. In MHD, the flow is distorted by the magnetic field as a result of the Lorentz force, which is produced by the interaction of the fluid's electric current and the applied magnetic field. This Lorentz force acts as an opposing force to the flow, creating resistance within the momentum barrier layer. Fluids are a crucial component in the design and manufacture of all contemporary technology and technologies. In engineering and industry, many structures and equipment are designed with the aid of a solid knowledge and use of universal mechanics. Engineering and industrial applications including thermal protection, targeted medicine administration, pumps, and power generation all heavily rely on the field of MHD. The MHD phenomena interacts with specific gadgets and the natural world often. For instance, magnetohydrodynamic flow happens inside the planet, in stars, and in the sun. Numerous laboratory tools, including propulsion systems, electrical discharges, and fluid electromagnetic fields, have been developed to interact with MHD directly. Targeted drug delivery and magnetic resonance imaging (MRI) are two biomedical fields in which the application of MHD is particularly significant. Magnetohydrodynamics is another name for the discipline that studies coupled conducting magneto-fluid phenomena (MHD). Astrophysics is another subject where MHD is used. MHD is used to model or explain how solar winds or other stellar winds behave, and it has grown to be a crucial tool for identifying and predicting space weather. For instance, knowing when a solar storm blast is coming will help you be ready for a sudden break in radio wave communications. The nuclear fusion reactor is one of the main MHD applications. The MHD equations do a good job of describing the plasma within the reactor. The public is increasingly coming to understand the value of alternative energy sources like nuclear energy as the supply of fossil fuels continues to decline. It is becoming more and more crucial to optimise and construct safer, more effective reactors. We might see the value and necessity of taking into account the coupled magneto-fluid problem from the applications of the coupled magneto-fluid phenomenon stated for conducting and non-conducting situations. When the magnetic field and velocity field are connected, an electrically conducting fluid is said to be in magnetohydrodynamics (MHD). It also refers to magneto fluid dynamics or hydromagnetics. Well-known examples of electrically conducting fluids include plasmas and liquid metals, mercury, sodium or molten iron, gallium, and ionised gases. Electrolytes, or salt water, are another type. Hannes Alfven created the notion of magnetohydrodynamics. In three words, he summarised the operational definition of magnetohydrodynamics: "magneto" denotes a magnetic field, "hydro" denotes a liquid, and "dynamics" denotes motion. An integral and important part of the study of magnetics is played by magnetohydrodynamics. The core idea of magnetohydrodynamics is that current may be produced in conductive fluids by a magnetic field. It is also in charge of applying forces to the fluid and has an impact on the magnetic field itself. There are countless uses for magnetohydrodynamics (MHD), including the analysis of the ionosphere, the creation of the Earth's magnetic field, and the use of electromagnetic forces to pump liquid metals. Magnetohydrodynamics and other methods are frequently used to investigate induction furnace and casting processes. MHD, or magnetohydrodynamics, studies the dynamics of fluids with non-zero electrical conductivity that interact with a magnetic field. A magnetic field and the velocity of an electrically conducting fluid cause the fluid to create an electric current. a magnetic field (transverse) and an electrically conducting fluid flowing together produce a force known as the Lorentz force. The initial motion of the conducting fluid is likely to change as a result of this force. Additionally, the primitive magnetic field is supplemented by the magnetic field produced by the induced currents. As a result, the conductor's velocity and the electromagnetic field are intertwined. Applications for MHD include fusion research and MHD accelerators, delay in the transition from laminar to turbulent flow due to the power generator. Due to the enormous impact of magnetic fields on boundary layer flow regulation and the participation of several systems utilising electrically conducting fluids, MHD natural convection flow in a permeable medium is particularly noteworthy. Flow metres and accelerators, plasma research, thermal insulation, MHD generators, heat storage beds, MHD pumps, nuclear reactor cooling, and geothermal energy extraction are further applications for this type of flow. The hydromagnetic behaviour of boundary layers over stationary or moving surfaces in the presence of a transverse magnetic field is one of the fundamental and important concerns in this discipline. MHD boundary layers have been found in a number of technological systems that include fluid metal and transversely flowing plasma. Many scientific and technical fields, including glass production, power generation, furnace design, thermo-nuclear fusion, space flight, plasma physics, high-temperature aerodynamics, and solar energy systems, depend on natural convection and radiation heat transfer. Depending on the properties and arrangement of the surface, radiant heat transfer is frequently equivalent to convective heat transfer in many real-world circumstances. Designing numerical techniques that provide answers that are physically accurate has been an exciting but difficult job. Entropy principles and positivity preserving characteristics have been investigated for several schemes in the study of non-conducting fluids, such as those modelled by the compressible Euler equations. When a fluid is exposed to magnetic and electromagnetic forces, the behaviour of the flow is referred to as "magnetohydrodynamic" (MHD). Applications of the MHD include highly conductive boilers, solar panels, and the polymer sector. To preserve nanofluids under the effect of electromagnetism, researchers have conducted a wide range of investigations in this field.

This dissertation is further arranged as follows:

Chapter 2 comprises the review of past published literature. In order to facilitate the readers, fundamental definitions and concepts has been discussed in Chapter 3. Chapter 4 is devoted to a detailed analysis of Ishak's [60] work. It discusses the study of radiative energy in viscous hydromagnetic liquid flow deformed by exponentially stretchy plate. The stretching propulsion of Casson material through a sheet is examined in Chapter 5 under the impacts of the magnetic field and velocity slip. The analysis retains nonlinear stratification as well as radiative effects. The viscous dissipation and convective heating processes serve as representations for the heat transfer phenomenon. In order to get equations in their dimensionless form, similarity variables are included. By using the homotopy approach, the obtained equations are evaluated analytically [$61-63$]. The physical explanation of velocity and temperature fields is illustrated graphically and thoroughly described. Conclusions are contained in Chapter 6.

Chapter 2

Literature Survey

The heat transport mechanism has a substantial significance regarding industrial view point. Several scientists and explorer focus on this area. In technically advanced technologies and numerous industrial related fields, the theory of non-Newtonian liquid has great influence because model of Newtonian liquid unable to express diverse flow features. A non-Newtonian liquid that satisfies the nonlinear type relationship among stress (shear) and strain (shear) strain. Such type of fluid theory have impactful implementation in advanced engineering, for instance, in industry of petroleum utilized to draw out crude oil from diverse petroleum productions. There are many cases where the Newtonian fluid characteristics are not valid but owing to this fact for non-Newtonian fluid the researchers desire to develop the complex models. From the last few decades, the significance of fluid of non-Newtonian has been increased, especially in the field of research. The fluids of non-Newtonian have several ever-incrementing applications in industry related sectors, but few particular are here mentioned, such as polymer processing, biological materials, enhancing and reducing systems of cooling/heating at larger-scale, flow tracers, processing plastic foam, muds handling, engineering of process and biochemical, well drilling, lubrication processes, reducing friction in oil pipeline, chemical processing, molten plastic extrusion in industry, food processing related industries, flow analysis regarding biomedical, all emulsions, reducing fluid friction, complex mixtures and slurries. Eminent scientists and investigators concentrated on non-Newtonian liquids under the diverse fluid geometries. Therefore, simulation and formulation of phenomena of non-Newtonian fluid flow help in facilitating, and play the significant role towards the life of human. Researchers analyzed diverse models of non-Newtonian fluid regarding physical and computational features such as power law model, viscoplastic model, model of second grade, Jeffrey model, model of Bingham plastic, Brinkman model, Oldroyd-B fluid, Walters-B fluid, Maxwell model and Casson model, that different models of fluid exist in the past literature have numerous features or definite limitations, for example, model of second grade fluid explains efficiently the elasticity but unaware of viscosity, on contrary, power-law model discussed viscosity but unable to describe elasticity characteristics, these facts attracts the mathematicians and researchers towards the analysis of such complex fluid models. Well-managed investigation of such type of liquid flow models have notably significant for theoretical analysis and practical executions regarding modernistic mechanization. Out of such fluids, Casson fluid gained exceptional attention, also labeled as a shear-thinning fluid, which is the conventional non-Newtonian substance because of its numerous widespread applications and impactful role in various field related to metallurgy, mechanical, bio and chemical engineering. The model of Casson fluid shows different characteristics of matters, when yield stress dominates the shear stress then static flow exists and fluid describes the elastic solid properties whereas on contrary when shear stress shows dominancy over yield stress then it guarantees to be the Casson fluid flow. Diverse products, like synthetic lubricants, coal, jelly, concentrated fruit juices, honey, paints, tomato sauce, artificial fibers, china clay, pharmaceutical chemicals, soup etc., are few practically implemented description of such fluid. Blood also considered in the category of Casson liquid [1], [2] since it incorporated various substances such as globulin, red blood cells, protein and fibrinogen in plasma (aqueous base). Casson fluid first develops in 1959 by Casson [3] in order to understand the flow behavior of suspended pigment-oil. After that many scientists, investigators, engineers extend the Casson's work under diverse situations. Mukhopadhyay et al. [4] described the stretching phenomenon in two-dimensional motion of non-Newtonian material under the heating features. Here Casson fluid is utilized in order to observe the non-Newtonian characteristics. The modeled problem is evaluated through closed form solution and with numerical procedure. Results tells that Casson parameter enhanced the temperature profile. Nadeem et al. [5] explained the influence of magnetic properties to determine the flow of Casson type liquid. The motion is analyzed under the stretching mechanism. Here the sheet is porous in character and responsible for the suction/ injection of fluid. The exact solutions were acquired in order to determine the impacts of involved effects. Ramesh and Devakar [6] disclosed the basic flow features of Casson incompressible fluid. The study incorporated Couette, generalized Couette and Poiseuille type of flows through the sheet held parallel to each other. To ehance the flow analysis, slip phenomenon has been considered. Outcomes suggested that slip features enhance the volume flow rate in two cases i.e., in generalized Couette and Poiseuille. Reddy et al. [7] disclosed the Hall impacts in non-Newtonian liquid motion through vertically held channel with free convection. The features of hydromagetic taken into account to witness the effect of magnitization. The heating profile is explored under the aspects of Joule heating and viscos dissipation. The whole analysis is evaluated utilizing homotopic technique. Results are shown through the graphical analysis. Rehman et al. [8] investigated the mixed convective impacts in Casson liquid flow caused by both stretchable cylinder and flat surface considering MHD. Thermal stratification impact is accounted to study the temperature field analysis. Numerical solution had been presented. Skin friction and rate of heat transport were analyzed. Ahmed et al. [9] described the slip and irreversibility phenomena in flow of non-Newtonian material through thicked stretchable surface considering mixed convection. Concentration profile is analyzed under the chemical reaction. These aspects have significant impact on flow fields. Raza [10] discussed the velocity slip phenomenon in the motion of fluid saturated in the point of stagnation. The flow of liquid is caused by stretchable sheet under the impact of magnetic strength. Heat characteristics are studied under the aspects of thermal radiation and convective condition. Kataria and Patel [11] discussed the Darcy phenomenon in the motion of Casson fluid caused by vertical held sheet. The flow of liquid is caused by stretchable sheet under the impact of magnetic strength. Heat and mass characteristics are studied under the aspects of heat generation/sink and chemical reaction. Patel [12] discussed the mixed convection phenomenon in the motion of Casson fluid saturated in the permeable medium. The flow of liquid is caused by stretchable sheet under the impact of magnetic strength. Heat and mass characteristics are studied under the aspects of heat generation/sink, thermal radiation, cross-diffusion and chemical reaction. Mabood and Das [13] discussed the melting phenomenon in the motion of Casson fluid saturated in the permeable medium. The flow of liquid is caused by stretchable sheet under the impact of magnetic strength. Heat characteristics are studied under the aspects of heat thermal radiation. Hussain et al. [14] described the vary viscosity impacts on flow of Casson material.

