COMPARATIVE ANALYSIS OF MATHEMATICAL SKILLS OF ELEMENTARY SCHOOL TEACHERS

BY

Kalsoom Riaz

NATIONAL UNIVERSITY OF MODERN LANGUAGES

ISLAMABAD

MAY, 2023

COMPARATIVE ANALYSIS OF MATHEMATICAL SKILLS OF ELEMENTARY SCHOOL TEACHERS

By

Kalsoom Riaz

M.Ed., University of Peshawar, 2016

A THESIS SUBMITTED IN PARTIAL
FULFILMENT OF THE REQUIREMENTS FOR THE
DEGREE OF

MASTER OF PHILOSOPHY

In **Education**

To

DEPARTMENT OF EDUCATIONAL SCIENCES
FACULTY OF SOCIAL SCIENCES

NATIONAL UNIVERSITY OF MODERN LANGUAGES, ISLAMABAD

NATIONAL UNIVERSITY OF MODERN LANGUAGES FACULTY OF SOCIAL SCIENCES

THESIS AND DEFENSE APPROVAL FORM

The undersigned certify that they have read the following thesis, examined the defense, are satisfied with the overall exam performance, and recommend the thesis to the Faculty of Social Sciences for acceptance.

Thesis Title: Comparative Analysis of Mathematical Skills of Elementary School Teachers

Submitted by: Kalsoom Riaz **Registration #:** 1789-MPhil/Edu/F-19

Master of Philosophy
Degree name in full

Education
Name of Discipline

Dr Hukam Dad Malik
Name of Research Supervisor

_____
Signature of Research Supervisor

Prof. Dr. Khalid Sultan
Name of Dean (FSS)

_____
Signature of Dean (FSS)

_____
Date

AUTHOR'S DECLARATION

I Kalsoom Riaz

Daughter of Raja Riaz Hussain

Registration # 1789-MPhil/Edu/F-19

Discipline Education

Candidate of **Master of Philosophy** at the National University of Modern Languages does hereby declare that the thesis **"Comparative Analysis of Mathematical Skills of Elementary School teachers"** submitted by me in partial fulfilment of my MPhil degree, is my original work, and has not been submitted or published earlier. I also solemnly declare that it shall not, in future, be submitted by me for obtaining any other degree from this or any other university or institution.

I also understand that if evidence of plagiarism is found in my thesis/dissertation at any stage, even after the award of a degree, the work may be cancelled, and the degree revoked.

_____

Signature of Candidate

_____

Name of Candidate

_____

Date

ABSTRACT

Title: Comparative Analysis of Mathematical Skills of Elementary School teachers

Mathematics is a core subject with integration into other sciences. Despite its wide application and usability in every field, the students' understanding does not meet the benchmark. The objectives of the study were to find the similarities and differences of teachers' competency at different schools in the public and private sectors at the elementary level, and to compare teachers' teaching mathematical skills regarding their effect on students' learning mathematics skills at the elementary level in public and private sector schools. Hence, a convergent-parallel mixed method study was conducted, and data collection was done in three phases simultaneously. Qualitative data was collected through the semi-structured interview with teachers; an observational checklist was filled during a classroom observation, and Quantitative data were collected through tests from students after the completion of each session. The population of the study constitutes 30 and 10 teachers, while 600 and 400 students from public and private sector school respectively. A total of six schools were selected for the study; three each from the public and private sectors. The sample selected was six teachers through purposive sampling technique and 300 students of public and private sector schools through using sample size determination techniques. From each school, 50 students were selected through Stratified Random Sampling using equal allocation technique sampling. The teacher-made tests were validated, and the reliability of the instrument was checked through Cronbach alpha. The qualitative data collected through semi-structured interviews and an observational checklist was analysed through a thematic approach while the quantitative data collected through tests were analysed using one-way MANOVA. The results revealed that the compromised mathematical understanding of students does not depend upon public and private sector cultural differences but the professional competence of the teachers. Teachers do not possess the required qualification and experience for teaching the subject of mathematics, the teaching methods employed were not effective, the mathematics subject was not assigned to the concerned teachers with relevant qualifications and experience, and a significant difference in learning mathematical skills in terms of numbers as the significance level was less than 0.05 (0.001), algebraic expressions (0.001), geometry (0.001), data handling (0.001) was found. The study recommends that the pre-requisite qualification of teachers needed to be assessed, allocation of subjects to relevant teachers is imperative, and proper grade monitoring is required.