The two-dimensional steady state flow is caused using the stretching phenomenon. The whole analysis is performed under the impact of magnetic field. The flow behavior of a Casson fluid with variable viscosity flowing past an extending or contracting surface is examined under the influence of some magnetic field. Goud et al. [16] described the vary heat source impacts on flow of Casson material. The two-dimensional steady state flow is caused using the stretching phenomenon. The whole analysis is performed under the impact of magnetic field. The flow behavior of a Casson fluid with suction/injection flowing past an fluctuating surface is examined under the influence of some magnetic field. Nandeppanavar et al. [16] described the Brownian diffusion impacts on flow of Casson nanomaterial. The two-dimensional steady state flow is caused using the stretching phenomenon. The whole analysis is performed under the impact of thermophoretic diffusion. Naqvi et al. [17] described the nanoparticles impacts on flow of Casson material. The two-dimensional steady state flow is caused using the stretching phenomenon. The whole analysis is performed under the impact of magnetic field. The heat flow behavior of a Casson fluid with Joule heating is examined under the influence of some viscous dissipation. Zhou et al. [18] disclosed the unsteady two dimensional motion of Casson liquid near the point of stagnation. The stretchable sheet is permeable and magnetic as well as slips impacts affect the flow phenomenon. The suction/injection features incorporate in the analysis. Heating profile is studied considering heat absorption and thermal radiation. The analysis found that magnetic field increments the drag forces whereas rate of heat transfer grows due enlarge suction and radiation effects. Chen et al. [19] disclosed the two dimensional motion of Casson liquid caused by stretching sheet. The stretchable sheet is impermeable and magnetic as well as duble-diffusive impacts affect the flow phenomenon. The thermophoretic features incorporate in the analysis. Heating profile is studied considering thermal stratification as well as solutal stratification. The analysis found that thermophoretic phenomenon increments the mass trasport rate whereas thermal field grows due enlarge thermophoretic effects. Banerjee et al. [20] disclosed the two dimensional motion of Casson liquid divergent channel. The divergent channel is permeable and suction/injection impacts affect the flow phenomenon. Heating profile is studied considering viscous dissipation. The analysis found that viscous dissipation and Casson material show significant impacts on termerature field. Hussain et al. [21] disclosed the two dimensional motion of Casson liquid through wedge. The stretchable wedge is permeable and magnetic impacts affect the flow phenomenon. The suction/injection features incorporate in the analysis. Heating profile is studied considering Joule heating and convective surface constraint. The analysis found that Casson parameter decrements the velocity whereas rate of heat transfer grows due enlarge convective parameter. Rehman et al. [22] reported the mixed convection features in order to study the Sutterby fluid flow due to stretchable plate. Further radiative and stratification aspects are incorporated to examine the heat flow. Results revealed that stratification phenomenon reduces the heating profile. Prameela et al. [23] discussed the mixed convective phenomena in flow of magneto Casson liquid deformed by permeable vertically held oscillating plate. Thermal diffusion is also accounted. Here Casson liquid parameter reduces the velocity field. Bejawada et al. [24] explained the heat and mass transportation are affected by chemical reaction as well as radiation. The Casson liquid flow is caused by inclined sheet under the nonlinear stretching mechanism. The slip and non-Darcy phenomena are also incorporated to study flow characteristics. Study revealed that porosity decays the temperature. Reddy et al. [25] discussed the heat transport via varying thermal conductivity and varying liquid viscosity. The motion of Casson fluid is observed through movable sheet. The heat transport problem is identified using the varying plastic absolute viscosity. Also heat source and convective surface condition are incorporated to see the flow. The slip effects are accounted to study flow assessment. Hussain et al. [26] reported the magnetic and dissipative features in order to study the flow and heat characteristics. For this purpose, Casson fluid model is utilized. The flow is deformed through the inclined permeable cavity. The natural convection is also included to modelling the momentum equation. Energy loss is estimated using the irreversibility phenomenon. Study witnessed that heat transport rate decreases by incrementing cavity inclination. Ali et al. [27] disclosed the free convection features in flow of Casson material deformed through parallel sheets. The fluid taken of dusty type material shows the characteristics of Casson liquid. The magnetic field impact is also accounted in the analysis. Swarnalathamma et al. [28] explored the suction/injection characteristics in Casson liquid. The flow phenomenon is analyzed using the vertically held inclined sheet under the impact of magnetic strength. Heat and mass transport are investigated considering the radiative, chemically reactive and absorption features.

Large-temperature environment can cause the thermally influenced damage in-flow regimes.

Usually the common sources of thermally influenced damage may be the thermal striping, thermal cycling, and thermal stratification. It commonly occurs in constrained subjected flow regimes where the fluid movement happens with weak velocity and has a reasonable temperature difference. The phenomenon of thermal stratification happens when diverse kinds of stream under differing temperature in contact with each other. Due to such variation in temperature, thick water immerses, colder, whereas warmer and less dense water stay over the colder surface of water. One of the significant features of thermal stratification is that the water layer situated deep into the ocean which do not have any surface contact, loses dissolved oxygen and therefore unabled to support aquatic life. Owing to this fact, the species of cool water such as trout limited to the zone beneath the water surface of greater temperature because lower cold layer in deep reservoirs and lakes become less oxygenated zone. Therefore, noteworthy layer of fish with less movement and food can be seen, and a good angler take full advantage of that. Moreover, energy related storage capacity such as utilization of solar thermal systems and many others fields witness the significance of thermal stratification. Kandasamy et al. [29] described the nanomaterial flow towards stretched vertically held sheet. The magnetic field influence has been taken into account. The flow and heating profiles are explored utilizing the stratification phenomenon, Brownian and thermophoretic diffusions. The study concluded that flow and thermal profiles are impactfully influenced by considered aspects. Srinivasacharya and Upendar [30] discussed the micropolar liquid flow towards vertically held sheet. The magnetic field influence has been taken into account. The flow and heating profiles are explored utilizing the dual stratification phenomena and free convection. The considered problem is evaluated through numeric technique. The study concluded that flow and thermal profiles are impactfully influenced by considered aspects. Srinivasacharya and Surender [31] disclosed the viscous liquid motion towards vertically held flat sheet. The mixed convection influence has been taken into account. The flow, concentration and heating profiles are explored utilizing the non-Darcy and dual stratification phenomena. The considered problem is evaluated through numeric technique. The profiles are estimated through graphical description. Mishra et al. [32] depicted the micropolar liquid motion towards vertically held flat sheet. The magnetic influence has been taken into account. The flow, concentration and heating profiles are explored utilizing the heat source and dual stratification phenomena. The considered problem is evaluated through numeric technique. The profiles are estimated through graphical description. Muhammad et al. [33] described the motion of second grade type ferrofluid using stretchable plate. The flow features are disclosed via magnetic dipole near the region of stagnation point. The aspect of thermal stratification is also incorportaed. The investigation reveals that stratification phenomenon significantly impacted in controlling the heat transport rate. Hayat et al. [34] depicted the motion of Oldroyd -B material using stretchable plate. The flow features are disclosed via mixed convection. The aspects of heat source/absorption and thermal stratification are also incorportaed. The investigation reveals that buoyancy phenomenon significantly impacts the heat transport rate. Hamid et al. [35] described the Williamson material flow deformed by inclined stretchable cylinder. The magnetic field and nanoparticles are incorporated in the analysis. Mixed convective, double stratified and radiative features are implemented in order to study heat and mass profiles. Stratification phenomenon diminishes both the concern profiles. Khan et al. [36] disclosed the varying viscosity Maxwellian nanomaterial flow deformed by thicked varying surface. The Brownian and thermophoresis diffusions are incorporated in the analysis. Double stratified and activation energy features are implemented in order to study heat and mass profiles. The analysis is evaluated numerically using RK-5 method. The considered aspects influences the flow profile significantly.Ijaz and Ayub [37] discussed the modified fluxes in stratified flow of non-Newtonian nanomaterial flow deformed by permeable stretchable surface. The Brownian and thermophoresis diffusions are incorporated in the analysis. Slip, chemical reaction and activation energy and heat source features are implemented in order to study heat and mass profiles. The analysis is evaluated analytically using homotopic method. The considered aspects influences the flow profile significantly. Khan et al. [38] calculated the magnetic intensity impacts on Prandtl liquid flow deformed through the stretchable sheet. The flow is two-dimensional in nature. Thermal stratification and heat generation phenomena are accounted in order to discuss the heat transport features. This work concluded that stratification phenomenon increments the heat transmission rate. Irfan et al. [39] calculated the magnetic intensity impacts on Oldroyd-B nanoliquid flow near the region of point of stagnation. The flow is two-dimensional in nature. Dual stratification, radiation and heat generation phenomena are accounted in order to discuss the heat and mass transport features. This work concluded that stratification phenomenon increments the heat transmission rate. Ramzan et al. [40] discussed the nanofluid motion through movable disks. Nanotubes are utilized as nanoparticles. Radiative and stratified features also incorporated. Magnetic field is utilized to study flow field. Malik et al. [41] investigated the influence of numerous design parameters on Falkner-skan stratification flow performance. A mixed convection aspect is incorporated to see its impact on Jeffrey fluid flow performance near about the point of stagnation. Viscous dissipation is also considered to see thermal performance. The results provide a quantitative estimation of profiles under pertinent parameters. Khan et al. [42] investigated the influence of inclined MHD on squeezed stratification flow performance. A non-Darcy aspect is incorporated to see its impact on viscous fluid flow performance. Viscous dissipation, convective conditions and Joule heating are also considered to see thermal performance. The results provide a quantitative estimation of profiles under pertinent parameters. Khan et al. [43] disclosed the Darcy phenomenon in hyperbolic type tangent liquid flow caused by stretchable sheet. Radiation, dual statification and chemical reaction aspects are implemented to study heat and mass transport. Megahed and Abbas [44] examined stratification influences on flow profile by way of the stretchable sheet. Flow equations are reduced using a suitable transformation and later numerically evaluated through shooting procedure. The outcomes demonstrate that the existence of the Darcy medium, varying thermal conductivity, chemical reaction, thermal stratification, and heat generation have a significant influence on the flow fields. On a inclined stretchable cylinder with double stratification impacts, Rehman and Shatanawi [45] investigated the nonlinear radiative flow. The constitutive equations have been computed numerically for radiative stratified flow. The outcomes display that the existence of stagnation point, heat generation, and magnetic field have a impactful role on the flow profile. Farooq et al. [46] analyzed the Powell-Eyring nanofluid flow in the region of stagnation point. The stretching phenomenon is utilized to deform the fluid. Heating aspects are explored using the viscous dissipation, stratification of thermal type, Joule heating. Magnetic effects are also accounted to see its impact on flow field under the irreversibility features. Rehman et al. [47] disclosed the phenomenon of Joule heating in Powell-Eyring stratified material with advanced melting constraint. Few more investigation regarding this direction are explored in references $[48-50]$.

The exploration on the flow associated to the boundary layer phenomenon along with heat transport past the linearly stretchable sheet is quite successful over the past few decades because of its extensive utilization in field of industry and science. Hot rolling, polymer extrusion, production of materials through spinning of metal, continuous plastic films stretching, artificial fibers, glass fiber, drawing of copper wires, wire drawing and metal extrusion, etc., are the few of these utilization. The current work emphasizes on the boundary layer related flows, transportation of heat with stratification of Casson fluid across a stretchable surface. The liquid which streams is indicated as fluid. The characteristics of heat transmission are key factor as per the flow related to boundary layer of a Casson fluid through stretching (moveable) sheet. The heat transport features via stretchable surfaces are noteworthy to acknowledge in order accomplishing the required quality. This is because to the utility of the final product primarily depends on rate of stretching and heat transfer. The important problem of boundary layer flow across a stretched sheet occurs in a number of industrial production processes. Making paper and drawing sheets out of plastic, glass, fiber, and rubber is one example of this. Crystal growth while extending polymers for manufacture, chilling metallic sheets in a cooling solution, etc. Because it has applications in polymer processing technologies, this flow issue with heat transmission has attracted the attention of researchers. Flows maintained by stretching (shrinking) sheets or injecting or suction fluid through porous surfaces have a vast and profound range of uses in engineering and business. As a result, scientists, engineers, and mathematicians have given the topic a lot of thought in an effort to broaden the use and understanding of such flows. The renowned work of Sakiadis is where the history of extending flows begins [51]. Researchers and scientists later studied and examined this issue, and many facets of the model problem were investigated by taking into account both Newtonian and Non-Newtonian fluid. The modeling and resolution of the fluid flows caused by stretching (shrinking) plates in various circumstances has been the subject of extensive investigation. Numerous experts have examined the well-known work of Crane and expanded the precise answer for situations of stretching and shrinking. But the precise analytical solution to only few stretching (shrinking) issues is known. The physical effects of the boundary layer close to the sheet surface are understood through the mathematical study of the stretching (shrinking) issues. Stretching (shrinking) issues are always approached by first converting the governing strong non-linear boundary value problem into a self-similar ODE, which is then precisely, analytically, or numerically solved. Numerous numerical techniques were used to arrive at numerical solutions to these extremely nonlinear issues in this field. The stretching rate's magnitude must be minimal due to the delicate nature of the problem of stretching sheets. This also guarantees that the stretching material discharged into the liquid from the space between the two solid blocks remains a level surface rather than curving. The issue therefore exhibits the finest mathematical manageability. In many physical settings, the sheet may be extended vertically into the cooling liquid rather than horizontally. Under these conditions, two mechanisms—the motion of the stretched sheet and the buoyant force—determine the liquid flow and the heat transfer properties. The flow is significantly influenced by the thermal buoyancy that results from heating and cooling a stretched sheet that is travelling vertically and heat transfer characteristics. The most common method of stretching the sheet that aids in regulating the liquid flow utilized for cooling purposes is inclined stretching. Both the horizontal and vertical stretching sheet issues are subsets of the inclined stretching sheet problem. The fluid mechanical characteristics that are sought for the final product (a stretched sheet) in a polymer extrusion process are mostly dependent on the pace of cooling. The liquid that is primarily intended to cool the stretched sheet is crucial in establishing the desired final product attribute. Due to the numerous engineering applications, including those in bio-engineering, the analysis of the magneto-hydrodynamic (MHD) flow field of an electrically conducting fluid in a boundary-layer caused by the stretched sheet/surface has become crucial in fluid dynamics and heat transfer for instance The cooling of metallic plates in a cooling bath, the aerodynamic extrusion of plastic sheets, the continuous extrusion of polymer sheets from dyed and heat-treated materials, and other processes all exhibit the properties of moving continuous surfaces. All of the body's capillaries, arteries, and veins carry blood. Blood is carried by capillaries via skin and muscles, arteries convey it away from the heart, and veins bring it back there. Additionally, we are aware that arteries, veins, skin, and muscles are all continually stretched. We refer to the skin, muscles, and cylinders of arteries and veins as stretching and contracting surfaces, respectively. Blood flow can therefore be used to stretch or contract surfaces. The assumption of constant flow is a major constraint in many physical settings since it leads to uneven flow in some cases, such as when a sheet is suddenly stretched or when its temperature changes suddenly. Thus, it becomes important to take into account how velocity, temperature, and other physical properties of flow change with time. In these circumstances, the accompanying technical and industrial processes affect the flow behavior as well as the rate of heat transmission. Velocity, pressure, temperature, and other time-dependent flow parameters are crucial for unsteady fluid flow. Due to excessive impulsive body movement, vibration, inadvertent rapid body acceleration when travelling in a car or participating in other athletic competitions, this sort of fluid flow can be seen in the human body. Fluid dynamics faces a serious challenge when a viscous fluid flows through a stretched sheet and transfers heat. Chemical engineering and metallurgy both have use for the investigation of the flow across a stretched sheet. However, under some circumstances, a rapid stretching of the flat sheet or a step change in the sheet's temperature might cause the flow field, heat transfer, and mass transfer to be unstable. On the boundary layer flow and heat transmission issues when the stretching force and surface temperature are changing over time, a few studies have been written. Patel et al. [52] investigated how irregularities in thermal radiation, chemical reaction, and heat generation impact the flow of a micropolar fluid. With the use of graphical depiction and tabular forms, the impact of diverse and significant dimensionless factors on the momentum, micropolar, energy, and concentration equations has been examined. It has been shown that the micropolar parameter decreases the velocity profile while the buoyancy force parameter, mixed convection parameter, and permeability parameter increase the velocity distribution. Ahamd et al. [53] presented to explore the stagnation flow process in a rotating disc and is based on the Darcy Forchheimer theory. The constitutive relationship of a second-grade fluid serves as an illustration of the nanoparticles to be saturated. Additionally, a hydro-magnetic phenomenon affects the flow. To comprehend fluctuations in the heat transport process, nonlinear stratification and different thermal conductivity properties are combined. Also covered is the variable influence of thermophoretic and Brownian diffusions on the movement of nanoparticles in the base fluid. To analyze mass and heat transport, the characteristics of singular slip and thermal leap are put to the test. Transport equations also take into account chemical reaction and thermal radiation. The study came to the conclusion that the lower thermal stratified and temperature jump characteristics related to the minimum temperature. The tangential and radial velocity components are both improved by a larger inertia parameter. The concentration function is intensified by higher diffusive parameters. Manzoor et al. [54] investigated at the impacts of a magnetic field on energy flux via concentration panel, mass flux via temperature distribution, and the enhanced radius of nanoparticles on fluid dynamics. It is possible to construct the dimensionless form of flow equations by developing relevant similarity techniques. The shooting approach is utilized to diminish ordinary differential equations (ODEs) from higher to lower order for the estimated model of the relevant governing system and then Keller box method ia utilized to evaluate lower order equations. Addressed are the characteristics of major dimensionless components in opposition to the velocity concentration, volumetric concentration, and temperature panels. As the heat source parameter, nanoparticle radius, nanoparticle concentration, and thermal radiation are rated more highly, the temperature distribution increases. Reddy and Maddileti [55] investigated the influence of the joule parameter and the Casson nanofluid on the variable radiative flow of the MHD stretching sheet. The governing equations (PDEs) for the flow are transformed into ordinary differential equations by using similarity transformations (ODEs). The resultant equations are numerically solved using the Keller box method. The findings of the most important physical characteristics have been visually shown and discussed, including fluctuations in concentration, temperature, and velocity profiles. For various sorts of variables, Sherwood number, Skin friction, and Nusselt number are presented together with their numerical values and explanations. The problem of heat and mass transfer in the hydromagnetic flow of a micropolar fluid via a stretched sheet with the presence of viscous dissipation and chemical reaction was investigated by Saidulu and Reddy [56]. It has been examined how important variables affect the velocities, micropolar, temperatures, and concentration functions. Measurements of the magnetic field have revealed that the micropolar factor affects all functions in the opposite direction, with the exception of temperature, whereas the microrpolar parameter causes objects to move more slowly and increases the temperature and concentration functions. With improved values of the Schmidt number, the temperature rises and the concentration declines as the Eckert number grows. Under the conditions of slip velocity and convective heating, Alrehili [57] considered the heat and mass transfer flow of a viscoelastic thermal across a stretched sheet. First, a dimensionless transformation is used to change the model's nonlinear partial differential equation governing equations. Here, we propose the possibility of the presence of an extremely hot additional fluid beneath the bottom surface of the stretched sheet, which may aid in warming the surface via convection. The suggested model is controlled by the viscoelastic non-Newtonian nanofluid that satisfies Walter's liquid B' fluid model. Brownian motion and thermophoresis' effects are taken into account. The shooting method-based numerical technique is used. A number of recently noteworthy measurements against velocity, temperature, and concentration distributions are displayed in graphs as a representation of the results. In light of the results, it has been noticed that the thickness of the thermal boundary layer obviously increases as the effects of Brownian motion and thermophoresis strengthen. However, the thermal boundary layer becomes thinner due to viscoelasticity. The analysis by Mishra et al. [58] of the MHD boundary layer flow of Williamson micropolar fluid pasting a non-linearly stretching sheet under the presence of nonlinear heat absorption/generation term, which arises in convection due to high temperature and is the novelty of the present work, is what makes this work unique. With proper boundary conditions, the governing equations for the aforementioned physical setup have been taken into consideration. Following the introduction of a stream function and an appropriate similarity transformation, the updated governing equations in the form of an ODE with boundary conditions have been derived. The shooting method is applied to solve the considered boundary value problem. Graphs have been used to show how different factors affect flow variables including temperature, microrotation, and velocity. The Hartmann number and Williamson parameter in particular are shown to have an influence on increasing skin friction. The temperature dependent relaxation time factor in the Cattaneo-Christov concept was examined by Ahmad et al. [59]. Modeling and analysis are done for the non-isothermal flow of a viscous fluid through a porous material. Under the heading of stretching mechanisms, the fluid deformation is covered. As part of the Cattaneo-Christov heat flow model, a research is also conducted to examine the characteristics of heat transport. Temperature affects thermal relaxation time, which is also taken into account. To observe temperature changes, the aspect of thermal stratification is also used. The right variables are used to simplify the energy and flow equations. Then, using an analytical convergent method, the reduced constitutive flow equations are calculated. Through various plots, the major effects of the many factors on the flow field and aspects of heat transfer are thoroughly assessed. The mathematical evaluation of the skin friction coefficient. According to the findings, the temperature considerably drops as the effects of stratification rise. The temperature function exhibits rising behavior as the thermal relaxation time parameter increases.

Chapter 3

Basic Concepts

This chapter provides explanations of certain fundamental terms, theories, and laws to aid in understanding the analysis in the dissertation.

3.1 Fluid

A substance that deforms under the external applied force continuously is demonstrated as fluid.

3.2 Fluid Mechanics

The branch of science which examines the behaviour and properties of fluid both in motion and at rest is illustrated as fluid mechanics. It is further classified as:

3.2.1 Fluid Static

This branch illustrates the behaviour of fluids at rest.

3.2.2 Fluid Dynamic

It deals with the behaviour of fluids in motion.

3.3 Flow

It is the process of continuous deformation of fluid under an action of different forces. It is further categorized as:

3.3.1 Steady Flow

If the fluid flowing per second does not vary then it is characterized as steady flow.

3.3.2 Unsteady Flow

If the fluid flowing per second varies then it is classified as unsteady flow.

3.3.3 Laminar Flow

If the particles of fluid move over the well-defined path and do not cross each other then such a flow is laminar flow.

3.3.4 Turbulent Flow

If the particles of fluid do not move over the well-defined path and cross each other then it is called turbulent flow.

3.3.5 Compressible Flow

Variation in density during the flow is deminstrated as compressible flow.

3.3.6 Incompressible Flow

If density remains constant for fluid flow then it is categorized as incompressible flow.

3.4 Density

It is expressed as mass of the entire object divided by its volume. Mathematically,

$$\rho = \frac{\text{mass}}{\text{volume}}. \tag{3.1}$$

In SI system, the unit and dimension of nanofluid density is $\frac{kg}{m^3}$ and $[\frac{M}{L^3}]$ respectively.