TABLE OF CONTENTS

Chapter **Page**

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATIONS

2D	Two Dimensional
3D	Three Dimensional
Df	Degree of Freedom
F	Factor Distribution
ICT	Information Communication Technology
IR	Interquartile Range
MANOVA	Multiple Analysis Of Variance
N	Population
NCTM	National Council of Teaching Mathematics
SD	Standard Deviation
SE	Standard Error
Sig.	Significance

ACKNOWLEDGEMENTS

Allah Almighty is the only ONE, WHO is the most Gracious and Compassionate and Admirable of all the admirations and praises. I am greatly obliged to my Almighty Allah, HE is THE one and only WHO consecrated me with the opportunity to acquire and communicate with the very knowledge to do effort in this field. It is merely the consecration of the All-knowing being to give upon us His Holy Prophet (SallallahuAlaihayWa'alihiWasalam) the last human appearance of his complete information, who has deposited all ground of knowledge and will continue the source of all intelligence that is to transpire in the forthcoming. Here, I would like to acknowledge the professional judgment of Dr. Hukamdad Malik, Associate Professor, Department of Education, who gave me a thought-provoking chance to conduct this research study. I am also thankful to Dr. Obaid Ullah for his kind guidance and support in the entire process of completing the thesis. I am also thankful to Dr Wajeeha Shahid, Head, Department of Education for providing departmental permission for data collection from the schools and Dr Saira Nudrat, Coordinator, Department of Education for her guidance and for sharing timely important information regarding the thesis. In addition, I am thankful to the principals, coordinators, teachers, and students at those schools and all those individuals who one way or another way gave their time and cooperation in the completion of this research and become the part and parcel of this research.

DEDICATION

This thesis is dedicated to my late Father Raja Riaz Hussain, and Mother and Sister Qaisra Riaz for their love, endless support, and encouragement. Furthermore, I extend my dedication to my husband Dr Obaid Ullah, and my beloved daughter Moazma Shahid. Due to their encouragement, support, and guidance, I reach to accomplish this academic milestone.

CHAPTER 1

INTRODUCTION

Education leads nations toward great success and glory. The progress of a country or nation depends on its education system mainly. If the education system is the latest, well-equipped in sense of technologies, well administration, and all honest staff it means that the nation is developing smoothly. Education is necessary for all people to have a good life and for awareness about how to live and move in society.

Mathematics is predominantly considered the gateway to science and technology that has a pivotal role in the economic stability of any country (Mbugua, 2012; Tshabalala & Ncube, 2016). The capability to learn the subject of mathematics is essential to have a subject command of other subjects i.e., science, technology, and all fields of engineering. These subjects are considered as a backbone in the soughing innovation and producing professional experts that significantly contribute to the knowledge of the economy of the country (Ker, 2013).

Being a dominant role in the uplifting of a country's status in the 21st century, the stance of getting exponential growth in the fields of science and technology is at its peak. Most countries are taking pain in delivering mathematics education and multiple strategies are taken by their governments to produce quality education (Maliki, Ngban, & Ibu 2009). Due to strong concern, mathematics subject is included at all levels i.e., from primary to post-graduate level.

Being an important subject, a large number of students make strong efforts to understand the concepts of mathematics (Galadima & Yusha'u 2007; Mazana, Montero, & Casmir, 2019). These strong efforts reveal that the understanding of students is not clear and show poor performance in the subject of mathematics. This results in the evolution of severe consequences for the nation in the coming years regarding the education status of youth. The learning of the subject of mathematics is not a matter of getting higher qualifications but also preparing the students for future endeavors. Thus, students who fail in the subject of mathematics face difficulties in learning along with related subjects at higher levels primarily science, engineering, healthcare, and business-related subjects. This leads to the scarcity of skilled people in the country and affects the knowledge economy. Sa'ad, Adamu, and Sadiq (2014) emphasized the importance of mathematics and claimed that the subject of mathematics is the backbone of all subjects and this subject plays an important role in the promotion of technology, which leads to advancement in society.

Educational policymakers must focus on the development of capacity building of students to foster the development of science and technology.

In the context of making mathematics subject easy, many stakeholders conducted research to explore the factors responsible for affecting the performance of students at all levels. Many studies proclaim that various factors are responsible for the poor performance of students in the subject of mathematics and among them, the most prominent is the availability of competent and dedicated teachers (Ponera, Mhonyiwa, & Mrutu, 2011), provision of Audio-Visual Aids for teaching mathematics and least number of staff in various schools (Kitta, 2004). Besides these, the aptitude of students and their motivation level also play an important role in their performance. In continuation to that, the factors i.e., teachers' least motivation to teach the mathematics subject, weak attitude of teachers towards the students and the subject, weak teaching strategies, and least command of a teacher on the subject matter affects the performance of students in the subject (Mazana et al., 2019; Michael, 2015). In addition, the level of emotions of the teacher in the classroom has a major effect on students' academics. If the teacher is emotionally stable, the performance of students will be up to the mark and if for instance, the teacher is emotionally not stable, the performance of students along with their personalities will be drastically damaged (Frenzel et al., 2009; Klusmann et al., 2008). Therefore, the emotional stability of the teacher matters a lot and is recommended to recruit a teacher with stable emotions and personality. Other factors like poor management of conducting an examination, curriculum-related issues and weak implementation plans, compromised monitoring system, weak training to teachers for implementation of the curriculum, and lack of potential to identify the weak aspects to concerned stakeholders affect the academia of students (Uysal & Banoglum, 2018).

1.1 The rationale of the Study

Many initiatives were taken by the government to increase the capacity of teachers and students to facilitate and equip the teaching-learning process on modern parameters. These initiatives, which include the induction of qualified teachers who have a strong command of the subject matter, the ability to teach the content effectively and efficiently using modern pedagogical skills and teaching methodologies (Kitta, 2004), provision of required Audio-Visual Aids to teachers for teaching the subject of mathematics, availability of mathematics laboratory (Sumra & Katabaro, 2014) progressively helps in reducing the cases of failure in the mathematics subject. In addition, the provision of rigorous training on subject matters, and the introduction of new subject Information Communication Technology (ICT) at elementary, secondary, and higher secondary levels motivates students to learn mathematics (Kafyulilo, Fisser, Pieters, & Voogt, 2015).

Despite taking strong initiatives by Government and other stakeholders, the performance of the students in the subject of mathematics remains unsatisfactory and numerous failure cases in mathematics were reported. This eventually leads to the underdeveloped economic condition of a country and the unsatisfactory performance of students in mathematics acts as a barrier to lead in economic growth and prosperity. The global rank of students in the subject of mathematics was below average (Bethell, 2016), which create several questions on the quality of teaching the subject. The quality of the education system depends upon the provision of education at the secondary level because it leads to the development of the educational career of students (World Bank, 2013). A study reveals that the most important subject at the secondary level is mathematics (Sullivan, P., 2011). Mathematics enables students to score outstanding results in their academia; that is why the subject of mathematics has an utmost importance (Bruhlmeier, 2010).

According to HakiElimu (2012), the quality of teaching could be assessed in terms of students' performances in their formative and summative assessments. Weak performances of students reveal that the teaching is compromised and may be some confounding variables are manipulating the results. For improving the quality of teaching, firstly, the learning experiences of students need to be improved and lastly, their performances are taken into consideration for improvement (Topçu, Erbilgin, & Arikan, 2016). Hence, the performances of students and the factors associated with them are worthwhile, however, several studies were conducted to identify such factors at certain specific education levels (Joseph, 2013) and to explore the perception of students regarding the subject of mathematics (Kilasi, 2017; Mazana et al., 2019).

The importance of mathematics was tremendously recognized by many stakeholders at a different time fashion. The focus was on the provision of education in mathematics at the secondary level (Tu, R., & Shen, W., 2010) and the research studies indicate that the subject of mathematics at the secondary level in Pakistan was not taught up to the mark and the performance of students was not satisfactory. The teacher taught the subject of mathematics through traditional teaching methods due to which the motivation of students was not high. In addition, in the traditional teaching method the involvement of students was also minimum, which ultimately affect the academic performance of students in this subject (Akhtar, M. & Ahmad Saeed, A., 2014). On the other hand, the countries like China, Singapore, and Japan have strongly emphasized the teaching of mathematics through the involvement of students and providing opportunities to engage in the problem-solving style of learning. Besides these, the students are motivated to learn the subject of mathematics using different strategies (Zhang et.al, 2004). The intention of developing mathematical skills among students is exponentially increasing in recent time reason being to ensure the advancement in the arena of technology

and science, and under circumstances, the teaching and learning of mathematics is a challenging assignment (Conway & Sloane, 2005; European Commission 2011).

The timely dissemination of education to their stakeholders significantly increases the knowledge economy of any country. The subject of mathematics is considered a significant subject in the fields of engineering and ensures the promotion of essential skills (Conway & Sloane 2005; Hourigan & O‟Donoghue 2007; Petocz et al. 2007). Subsequently, the technical mindsets are encapsulated with logical sequences, which is the main aim of mathematics subject (Blockley & Woodman 2002; Pyle, 2001) and hence become an integral part of technical education promoting critical thinking abilities among the students (Mustoe, 2003).

1.2 Statement of the Problem

There is a research gap to explore and address the factors responsible for the weak performance of students in their academics in the subject of mathematics. Walberg's theory of productivity was employed to understand the factors which are responsible for the learning of mathematics by students. This theory indicates the attributes of individual students and the psychological environment influence the cognitive and affective aspects of learning outcomes (Walberg, Fraser, & Welch, 1986).