3.5 Stress

It is defined as internal force per unit surface area within a deformable body. The SI unit and dimension of stress is $Kg/m.s^2$ and $\left[ML^{-1}T^{^{-2}}\right]$ respectively. It is further categorized as follows:

3.5.1 Shear Stress

The stress component which acts in tangential direction to the surface is termed as shear stress.

3.5.2 Normal Stress

The component of stress which acts in perpendicular direction to unit surface area is known as normal stress.

3.6 Strain

It measures the deformation per unit length under an applied force.

3.7 Viscosity

It is defined as the measurement of the resistance to the deformation rate. Mathematically, it can be written as

$$\text{viscosity} = \frac{\text{shear stress}}{\text{rate of shear strain}}, \tag{3.2}$$

or

$$\mu = \frac{\tau}{du/dy}. \tag{3.3}$$

Where $\boldsymbol{\tau}$ denotes shear stress, $\frac{du}{dy}$ is the shear strain rate. Its SI Unit and dimension are $kg/m.s$ and $\left[ML^{-1}T^{-1}\right]$ respectively.

3.8 Kinematic Viscosity (υ)

The absolute fluid viscosity per unit density of the fluid. i.e.,

$$\nu = \frac{\mu}{\rho} \tag{3.4}$$

In system of SI, it's unit and dimension are m^2/s and $\left[L^2T^{-1}\right]$ respectivley.

3.9 Viscosity Law (Newtonian)

This theory directly and linearly connects the shear stress to the gradient of velocity.

Mathematically, it can be written as:

$$\tau_{yx} \propto \frac{du}{dy}, \tag{3.5}$$

or

$$\tau_{yx} = \mu \frac{du}{dy}, \tag{3.6}$$

where τ_{yx} depicts shear stress acting and μ used as constant of proportionality.

3.10 Newtonian Fluids

If the fluids agree the viscosity law (Newtonian) then such fluids are categorized as Newtonian fluids. These fluids include water, sugar solutions, air, glycerin etc.

3.11 Model of Power Law

According to this equation, the relationship between shear strain and shear stress in a fluid flow is nonlinear (velocity gradient).

Mathematically,

$$\boldsymbol{\tau}_{yx} \propto \left(\frac{du}{dy}\right)^n, \qquad n \neq 1, \tag{3.7}$$

where

$$\boldsymbol{\tau}_{yx} = \eta^* \frac{du}{dy}, \quad \eta^* = k \left(\frac{du}{dy}\right)^{n-1}, \tag{3.8}$$

here apparent viscosity is denoted by η^*, The flow index is called n, while the consistency index is called k.

3.12 Non-Newtonian Liquid

Fluids classified as non-Newtonian fluids satisfy the power law concept. Examples include things like toothpaste, soapy water, honey, blood, etc.

3.13 Model of Casson Liquid

Such a model that illustrates the shear-thinning properties is the Casson fluid. The following rheological equation, in which viscosity relies on shear rate, is satisfied by the Casson fluid model.

$$\boldsymbol{\tau}_{ij} = \begin{cases} 2\left(\mu_B + p_y/\sqrt{2\pi}\right) e_{ij}, & \pi > \pi_c \\ 2\left(\mu_B + p_y/\sqrt{2\pi_c}\right) e_{ij}, & \pi < \pi_c \end{cases}. \tag{3.9}$$

Here, the term e_{ij} stands for the $(i, j)th$ components of deformation rate, "" is the product of the deformation rate component with itself, plastic dynamic viscosity is specified by μ_B, fluid yield stress is p_y, and π_c specify critical estimation of product based on the non-Newtonian material.

3.14 Modes of Heat Transfer

Whenever the temperature difference exists between different bodies, heat transfer must occurs. We refer to three types of heat transfer phenomenon which are discussed below:

3.14.1 Conduction

Process of heat transfer in which heat transfer occurs by the collision of molecules or atoms is term as conduction.

3.14.2 Convection

Heat transfer by actual movement of molecules from hot place to cold place is categorized as convection.

Forced Convection

If the flow is generated by blowing or pumping the fluid due to some external agent then it is said to be forced convection.

Natural Convection

If the flow is induced by virtue of natural differences in densities due to variation of temperature within the fluid is categorized as natural or free convection.

Mixed Convection

When mechanism of force and natural convections act together to transport heat is termed as mixed convection.

3.14.3 Thermal Radiation

In radiation, heat transfers due to electromagnetic waves without or with any medium. Examples include sunlight, fire etc.

3.15 Body and Surface Forces

It includes all forces that require no physical contact but act upon the whole volume of a body are demonstrated as body forces. Coriolis, magnetic, gravitational and Euler forces are considered as body forces where as a force which acts only on the body's surface with direct contact is categorized as surface forces e.g. Stresses and pressure etc.

3.16 Stratification

Stratification is the formation of fluid's layers having different densities because of differences in temperature (thermal stratification), variation in concentration (solutal stratification) or mixture of different fluids having variation in densities is categorized as stratification. It plays a significant role in simultaneous phenomena of mass and heat transfer.

Various buoyant flow systems, volcanic flows, agriculture field, utilization of heterogeneous mixtures in industries, groundwater reservoirs and oceans etc. are some of the examples of such flows.

3.17 Joule Heating Law

The Lorentz force is exactly proportional to current density.i.e.,

$$\mathbf{J_1}{=}\sigma\mathbf{E}^*, \tag{3.10}$$

where,

$$\mathbf{E}^* = \mathbf{E} + \mathbf{V} \times \mathbf{B}, \tag{3.11}$$

$$\mathbf{J_1} = \sigma(\mathbf{E} + \mathbf{V} \times \mathbf{B}), \tag{3.12}$$

where $\mathbf{E}$ denoted electric field, $\mathbf{V}$ represents velocity field, $\mathbf{B}$ represents field of magnetic, $\mathbf{J_1} \times \mathbf{B}$ represents force of Lorentzand, $\mathbf{J_1}$ represents current density.

3.18 Thermal Conductivity

According to Fourier's law it measures the capacity of a material to conduct heat.

Mathematically,

$$k = -\frac{Q}{A}\frac{1}{dT/dx}, \tag{3.13}$$

where Q depicits heat transfer rate, k represents thermal conductivity, A shows area and

$\frac{dT}{dx}$ is the change in temperature .

In SI system, it's unit is $kg.m/s^3.K$ and dimension is $\left[\frac{ML}{T^3\theta}\right]$.

3.19 Thermal Diffusivity

It relates the thermal conductivity and heat capacity times density. Mathematically, it is the defined as follows

$$\alpha = \frac{k}{\rho c_p}. \tag{3.14}$$

In system international, it's unit and dimension are m^2/s and $\left[\frac{L^2}{T}\right]$ respectively.

3.20 Convective Boundary Condition

It is the combination of both convection as well as conduction. Suppose a fluid over a sheet which is placed horizontally along the $x-axis$. The lower face of the sheet is in contact with another fluid having temperature T_f. In this case heat transfer by conduction to the sheet of surface and within a sheet heat transfer by conduction. The associated condition at boundary is

$$-k\frac{\partial T}{\partial y} = h(T_f - T). \tag{3.15}$$

Given that both convection and conduction are heat flows, this condition states that they are equal. ($q_w = q_c$). First, in line with Newton's law of cooling, heat is transferred from the heated fluid to the plate through convection.i.e.,

$$q_w = h(T_f - T), \tag{3.16}$$

and then, using Fourier law, heat is transferred by conduction. i.e.,

$$q_c = -k\frac{\partial T}{\partial y}. \tag{3.17}$$

Thus, the convective boundary condition combines the Newtonian law of cooling and the

Fourier law of heat conduction. Convection and conduction, for instance, are both important in cooking.

3.21 Dimensionless Numbers

3.21.1 Reynold Number

It is stated as, "the ratio between the inertial force to viscous force". Physically, it tells whether the fluid flowing is turbulent or laminar. For small Reynolds number,viscous forces are dominant in comparison to invertial forces and it consequently represents laminar flow. If Reynolds number is high then inertial forces are dominant which in turn represents turbulent flow.

Mathematically, it is denoted by:

$$\mathrm{Re} = \frac{\text{Inertial force}}{\text{Viscous force}}, \tag{3.18}$$

$$\mathrm{Re} = \frac{Lv}{\nu}, \tag{3.19}$$

here, v depicts maximum velocity, ν represents kinematic viscosity, and L represents length.

3.21.2 Prandtl Number

The relation of viscous and thermal diffusions is categorized as Prandtl number.

$$\mathrm{Pr} = \frac{\text{rate of viscous diffusion}}{\text{rate of thermal diffusion}}. \tag{3.20}$$

Mathematically, it is represented by:

$$\mathrm{Pr} = \frac{\nu}{\alpha} = \frac{\mu/\rho}{k/\rho c_p} = \frac{\mu c_p}{k}, \tag{3.21}$$

where, ν represents momentum diffusivity and α is the thermal diffusivity. Physically, it gives the relative boundary layers thickness of momentun and thermal.

3.21.3 Biot Number

It connects the heat resistance of a body's inside and exterior. It has the following mathematical definition:

$$Bi = \frac{hl}{k}. \tag{3.22}$$

here, h represents coefficient of heat transfer, l represents length and k depicts thermal conductivity.

3.21.4 Nusselt Number

Nusselt Number is the relationship between coefficient of heat transfer due to convection and conduction.

$$Nu_L = \frac{\text{convective heat transfer coefficient}}{\text{conductive heat transfer coefficient}}. \tag{3.23}$$

Mathematically,

$$Nu_L = \frac{h\Delta T}{k\Delta T/L} = \frac{hL}{k}, \tag{3.24}$$

where, h is the heat transfer due to convection, L is the length and k is the fluid's thermal conductivity.

3.21.5 Skin Friction

Skin friction is the frictional force that occurs between the surface and fluid particles. It is defined as follows:

$$C_f = \frac{\boldsymbol{\tau}_w}{\rho U^2/2}, \tag{3.25}$$

where, $\boldsymbol{\tau}_w$ shows wall shear stress, ρ represents the density and U is free-stream velocity.

3.22 Basic Equations

3.22.1 Equation of Continuity

The conservation law of mass or continuity equation describes as total flux of the system. Physically, it follows that total mass of the system is conserved. The differential form of continuity equation is

$$\boldsymbol{\nabla}.(\rho\mathbf{V}) + \frac{\partial \rho}{\partial t} = 0 \tag{3.26}$$

For incompressible fluid i.e., ($\rho = constant$), the above Eq. gives

$$\nabla.\mathbf{V} = 0 \tag{3.27}$$

In two dimensional flow, it can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3.28}$$

3.22.2 Momentum Equation

The conservation law of momentum (*momentum equation*) or sometimes called Navier Stoke's equation can easily be derived from Newton's 2^{nd} law of motion. For incompressible viscous fluid momentum equation is given as

$$\rho\frac{d\mathbf{V}}{dt} = \operatorname{div}\boldsymbol{\tau} + \rho\mathbf{b}. \tag{3.29}$$

Here, the inertial forces is represented by $\rho\frac{d\mathbf{V}}{dt}$, body force is represented by $\rho\mathbf{b}$ and surface force is represented by $\operatorname{div}\boldsymbol{\tau}$. Physical evidence indicates that the system's entire momentum is preserved. In components form we have

x - component

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \upsilon(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}) + \rho b_x, \tag{3.30}$$

y - component

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \upsilon(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}) + \rho b_y, \tag{3.31}$$

z - component

$$u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \upsilon(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}) + \rho b_z. \tag{3.32}$$

3.22.3 Energy Equation

The conservation law of energy is obtained from 1^{st} law of thermodynamics. Energy equation depicts that total system energy is conserved. Mathematically, it can be defined as

$$(\rho c_p)_{nf} \frac{dT}{dt} = -\operatorname{div} \mathbf{q} + \boldsymbol{\tau}.\mathbf{L}. \tag{3.33}$$

Where $(\rho c_p) \frac{dT}{dt}$ is the total internal energy, $\operatorname{div} \mathbf{q}$ is the total heat flux where $\mathbf{q} = -k\boldsymbol{\nabla} T$, ρ represents density, (c_p) represents specific heat, $\boldsymbol{\tau}$ represents stress tensor, $\mathbf{q}$ denotes heat flux, T denotes temperature, k denotes thermal conductivity,and $\mathbf{L}$ stands for rate of strain tensor.