The national curriculum focuses on promoting the conceptual understanding of students regarding the development of critical thinking, measuring, and developing problem-solving approaches among students. It is generally believed that at the elementary level, the mathematical skills of students are not given due consideration due to which the performance of such students is not satisfactory even for simple mathematical concepts when required. This research focused on instructional quality and classroom management skill of the teacher that affects the learning achievements of students. Therefore, it is imperative to conduct a formal comparative mixed-method study on the teaching and learning of mathematics at the elementary level to find the factors responsible for unsatisfactory performance of students in mathematics.

1.3 Research Objectives

The objectives of the study were to:

1. Find the similarities of mathematics teachers' competencies in the public and private sectors' schools at the elementary level.
2. Find the differences in mathematics teachers' competencies in the public and private sectors' schools at the elementary level.

3. Compare the teachers' teaching mathematical skills in classrooms at the elementary level in public and private sector schools.
4. Compare teachers' teaching mathematics skills regarding their effect on students' learning mathematics skills at the elementary level in public and private sector schools.

1.4 Research Questions

The research questions of the study were:

1. Does the teacher's academic and professional qualifications according to Federal Board?
2. Does the teachers have math teaching experience?
3. Is the teacher's subject specialized?
4. Does the teacher have complete awareness regarding Pakistan National Curriculum 2006 of Math for grade 7th?
5. Does the teacher follow the Pakistan National Curriculum 2006 of Math for grade 7th?
6. Is the teacher provided with a Teacher Guide?
7. Does the teacher have a prepared lesson plan?
8. Does the teacher deliver the clear objectives of the lesson in the classroom?
9. Does the teacher have started the topic actively?
10. Are audio/visual aids present in the grade according to the topic?
11. Does the teacher select the proper method of teaching related to the topic?
12. Does the teacher adopt a clear teaching strategy in the classroom environment?
13. Does the teacher use suitable teaching techniques?

1.5 Null Hypotheses

H_{o1} There was no significant difference in learning the concept of numbers in public and private sector schools.

H_{o2} There was no significant difference in learning the concept of algebra in public and private sector schools.

H_{o3} There was no significant difference in learning the concept of geometry in public and private sector schools.

H_{o4} There was no significant difference in learning the concept of data in the public and private sectors in schools.

1.6 Theoratical Framework

Nine models of school learning were explored by Haertel, Walberg, and Weinstein (1993), which are based on the psychological learning theory (Glaser, 1976) or learning model concerning time-based (Bennett, 1978; Bloom, 1976; Carroll, 1963; Cooley & Leinhardt, 1975; Harnischfeger & Wiley, 1976). Among these models, there were variations present in the constructs, despite that Haertel et al. (1983) identified variables i.e., students' ability, motivation, quality of teaching, and the quantity of teaching. In addition, the items reflecting the classroom and home environment, the influence of peers, and mass media were less addressed (Watson & Keith, 2002). According to the Theory of Educational Productivity proposed by Walberg (1981), learning in the classroom environment often increased based on four important factors, which include the ability of students and their motivational level, the classroom psychological environment, a conducive environment in the home, and peers influence and confronting mass media. All the factors are mandatory for creating and developing the learning of students and the presence of these factors is required at a minimal level. In addition, the variables i.e., ability and motivation the reflection of students' attributes, while other variables i.e., classroom and home environment, peer groups, and media exposure address the psychological environment (DiPerna et al., 2002).

Walberg et al (1981) identified key factors which affect learning, concerning the global theory of educational productivity. Among those key factors, the quality of instruction and classroom environment (classroom management) are taken into consideration in this study, which shows the reflection of a competent teacher. Thus focusing on the teaching skills and learner's learning skills are deducted from the quality of teaching.

1.7 Conceptual Framework

The conceptual framework for the current study is divided into two strata, which are teaching (Shulman, 1987) and learning (Bloom,1969) mathematics. In the process of learning, effective teaching plays a pivotal role in the development of the academic concepts of students. In this regard, each stratum is explained as under:

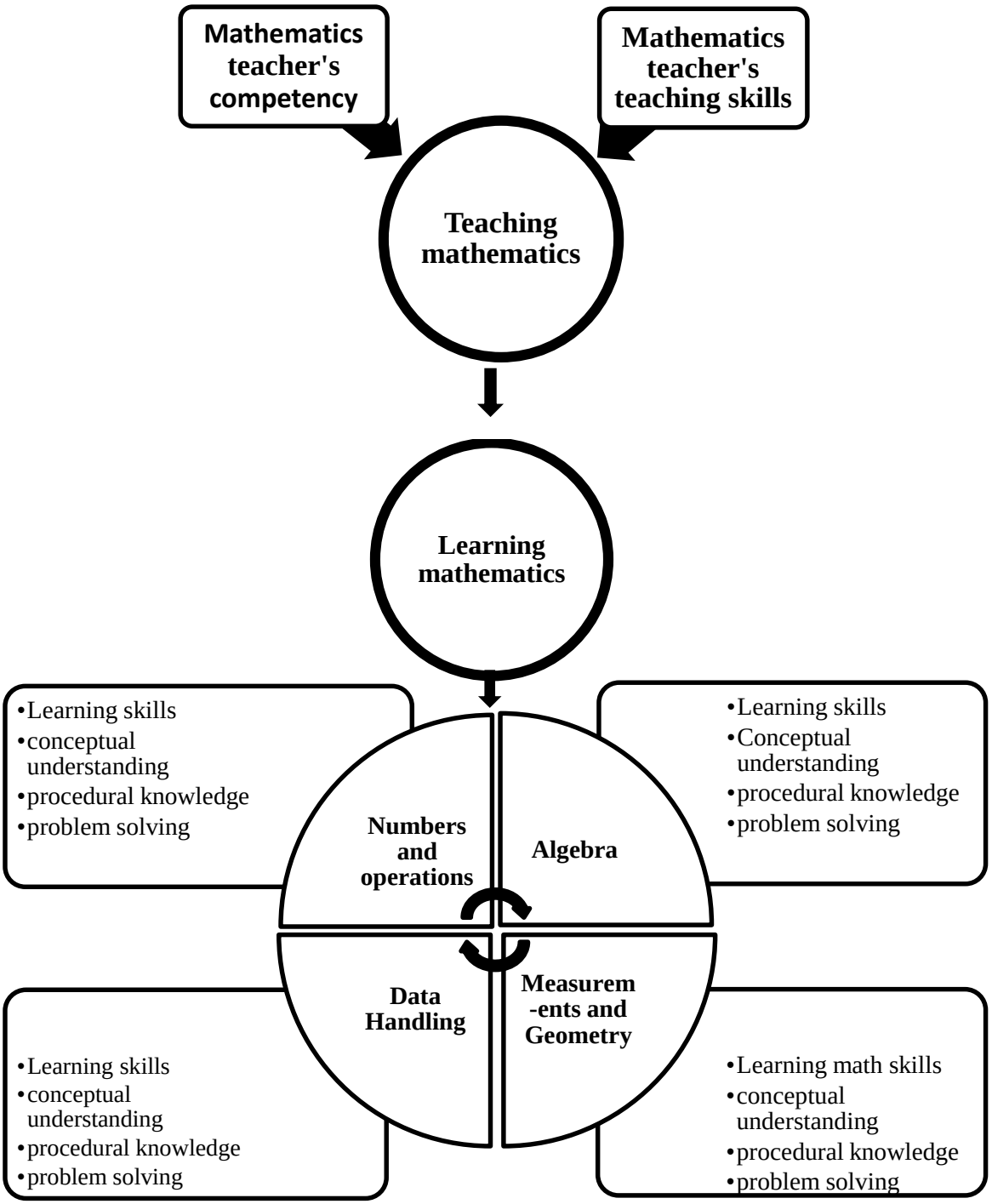


Fig. 1.1 ***Conceptual Framework for the development of teaching and learning mathematical skills****(Walberg et al,1981;Shulman, 1987; GoP,2006)*

1.7.1 Teaching Mathematics

The teaching of mathematics depends upon the teacher's professional competency and teacher's teaching skills.

1.7.1.1 Math Teacher competency

According to Shulman's taxonomy of teacher knowledge (1987), among seven categories only three categories, come under the purview of this study including content knowledge (denoted as an academic qualification), knowledge of pedagogy (denoted as a professional qualification), and knowledge of curriculum were given due consideration.

1.7.1.2 Math Teacher Teaching mathematical skills

The concept of teaching skills refers to the ability and potentialities of an individual to transfer or communicate knowledge to the learner in a comprehensive way so that the learner grasps knowledge without any confusion or problem also categorized by Shulman (1987) as knowledge of content pedagogy. In other words, if the communication skills of an instructor or teacher are strong, the students would be able to learn and understand the concept disseminated by the teacher. These attributes play an important role in making teaching effective and commendable and are categorized into three main domains i.e., comprehensiveness, repertoire (selection of skills from the diversified pool), and matching (Saphier, Gower, & Haley-Speca, 1997).

Comprehensiveness

Comprehensiveness does not apply to the mastery of subject matter only, but it covers all the domains, which are required for effective teaching in the grade.