3.23 Homotopy Analysis Method

Homotopic technique is recomended by Liao [52] in 1992. This technique is developed for the construction of highly non-linear equations of differential type. In general consider the non-linear differential type equation to address the basic idea of homotopic technique

$$N\ [f(x)] = 0,\ x \in \Omega, \tag{3.34}$$

where, N shows non-linear operator, x corresponding to independent variable where $f(x)$ shows the unknown function.

The zeroth order deformation equation is

$$(1-q)\,\mathcal{L}\left[\widehat{f}(x;q) - f_0(x)\right] = q\hbar N\left[\widehat{f}(x;q)\right], \quad x \in \Omega, \quad q \in [0,1], \tag{3.35}$$

where, $f_0(x)$ denotes the initial guess, $\mathcal{L}$ is an auxiliary linear operator, $q \in [0,1]$ is an embedding parameter, $\hbar$ is the non zero auxiliary parameter where as $\widehat{f}(x;q)$ represents unknown function of x and q.

At $q = 0$ and $q = 1$, we have

$$\widehat{f}(x;0) = f_0(x), \qquad \text{and} \qquad \widehat{f}(x;1) = f(x). \tag{3.36}$$

As when $q \in [0,1]$ increases from $0 \to 1$, the solution $\widehat{f}(x;q)$ varies from initial approximation

$f_0(x)$ to the final solution $u(x)$. Using Taylor series such that

$$\widehat{f}(x;q) = f_0(x) + \sum_{k=1}^{\infty} f_k(x)\, q^k, \qquad u_k(x) = \frac{1}{k!} \frac{\partial^k \widehat{f}(x;q)}{\partial q^k}\bigg|_{q=0}. \tag{3.37}$$

Differentiating Eq.(2.29) k- time corresponding to q, then dividing by $k!$ and finally setting $q = 0$, we get the k - th order deformation equation

$$\mathcal{L}\left[f_k(x) - \chi_k f_{k-1}(x)\right] = \hbar \mathcal{R}_k(x), \tag{3.38}$$

$$\mathcal{R}_k(x) = \frac{1}{(k-1)!} \frac{\partial^k N\left[\widehat{f}(x;q)\right]}{\partial q^k}\Bigg|_{q=0}, \tag{3.39}$$

with

$$\chi_k = \begin{cases} 0, & k \le 1 \\ 1, & k > 1 \end{cases}. \tag{3.40}$$

Chapter 4

Magnetic Features in Stretching Flow of Fluid with Thermal Radiation

4.1 Introduction

This chapter provides a thorough analysis of the work Ishak [60] presented. A study of the exponentially stretching process is presented in this chapter. Investigations are conducted on how radiation phenomena affect heat transport. Additionally included is the MHD (magneto-hydrodynamic) effect. By using the appropriate transformations, a system of ODEs is formed from a non-linear PDE. To construct the analytical solutions to the governing equations, a homotopy technique is used. The effects of significant factors, such as temperature and velocity, are illustrated graphically.

4.2 Mathematical Modeling

The current study takes into account the flow of magneto hydro fluid through a horizontally moveable surface (Fig. 4.1). It is assumed that the sheet is heated with exponential surface temperature $T_w$and exponentially stretched in the horizontal direction with a velocity $U{=}U_{w0}$ $^{l/x}e$. Additionally, the magnetic strength $B{=})x(B_0{}^{/x}e^{2\ l}$is employed in the perpendicular direction. The radiation phenomena is examined along with the heat transfer.

The governing equations take the form through boundary layer theory:

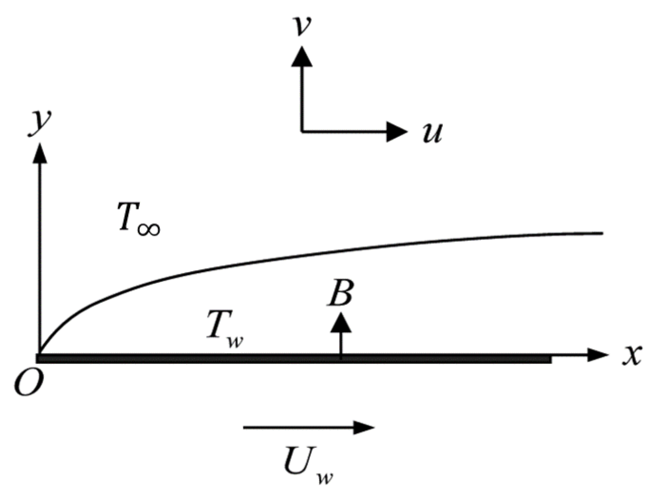

Fig. 4.1: Schematic of problem

$$\frac{\partial \widetilde{u}}{\partial \widetilde{x}} + \frac{\partial \widetilde{v}}{\partial \widetilde{y}} = 0, \tag{4.1}$$

$$\widetilde{v}\frac{\partial \widetilde{v}}{\partial \widetilde{y}} + \widetilde{u}\frac{\partial \widetilde{u}}{\partial \widetilde{x}} = \upsilon\frac{\partial^2 \widetilde{u}}{\partial \widetilde{u}^2} - \frac{\sigma B_0^2}{\rho}\widetilde{u}, \tag{4.2}$$

$$\widetilde{v}\frac{\partial \widetilde{T}}{\partial \widetilde{y}} + \widetilde{u}\frac{\partial \widetilde{T}}{\partial \widetilde{x}} = \frac{16\sigma_e T_\infty^3}{3\rho C_p k^*}\frac{\partial^2 \widetilde{T}}{\partial \widetilde{y}^2} + \frac{k}{\rho C_p}\frac{\partial^2 \widetilde{T}}{\partial \widetilde{T}^2}. \tag{4.3}$$

4.2.1 Conditions at Boundary

$$\begin{aligned}
\widetilde{T} &= \widetilde{T}_w = \widetilde{T}_\infty + \widetilde{T}_0 e^{\frac{X}{2l}}, \\
\widetilde{u} &= \widetilde{u}_w = \widetilde{u}_0 e^{\frac{x}{l}}, \widetilde{v} = 0 \quad \text{at} \quad \widetilde{y} = 0, \\
\widetilde{u} &\to 0, \widetilde{T} \to \widetilde{T}_\infty \quad \text{as} \quad \widetilde{y} \to \infty,
\end{aligned} \tag{4.4}$$

where, density is ρ, velocity (reference) is U_0 , temperature (reference) is T_0, thermal conductivity is k and velocity fields are u, v, heat capacity is C_p, viscosity (kinametic) is υ, magnetic strength is $\mathbf{B}_0$, temperature is T, conductivity (electric) is σ and here Stefan-Boltz man constant, mean absorption coefficient are σ_e, k^*, lenght (reference) is l, ambient temperature is T_∞.

4.2.2 Suitable Variables

$$\begin{aligned}
\widetilde{T} &= \widetilde{T}_\infty + \widetilde{T}_0 + e^{\frac{X}{2l}}\theta(\eta), \eta = \left(\frac{\widetilde{u}_0}{2\upsilon l}\right)^{\frac{1}{2}} e^{\frac{x}{2l}}\widetilde{y}, \\
\widetilde{u} &= \widetilde{u}_0 e^{\frac{x}{l}} f'(\eta), \widetilde{v} = -\left(\frac{\upsilon\widetilde{u}_0}{2l}\right)^{1/2} e^{\frac{x}{2l}} \left(f(\eta) + \eta f'(\eta)\right),
\end{aligned} \tag{4.5}$$

Using the aforementioned modifications, the governing equations are simplified to:

$$f''' + ff'' - 2f'^2 - Mf' = 0, \tag{4.6}$$

$$Pr(f\theta' - f'\theta) = -\theta''\left(1 + \frac{4}{3}K\right), \tag{4.7}$$

associated boundary conditions are:

$$\begin{aligned}
f(0) &= 0, \theta(0) = 1, \; f'(0) = 1, \\
\theta(\eta) &\rightarrow 0, \; f'(\eta) \rightarrow 0, \; as \; \eta \rightarrow \infty.
\end{aligned} \tag{4.8}$$

In aformentioned equations $M = 2\sigma B_0^2 l/\rho\widetilde{u}_0$ depicts magnetic parameter, $\Pr = \rho\upsilon C_p/k$ depicts Prandtl number, $K = 4\sigma_e T_\infty^3/kk^*$ depicts radiation parameter.

Introduce the skin friction and Nusselt number as:

$$C_f = \frac{(\tau_{xy})_{y=0}}{\rho u_w^2(x)}, Nu = \frac{-kx\left(\frac{\partial T}{\partial y}\right)_{y=0}}{k\Delta T}, \tag{4.9}$$

In dimensionless form:

$$(\mathrm{Re})^{1/2} C_f = f''(0), (\mathrm{Re})^{-1/2} Nu = -(1+K)\theta'(0), \tag{4.10}$$

here, Reynolds number describes as $u_w l/v$.

4.3 Homotopic Procedure

It is well known that the homotopic technique's outcomes depend on initial approximations and auxiliary operators (linear) that are selected in the following manner:

$$\theta(\eta) = e^{(-\eta)}, f(\eta) = (1 - e^{(-\eta)}), \tag{4.11}$$

$$L_\theta\ (\theta) = \theta'' - \theta,\ L_f\ (f) = f''' - f', \tag{4.12}$$

with

$$L_f[r_1 + r_2 e^{(\eta)} + r_3 e^{(-\eta)}] = 0, \tag{4.13}$$

$$L_\theta[r_4 e^{(\eta)} + r_5 e^{(-\eta)}] = 0, \tag{4.14}$$

where, $ri(i = 1 - 5)$ are arbitrary constants.

4.3.1 Problem of 0th-Order

$$z\hbar_f N_f[\widehat{f}(\eta; z)] = (1 - z) L_f\ [\widehat{f}(\eta; z) - f_0(\eta)], \tag{4.15}$$

$$(1 - z) L_\theta\ [\widehat{\theta}(\eta; z) - \theta_0(\eta)] = z\hbar_\theta N_\theta[\widehat{\theta}(\eta; z)], \tag{4.16}$$

$$\begin{aligned} \widehat{f}(0; z) &= 0, \widehat{f}'(\infty; z) \to 0, \widehat{f}'(0; z) = 1, \\ \widehat{\theta}(\infty; z) &\to 0, \widehat{\theta}(0; z) = 1, \end{aligned} \tag{4.17}$$

here, auxiliary (non-zero) parameter is depicted by $\hbar_f, \hbar_\theta$, operatore (non-linear) are N_f, N_θ

given below::

$$-M\left(\frac{\partial\widehat{f}\left(\eta;z\right)}{\partial\eta}\right)+\widehat{f}\left(\eta;z\right)\frac{\partial^{2}\widehat{f}\left(\eta;z\right)}{\partial\eta^{2}}+\frac{\partial^{3}\widehat{f}\left(\eta;z\right)}{\partial\eta^{3}}-2\left(\frac{\partial\widehat{f}\left(\eta;z\right)}{\partial\eta}\right)^{2}=\ N_{f}[\widehat{f}\left(\eta;z\right)], \tag{4.18}$$

$$N_{\theta}[\widehat{\theta}\left(\eta;z\right)]=Pr\left(\widehat{f}\left(\eta;z\right)\frac{\partial\widehat{\theta}\left(\eta;z\right)}{\partial\eta}\right)-\Pr\left(\frac{\partial\widehat{f}\left(\eta;z\right)}{\partial\eta}\widehat{\theta}\left(\eta;z\right)\right)+\frac{\partial^{2}\theta\left(\eta;z\right)}{\partial\eta^{2}}(1+\frac{4}{3}K), \tag{4.19}$$

here, $z\in[0,1]$ specifies embedding parameter.