1. Conceptual understanding: The teacher is required to may not solely focus on the completion of the syllabus but ensure the conceptual understanding of each student from simple to complex without missing any individual in the grade.
2. The students are only anticipating the response from their teachers and also ought to be facilitated accordingly.
3. Teachers may be aware of students' problems and intellect levels.
4. They may have all the necessary details about students so are in a better position to handle the chronic and challenging situations regarding students' psyche.
5. The teachers often extend their help to students to connect their concepts to daily life to solve the confronted problems.
6. The teachers can design assessment tasks to check students' abilities.
7. Since the students mostly spend time in schools, the teachers may provide guidance and help them in resolving the confronted problem in a timely fashion.
8. When the students face any academic problems, the teachers initially engage them

in providing conservative guidance and motivate them todo brainstorming and identify any possible solution for the problem at hand. This approach helps the students to develop and enhance their self-confidence and sense of commitment toward their academics.

9. In addition, the teachers provide a conducive environment to the students by engaging them in classroom activities and motivating them to work in collaboration with their classmates. Since, when the students are engaged to work in collaboration, they help to assist the students who have any baffling or weak academic concepts.

Repertoire (Selection of Relevant Skills from the diversified pool)

The term repertoire could be defined as the application of different teaching strategies to handle different situations of teaching during the teaching-learning process and avoid any sort of disturbance in disseminating academic concepts. The teacher can plan a lesson and decide on some specific teaching strategies to make concept sound delivery in the classroom. In the process of teaching and learning, every day is a new day for students as well as for teachers, so the teachers may adopt different teaching strategies during the classroom to clarify the confusing and difficult academic concepts and motivate them towards their academics. For instance, the teacher has strong control over the subject matter and can adopt any teaching strategies in the classroom without any hesitation or confusion, the performance of such teachers is remarkable.

Matching

In the context of teaching skills, the matching is the expertise of the teacher, which enables them to opt for or select the suitable strategy during classroom teaching. The teachers often provide a blend of learning opportunities to their students when they have expertise in accommodating or opting content relevant suitable teaching strategies. This approach motivates the students to learn new concepts and also makes them inquisitive about new teaching strategies.

1.7.2 Learning skills

Learning skills refer that the ability to learn something new in a controlled environment. Conceptual understanding, procedural knowledge, and problem-solving are three important mathematical skills to fully understand the basic mathematical concepts i.e., numeracy, time management, accountancy, percentage finding, presenting data, measurement, and geometry, and applying acquired knowledge to daily life.

Subsequently the mathematical skills has close association with the Blooms' taxonomy, which has six stages such that knowledge, comprehension, Analysis, application, synthesis, Evaluation. Conceptual understanding, procedural knowledge, and problem solving come under the purview of six phases of Bloom' taxonomy.

1.7.2.1 Conceptual understanding

Conceptual understanding refers to an integrated and functional grasp of mathematical ideas. Students with conceptual understanding know more than isolated facts and methods. They understand why a mathematical idea is important and the kinds of contexts in which is it useful.

1.7.2.2 Procedural knowledge

The type of knowledge exercised in the performance of a task. Procedural knowledge makes the student solve the problem step by step in a logical way.

1.7.2.3 Problem-solving

Problem-solving is the act of defining a problem; determining the cause of the problem; identifying, prioritizing, and selecting alternatives for a solution; and implementing a solution.

As the conceptual framework shows the interconnection between the two strata which are teaching skills and learning skills. The conceptual framework demonstrates that teaching skills have prior importance for any learning process. The learning skills of students could only be developed through effective teaching strategies if employed in the classroom. The numerical skills specify the concepts of information that are ideas involved in effectively accommodating the handling of numerical problems, which incorporates the understanding of key concepts along with their applications, and processing procedure for resolving the problems. The numerical skill refers to the capacity of students to develop their reasoning skills, to share and disseminate, and able to develop associations of the notions and potential capabilities in mathematics. Implementation of acquired knowledge on daily life problems regarding math successfully. For example able to calculate the percentage, measure area, volume, etc.

1.8 Significance of the Study

The current study helps the curriculum developers, teachers, and other concerned stakeholders about the importance of the subject of mathematics and the degree of development of mathematical skills among the students. This research also elucidates the

teaching of mathematics at the elementary level and the mechanism of learning by students. It is expected that the findings of this research study fascinate mathematics teachers towards the adoption of attractive and effective teaching methodologies and strategies to teach the subject of mathematics at the elementary level. This study will provide insight to the relevant stakeholders regarding various teaching strategies for mathematics subjects, a strong and transparent teacher evaluation mechanism, induction of qualified teachers, and the development of professional aspects of teachers.

1.9 Methodology

The conceptual framework of the current study is based on the "Theory of Educational Productivity" by Walberg et al (1981). Among the key factors identified by Walberg et al (1981, the quality of instruction and classroom environment was taken into consideration in this study, which shows the reflection of a competent teacher. Thus focusing on the teaching skills and learner's learning skills are deducted from the quality and quantity of teaching. Objectives were designed and according to objectives, research questions and hypotheses were formed. Research instruments were designed in light of the literature review and conceptual framework. Through focus group designs the research instruments were validated. Reliability was verified through a pilot study. The whole data was collected by the researcher herself from public and private sector selected schools in Islamabad Sector I-9 and I-10.

1.9.1 Research Design

The researcher selected a convergent parallel-mixed method research design to get meticulous details about the subject researched. In this study design qualitative and qualitative data were collected simultaneously and then analysed separately. After analysis, both qualitative and quantitative data were compared, triangulated, and then interpreted.

1.9.2 Population

The population of the study constitutes 5 registered private and 12 public schools of elementary level located in I-9, and I-10 sectors. The population of 7^{th}-grade students was 1050, 600 students in public sector schools and 450 in private sector schools. 40 teachers, 30 teachers in public sector schools, and 10 teachers in private sector schools (Pakistan Education Statistics, 2017).

1.9.3 Sampling

From the population of the study, a total of 06 schools (03 public sector, and 03 private sector schools) were selected. The students who constituted the sample of the study were 278 $\approx$ 300 selected through proportionate sampling using sample size determination techniques (Krejcie & Morgan, 1970).

Furthermore, 06 teachers were selected from the selected schools i.e., 03 teachers from the public sector and 03 teachers from the private sector respectively, who were teaching mathematics to grade 7th.

1.9.4 Instrument

For data collection, three research instruments were used i.e., semi-structured interview (Appendix-I), observation checklist (Appendix-II), and tests (Appendix -3, Appendix -4, Appendix -5, Appendix -6), encompassing five basic standards of mathematics subject. The semi-structured interview and observational checklist were developed in the light of the empirical approach, literature review, and reflection on the research objectives.

1.9.5 Validity of the Research Tool

The validity of the semi-structured interview, observational checklist, and tests were checked through Focus Group Design (given observational checklist and tests to experts in the relevant field) emphasizing face, content, and construct validity.

1.9.6 Reliability of the Research Tool

After the pilot study phase, the reliability of the tests was checked through the Coefficient of Cronbach Alpha.

1.9.7 Data Collection

The research approach was a mixed method, two types of data were collected. The qualitative data was collected through the semi-structured interview with 7th-grade math teachers before the observation of their sessions in the classroom and the observation checklist was filled during the math period of the 7th grade after the session tests were given to the students of the same grade to collect quantitative data. Both data were collected actively by the researcher herself.

1.9.8 Data Analysis

Qualitative data were analysed using a thematic approach while the quantitative data collected through tests were analysed through One Way MANOVA. After analysis, the results were triangulated.

1.10 Delimitations

The study was delimited on the following points because of a lack of resources, covid-19, and limited time.

1. The study was conducted in Islamabad city only.
2. The schools of H-8, I-9, and I-10 were selected because of availability and ease to access.

3. A total of 06 schools were selected in which each of the 03 schools was selected from the Public and Private sectors.
4. The teachers of elementary school levels, teaching math to grade 7^{th} were selected from the public and private sector schools.
5. Only the students in grade 7th were selected for the study.
6. There were two shifts in the schools of Islamabad i.e., Morning and Afternoon. Only the students on the morning shift were selected for the study.
7. Among nine theoretical constructs of Walberg et al (1984) Theory of Educational productivity two constructs that are Instructional quality and Classroom Psychological Environment were selected for the current study
8. The factors i.e. comprehensiveness, repertoire, and matching were selected concerning teaching skills from the skilful teacher model by Saphier, J., Gower, R. R., & Haley-Speca, M. A. (1997).
9. The learning skills i.e., conceptual knowledge, procedural knowledge, and problem-solving were selected concerning learning skills Pakistan National curriculum of Mathematics.

1.11 Operational definitions

1.11.1 Teaching Skills

The ability of a teacher to transfer knowledge to the students in an effective way so they can easily absorb it and be able to use it in daily life.

1.11.2 Teaching Methodologies

Teaching methodologies refer to any set of instructions and procedures which are employed by teachers or instructors in formal or informal settings aiming to disseminate learning effectively to the students

1.11.3 Lesson Plan

A lesson plan refers to the planned activity for delivery of knowledge to the students in which certain parameters are given due consideration like classroom management, time management, the usability of Audio-Visual Aids, articulating learning objectives, identifying learning outcomes, and focusing on achieving in a timely fashion, and mentioning classroom assessment

1.11.4 Classroom Management

Classroom Management refers to the techniques in which the classroom is arranged in an effective pattern to disseminate learning among the students

1.11.5 Classroom Assessment

Classroom assessment refers to the techniques where the student's learning is assessed or checked after the lectures delivered by the teachers regarding the subject matter.