4.3.2 Problem of mth-Order

$$\hbar_{f}\ R_{m}^{f}\left(\eta\right)=L_{f}\ [f_{m}\left(\eta\right)-\chi_{m}f_{m-1}\left(\eta\right)], \tag{4.20}$$

$$L_{\theta}[\theta_{m}\left(\eta\right)-\chi_{m}\theta_{m-1}\left(\eta\right)]=\hbar_{\theta}R_{m}^{\theta}\left(\eta\right), \tag{4.21}$$

$$f_{m}^{'}\left(0\right)=0,\ f_{m}^{'}\left(\infty\right)\rightarrow0,f_{m}\left(0\right)=0, \tag{4.22}$$

$$\theta_{m}\left(\infty\right)\rightarrow0,\ \theta_{m}\left(0\right)=0, \tag{4.23}$$

the non-linear operators are

$$-2\sum_{k=0}^{m-1}\left(f_{m-1-k}^{\prime}f_{k}^{\prime}\right)+f_{m-1}^{\prime\prime\prime}+\sum_{k=0}^{m-1}\left(f_{m-1-k}f_{k}^{\prime\prime}\right)-Mf_{m-1}^{\prime}=\ R_{m}^{f}\left(\eta\right), \tag{4.24}$$

$$R_{m}^{\theta}\left(\eta\right)=\Pr\sum_{k=0}^{m-1}(f_{k-1-k}\theta_{k}^{\prime})-\Pr\sum_{k=0}^{m-1}(f_{k-1-k}^{\prime}\theta_{k})+\left(1+\frac{4}{3}K\right)\theta_{m-1}^{\prime\prime}, \tag{4.25}$$

with

$$\chi_{m}=\begin{matrix}0, & m\leq1\\ 1, & m>1\end{matrix}. \tag{4.26}$$

Clearly for $z(=0)$ and $z(=1)$, gives

$$\widehat{f}(\eta;1) = f(\eta), \quad \widehat{f}(\eta;0) = f_0 \ (\eta), \tag{4.27}$$

$$\widehat{\theta}(\eta;0) = \theta_0 \ (\eta), \ \ \widehat{\theta}(\eta;1) = \theta(\eta), \tag{4.28}$$

Implementing Taylor's series,

$$f_m(\eta) = \frac{1}{m!} \frac{\partial^m \widehat{f}(\eta;z)}{\partial z^m}\bigg|_{z=0}, \widehat{f}(\eta;z) = f_m(\eta)\, z^m + f_0 \ (\eta) \ , \tag{4.29}$$

$$\widehat{\theta} \ (\eta;z) = \theta_0 \ (\eta) + \theta_m(\eta)\, z, \ \theta_m(\eta) = \frac{1}{m!} \frac{\partial^m \theta(\eta;z)}{\partial z^m}\bigg|_{z=0}, \tag{4.30}$$

for $q(=1)$, gives:

$$f_0(\eta) \ + f_m(\eta) = f(\eta), \tag{4.31}$$

$$\theta(\eta) = \theta_0 \ (\eta) + \theta_m(\eta), \tag{4.32}$$

The (f_m, θ_m) solutions in term of the (f_m^*, θ_m^*) solutions are:

$$f_m(\eta) = r_1 + r_2 e^{\eta} + r_3 e^{-\eta} + \ f_m^*(\eta), \tag{4.33}$$

$$\theta_m^*(\eta) + r_4 e^{\eta} + r_5 e^{-\eta} = \theta_m(\eta). \tag{4.34}$$

4.3.3 Analysis of Convergence

The convergence of iterative solutions that depend on auxiliary parameters h_f, h_θ is ensured using homotopy approach. Fig. 4.2 displays the pertinent h-curves for the momentum and energy equations. The acceptable range of auxiliary parameters h_f , h_θ is as: $-1.5 \leq h_f \leq -0.5$, and $-1.5 \leq h_\theta \leq -0.3$.

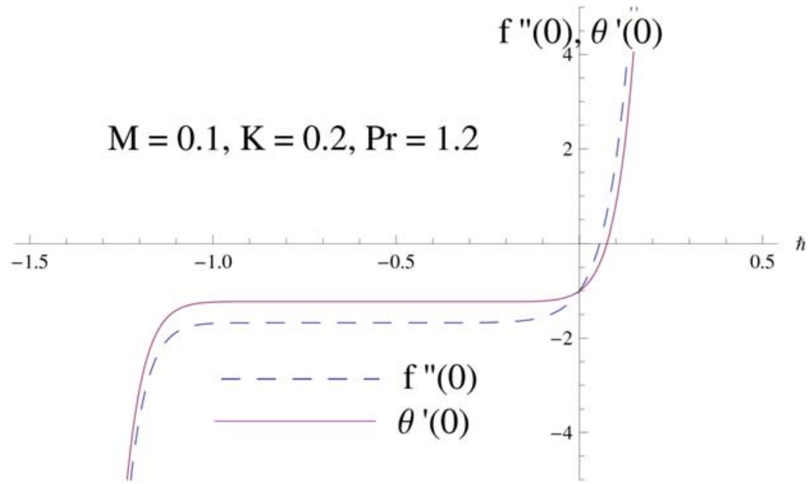

Fig. 4.2: h- curves of f and θ

4.4 Discussion

The graphical representation and explanation of rheological features, such as velocity and temperature profiles for included dimensionless quantities, are covered in this section. Fig. 4.3 elaborates magnetic parameter M effects on the velocity field. Here, we observed a drop in velocity subject to greater M. Obviously, raising the magnetic parameter causes the Lorentz force to increase, which in turn causes the resistive force to increase and the velocity field to decrease. Temperature field with the influence of the magnetic parameter M is seen in Fig. 4.4. Because raising M offers greater resistance and, as a result, increases heat generation, an increase in temperature is shown here. As a result, the temperature field rises. Fig. 4.5 explains the characteristics of the Prandtl number (Pr) vs temperature field relationship. Temperature decreases as Pr is intensified. Physically, raising Pr causes less thermal diffusivity, which in turn causes the temperature field to be smaller. In Fig. 4.6, the effects of the radiation parameter K on the temperature field are shown. For greater K, fluid temperature describes the dominant trend. Higher K undoubtedly results in a lower coefficient of mean absorption and a rise in temperature field is witnessed.

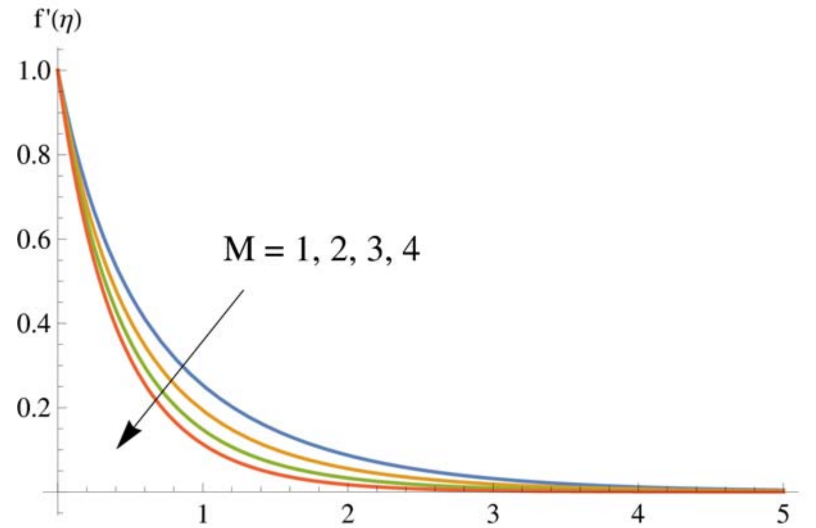

Fig. 4.3: Depiction of M on $f'(\eta)$

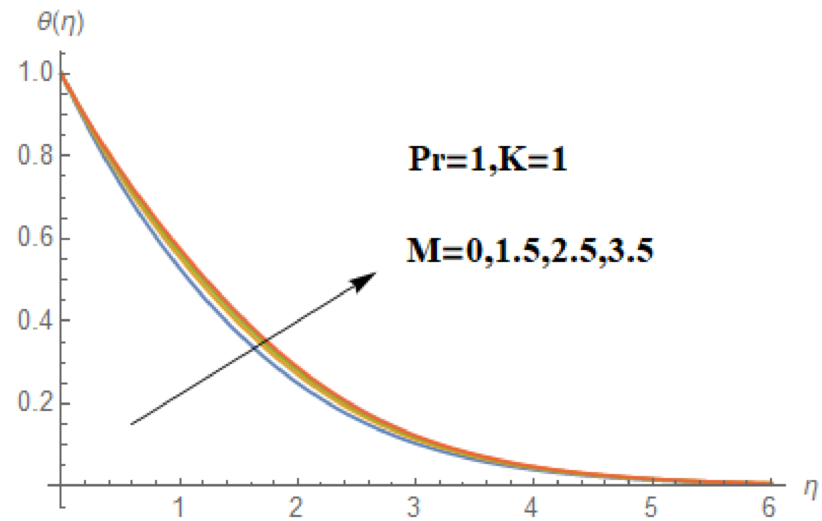

Fig. 4.4: Depiction of M on $\theta(\eta)$

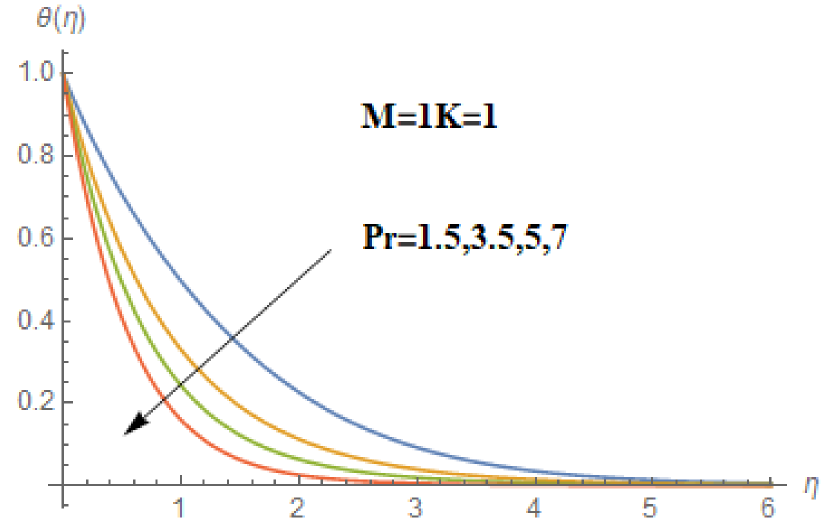

Fig. 4.5: Depiction of Pr on $\theta(\eta)$

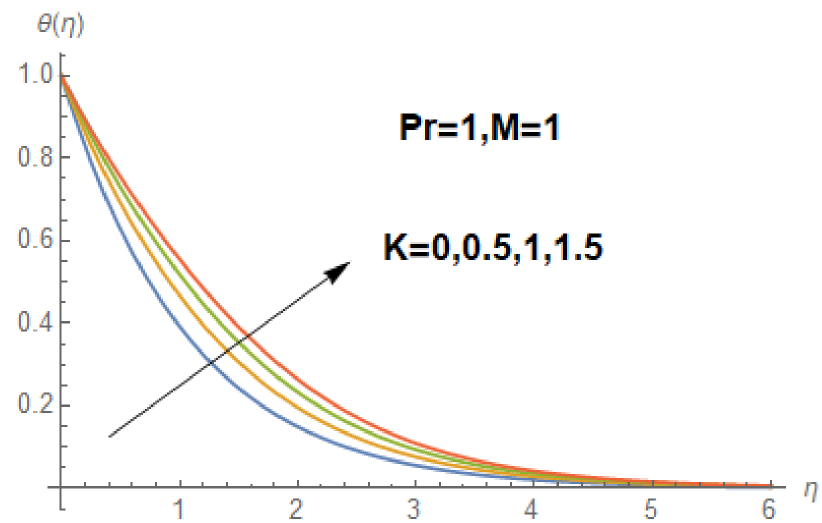

Fig. 4.6: Depiction of K on $\theta(\eta)$

Chapter 5

Characteristics of Thermally Stratified Casson Fluid Flow with Convective Boundary Conditions

5.1 Introduction

The stretching propulsion of Casson material through a sheet is examined in this chapter under the impacts of the magnetic field and velocity slip. The analysis retains nonlinear stratification as well as radiative effects. The viscous dissipation and convective heating processes serve as representations for the heat transfer phenomenon. In order to get equations in their dimen-sionless form, similarity variables are included. By using the homotopy approach, the obtained equations are evaluated analytically. The physical explanation of velocity and temperature fields is illustrated graphically and thoroughly described.

5.2 Mathematical Formulation

Assume a sheet that is exponentially stretchy is deforming an incompressible, continuous flow of Casson fluid. It is assumed that the magnetic field is applied in the y-direction to the flow and that the plate is situated in the x-direction of the Cartesian coordinate system. Heat transfer-related non-linear thermal stratification phenomena is taken into account. To further explain the characteristics of heat, viscous dissipation, thermal radiation, and convective surface conditions are used. Through the second order slip phenomena related to the Casson fluid, flow properties are examined. The governing equations take the form through boundary layer theory:

$$\frac{\partial \widetilde{u}}{\partial \widetilde{x}} + \frac{\partial \widetilde{v}}{\partial \widetilde{y}} = 0, \tag{5.1}$$

$$u\frac{\partial \widetilde{u}}{\partial \widetilde{x}} + \widetilde{v}\frac{\partial \widetilde{u}}{\partial y} = \upsilon(1+\frac{1}{\beta})\frac{\partial^2 \widetilde{u}}{\partial y^2} - \frac{\sigma B_0^2}{\rho}\widetilde{u}, \tag{5.2}$$

$$\widetilde{u}\frac{\partial \widetilde{T}}{\partial \widetilde{x}} + v\frac{\partial \widetilde{T}}{\partial \widetilde{y}} = \frac{k}{\rho Cp}\frac{\partial^2 \widetilde{T}}{\partial y^2} + \frac{16\sigma_e \widetilde{T}_\infty^3}{3\rho Cpk^*}\frac{\partial^2 \widetilde{T}}{\partial \widetilde{y}^2} + \frac{\mu}{\rho Cp}(1+\frac{1}{\beta})\left(\frac{\partial \widetilde{u}}{\partial \widetilde{y}}\right)^2, \tag{5.3}$$

5.2.1 Conditions at Boundary

$$\begin{aligned} \widetilde{u} &= \widetilde{u}_w(x) + L\upsilon(1+\frac{1}{\beta})\frac{\partial \widetilde{u}}{\partial \widetilde{y}}, \qquad \widetilde{v} = 0, \\ -k\frac{\partial \widetilde{T}}{\partial y} &= h_f(\widetilde{T}_f - \widetilde{T}) \text{ at } \widetilde{y} = 0, \\ \widetilde{u} &\to 0, \widetilde{T} \to \widetilde{T}_\infty \quad as \ \ \widetilde{y} \to \infty. \end{aligned} \tag{5.4}$$

Here, component of velocity specified by $u, \ v$, factor related to velocity slip factor is $L = L_0 e^{-\frac{X}{2l}}$ here factor related to initial velocity is L_0, fluid number is β, viscosity (kinematic) is υ, coefficient of mean absorption is k^*, temperature is $\widetilde{T}$, thermal expansion coefficient is h_f, heat capacity is C_p, conductivity (electric) and constant of Stephen Boltzman are σ, σ_e, strength of magnetic is B_0, stretching type velocity is $\widetilde{u}_w$, conductivity (thermal) is k , varying ambient temperature is $\widetilde{T}_\infty(x) = \widetilde{T}_0 + A_1 e^{\frac{X}{2l}}$,density is ρ, absolute viscosity is μ, and suspended fluid varying temperature is $\widetilde{T}_f\,(x) = \widetilde{T}_0 + Ae^{\frac{X}{2l}}$.

5.2.2 Suitable Variables

$$\begin{aligned} \widetilde{v} &= -\left(\frac{\upsilon U_0}{2l}\right)^{1/2} e^{\frac{x}{2l}}\left(f(\eta)+\eta f'(\eta)\right), \widetilde{u}=\widetilde{u}_0 e^{\frac{x}{l}} f'(\eta), , \\ \theta(\eta) &= \frac{\widetilde{T}-\widetilde{T}_\infty}{\widetilde{T}_f-\widetilde{T}_0}, \eta=\left(\frac{\widetilde{u}_0}{2\upsilon l}\right)^{\frac{1}{2}} e^{\frac{x}{2l}}\widetilde{y}, \end{aligned} \tag{5.5}$$

by employing these variables, the equations become:

$$\left(1+\frac{1}{\beta}\right) f''' + ff'' - 2f'^{2} - M^2 f' = 0, \tag{5.6}$$

$$\left(1+\frac{4}{3}K\right)\theta'' + Pr(f\theta' - f'\theta - Sf') + \Pr Ec\left(1+\frac{1}{\beta}\right) f''^2 = 0, \tag{5.7}$$

related boundary conditions are:

$$\begin{aligned} f(0) &= 0,\ f'(0) = 1+\lambda\left(1+\frac{1}{\beta}\right) f''(0), f'(\infty)\to 0, \\ \theta'(0) &= -Bi(1-S-\theta(0)), \theta(\infty)\to 0. \end{aligned} \tag{5.8}$$

Here, Prandtl number is Pr, magnetic parameter is M, Biot number is Bi, radiation parameter is K, Eckert number is Ec, stratified parameter is S, velocity slip parameter is λ. Mathematically, the parameters are specified as:

$$\begin{aligned} Pr &= \frac{\rho\upsilon C_p}{k}, M=\frac{2\sigma B_0^2 l}{\rho U_0}, Bi=\frac{h_f}{k}\sqrt{\frac{2\upsilon l}{U_0}}, S=\frac{A_1}{A}, \\ K &= \frac{4\sigma_e T_\infty^3}{kk^*}, Ec=\frac{U_0 e^{\frac{x}{2l}}}{C_p\triangle T}, \lambda = L_0\sqrt{\frac{U_0 v}{2l}}. \end{aligned} \tag{5.9}$$

Introduce the skin friction and Nusselt number as:

$$C_f = \frac{(\tau_{xy})_{y=0}}{\rho u_w^2(x)}, Nu = \frac{-kx\left(\frac{\partial T}{\partial y}\right)_{y=0}}{k\Delta T}, \tag{5.10}$$

In dimensionless form:

$$(\mathrm{Re})^{1/2}\,C_f = \left(1+\frac{1}{\beta}\right)f''(0)\,,(\mathrm{Re})^{-1/2}\,Nu = -\left(\frac{1}{1-S}\right)\theta'(0)\,, \tag{5.11}$$

here, Reynolds number describes as $u_w l/\upsilon$.

5.2.3 Homotopic Procedure

The initial guesses and linear operators are reported as follows:

$$\boldsymbol{\theta}_0(\eta) = \frac{Bi}{1+Bi}(1-S)^e(-\eta)\,,f_0(\eta) = \frac{1}{1+\lambda\left(1+\frac{1}{\beta}\right)}\left(1-e^{(-\eta)}\right), \tag{5.12}$$

$$L_f\ (f) = f''' - f'\ ,L_\theta(\theta) = \theta'' - \theta, \tag{5.13}$$

with properties

$$L_f[r_1 + r_2 e^{(\eta)} + r_3 e^{(-\eta)}] = 0, \tag{5.14}$$

$$L_\theta[r_4 e^{(\eta)} + r_5 e^{(-\eta)}] = 0, \tag{5.15}$$

here, $ri(i = 1-5)$ specify arbitrary constants.

5.3 Analysis of Convergence

The homotopic approach was used to compute the series type solutions in order to confirm their convergence. Therefore, the convergence is illustrated in (Fig. 5.1). It is witnessed that the suitable domains of auxiliary parameters $(h_f\ ,h_\theta)$ are from $-1.3 \leq h_f \leq -0.2$ and $-0.9 \leq h_\theta \leq -0.3$.

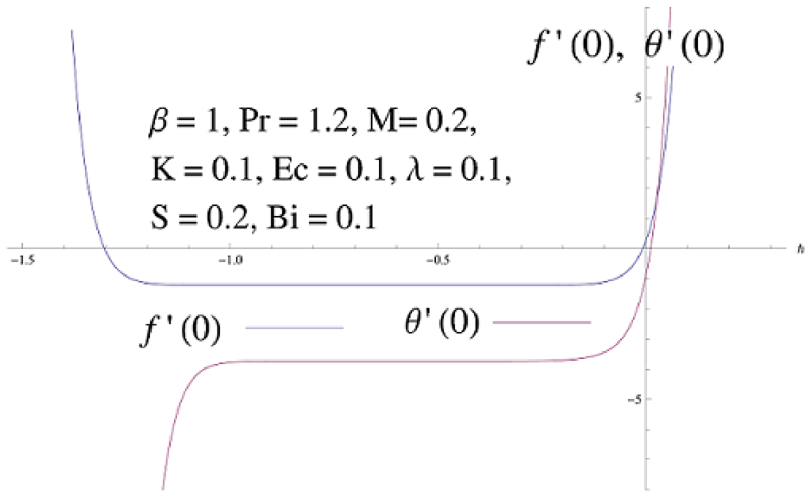

Fig. 5.1: h- curves of f and θ

5.4 Discussion

Here, the primary goal is to graphically explain a number of newly developing factors related to the horizontal velocity field (f'), and temperature variation (θ). The Casson parameter impacts corresponding to the velocity field is seen in Fig. (5.2). For the dominating Casson fluid parameter, the velocity field expands. It is made clear that the existence of the slip boundary condition causes velocity close to the sheet's surface to behave expansively. Physically, strong Casson fluid parameter strengthens the velocity field away from the plate due to weak viscous effects. So, velocity increments. Velocity slip characteristics in the velocity field is displayed in Fig. (5.3). A decrease in velocity is found for growing slip parameter.Physically, when slip happens, the friction between stretching plate and the fluid diminishes because partial transmission of stretching force to the fluid and so, increasing slip parameter declines the flow velocity. In Fig. (5.4), the Eckert number's behaviour in relation to the temperature field is shown. Higher Eckert numbers are associated with dominant temperature behaviour. Higher viscous dissipation caused by a dominant Eckert number, on the other hand, causes the temperature field to expand. Fig. (5.5) shows the analysis of the Biot number in relation to the temperature field. The temperature field is shown to exhibit dominating behaviour in accordance with the Biot number. In terms of physics, a greater Biot number causes the rate of heat transfer from a heated plate to a fluid to increase. The temperature field therefore improves. The behaviour of the thermal stratification parameter with respect to fluid temperature is seen in Fig. (5.6). For dominant S, decrement behaviour is found. The fluid was divided into different sections according to density, which hinders heat transport. Thus, temperature field degrades as a result of greater stratification parameter. Fig. (5.7) depicts the analysis of the skin friction coefficient C_f in relation to the fluid parameter β and the magnetic parameter M. C_f decreases as β and M get bigger. Fig. (5.8) is depicted for Nusselt number Nu against Biot number B_i and stratification parameter S. Here, Nu decreases as B_i and S increases.

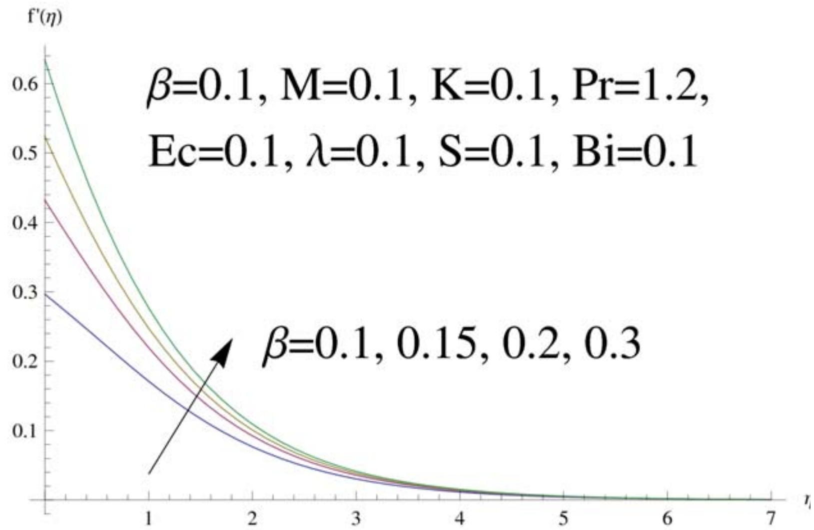

Fig. 5.2: Depiction of β on $f'(\eta)$

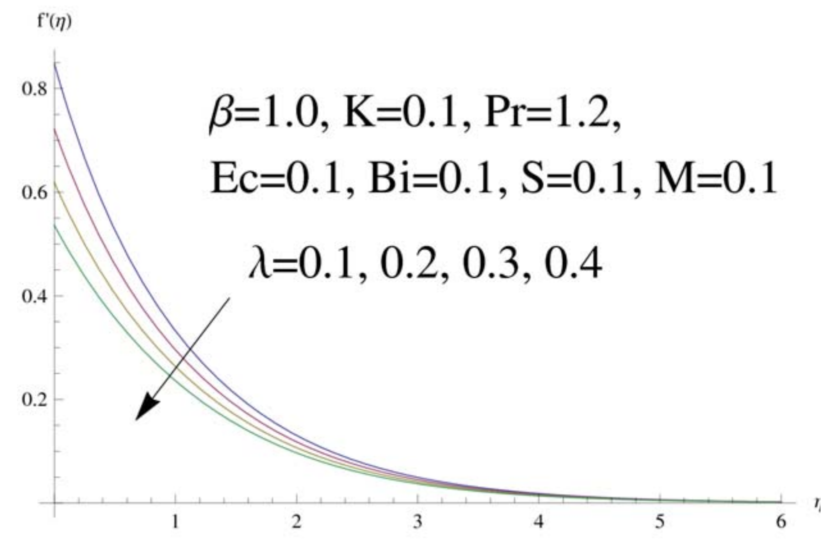

Fig. 5.3: Depiction of λ on $f'(\eta)$

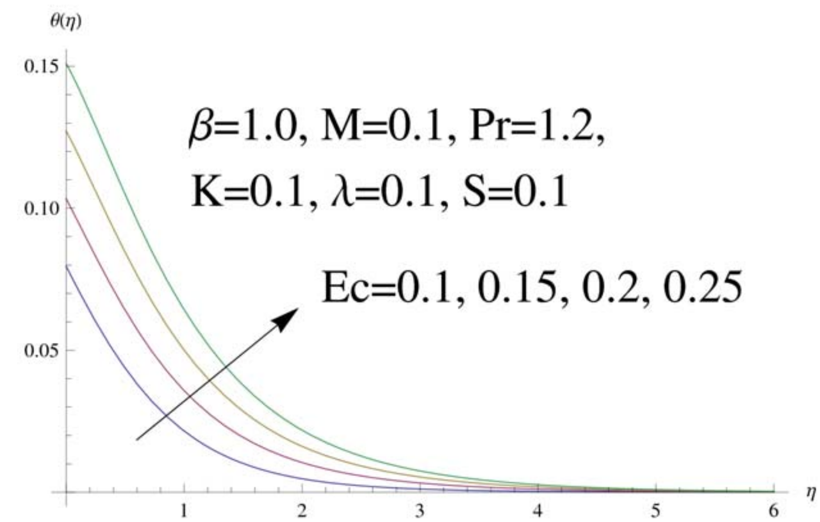

Fig. 5.4: Depiction of Ec on $\theta(\eta)$

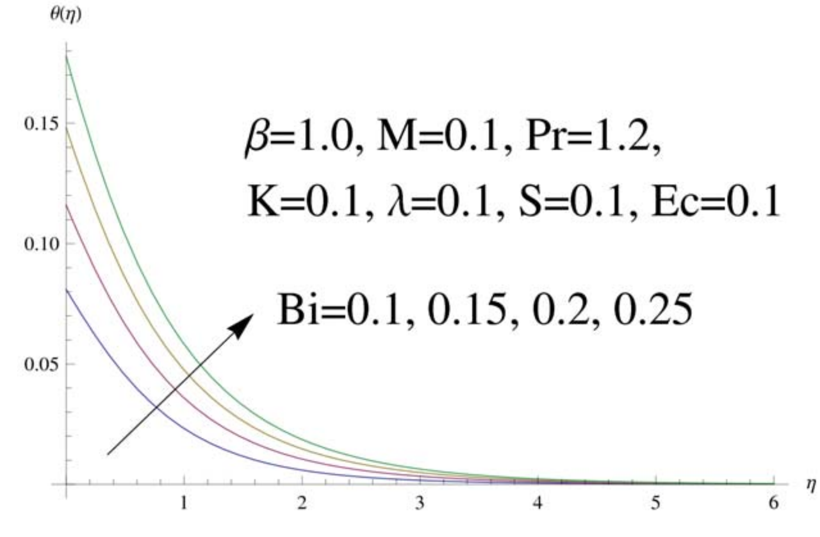

Fig. 5.5: Depiction of Bi on $\theta(\eta)$

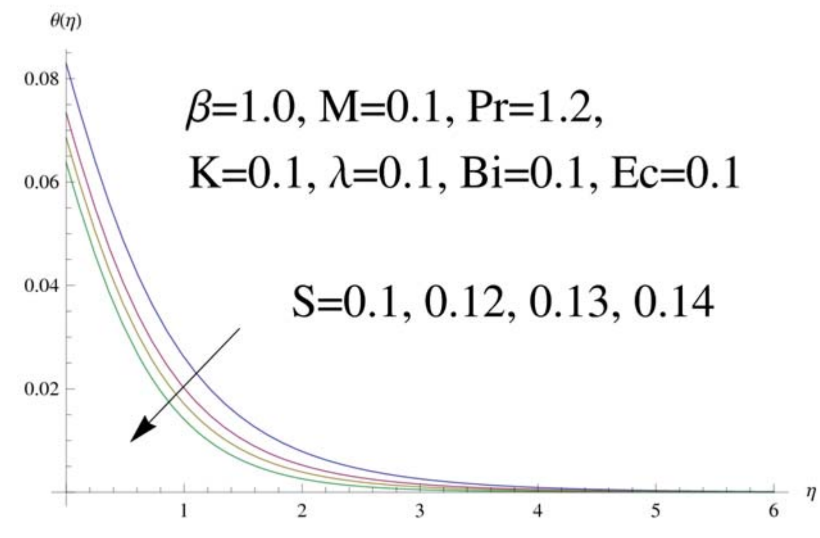

Fig. 5.6: Depiction of S on $\theta(\eta)$

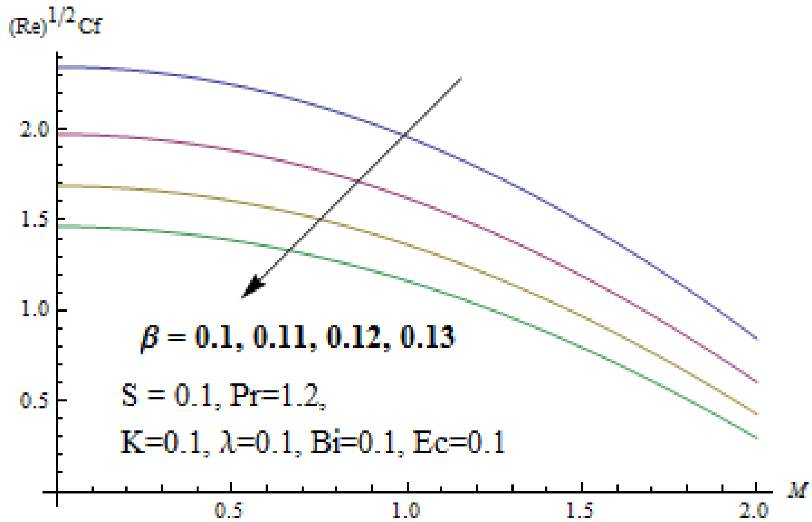

Fig. 5.7: Depiction of β & M on Cf

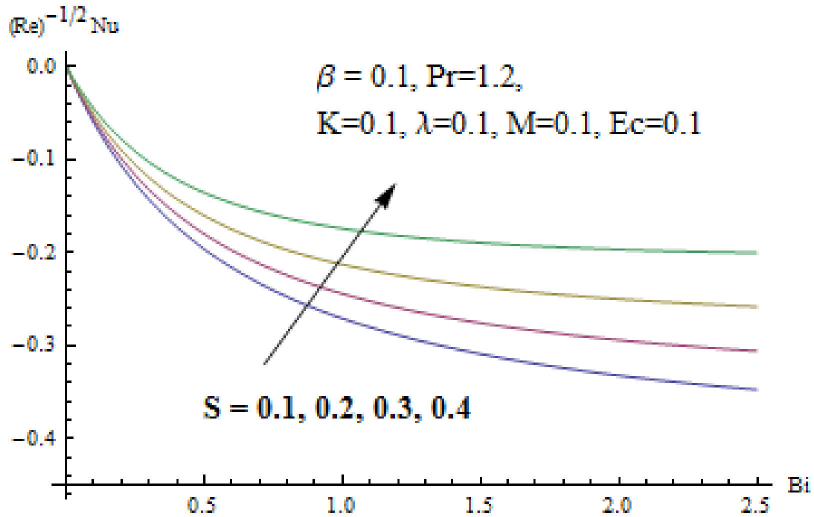

Fig. 5.8: Depiction of S & Bi on Nu

Chapter 6

Conclusions

Slip phenomenon analysis in motion of Casson fluid caused by exponentially stretchable plate is studied and systematically analyzed. The main points regarding current analysis are summarized as follows:

For the dominating Casson fluid parameter, the velocity field expands. For larger slip parameters, the velocity field decrements. With increased Eckert, the temperature field is enhanced. For the most important stratification parameter, the temperature field decays. With increased Biot numbers, the temperature field is enhanced. Skin friction decreases as Casson fluid and magnetic parameters get stronger. Nusselt number decreases as Biot number and stratification parameter increase.

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