1.11.6 Mathematical Skills

Ability to solve number-related problems in daily life, for example, Numeracy skill. Problem-solving skills, Time management, Measurement, logical reasoning, Data handling, etc.

1.11.7 Curriculum

A national formal document that contains all the designed sets of instruction to provide a clear guideline to educational institutions regarding the educational achievements of learners per the National Education Policy and the National Ideology of a country.

1.11.8 Learning skills

Learning skills refer to the production process which is acquainted with the involvement of students in their academia. This encompasses conceptual understanding, procedural knowledge, and problem-solving

1.11.8.1 Conceptual understanding

Ability to understand the essence of taught math topics and its implication regarding the need in practical life.

1.11.8.2 Procedural knowledge

The basic knowledge to solve a problem in logical sequence step by step to get the result.

1.11.8.3 Problem-solving

Ability to identify the nature of the problem and select the correct method to resolve it systematically in a concrete way.

1.11.9 Compromised mathematical understanding

Mathematical understanding that does not meet the benchmark of mathematical studies and are not easily understandable to the students. The mathematical concept is not clear or able to apply the knowledge to the confronted problem.

CHAPTER 2

REVIEW OF RELATED LITERATURE

A literature review is a pivotal segment in research since it provides an insight into the field of interest, and ensures intellectual freedom and fluency in information in a logical sequence (Boote & Beile, 2006). A literature review aims to provide formal justification for the study and it can be achieved by four main objectives. Firstly, the published literature is reviewed to identify and summarize relevant concepts and studies; secondly, to develop a critique of the literature in terms of developing stout arguments to support or deny the theories, assess the worth of research claimed by the previous researchers, and identify the limitations in the previous research studies it provides a systematic procedure. Thirdly, regarding the pattern of gap identification in the previous and current research studies and developing insight about the knowledge gap and areas being partially researched, literature review plays a pivotal role. Fourth, the literature review aims to yield the proposed research in terms of providing a rationale for the proposed study and to help in opting for a relevant research design and methodology (Younger et al., 2004). Since, to assess the gaps in a literature review, the current study must provide a rationale for conducting the review of relevant literature; hence, at the end of a review of relevant literature, the gaps have been identified in the synthesis paragraphs.

2.1 Mathematics

Mathematics is the language of the universe (Gallardo, 2018) that plays vital role in understanding the key concepts of life with logical reasoning. Mathematics is an integral part of all fields of life that is from the study of the universe to the calculation of money during buying groceries.

Mathematics is an area which promotes logical thinking and helps to portray abstract entities in qualitative terms. It helps in dealing with day-to-day life activities and provides the foundation for different areas of natural and social sciences. The exponential growth in different subjects has direct or indirect connections with mathematics. It helps to develop precision, accuracy, focus, critical thinking, and the ability to criticize for the sake of development (Iqbal, 2004).

According to Okereke (2006), mathematics portrays the actual picture of maintaining consistency and logical order in an entity. Mathematics provides a foundation for other sciences and is integrated into every other field. Besides its importance and significance, the teaching of mathematics subject is not up to the mark and mostly the learners' aptitude towards this subject is not justified. The learners face tremendous problems in conceptualizing the

content and then its applications in a real-life scenario. Female learners are not performing well; however, the performance of male learners in the subject of mathematics is satisfactory. The research further proclaims that factors like a rough and difficult concept about the subject of mathematics built in the society, availability of qualified and trained teachers, non-existence of mathematics laboratory, least level of attraction in the subject, lack of innovative approach adopted by teachers to disseminate the academic concepts, and a general concept of merely passing the examination affect the overall performance of the learners.

2.2 Mathematics Curriculum

In the context of the National Curriculum of Mathematics, three categories have been mentioned to promote literacy and develop the critical approach among learners in Pakistan. These categories include knowing and applicability of mathematics knowledge (conceptual knowledge), practicing mathematics (procedural knowledge) and practical implications of mathematics knowledge (problem solving). These indicators are associated with the standards, which lead the learners to learn mathematics. A focus on learning the subject of mathematics is given to the components like higher-order thinking, and in-depth and practical knowledge of mathematics (Govt. of Pakistan, 2006). Since the proposed results are mentioned in the curriculum, however, the pedagogical component needs alignment to teach the relevant subject matter with the appropriate teaching method. This ensures the clear teaching philosophy of teachers to develop centrality in the learning pattern of learners (Fosnot, 2005). Teaching the content with relevant and appropriate teaching methodology provides a forum for learners to learn the concepts and indeed provides an opportunity to grow.

From the literature, it is evident that the syllabus of mathematics subject taught in the schools shows a vast gap between the goals set in a curriculum and the actual syllabus taught in different schools in Pakistan. Being existence of a variety of schools i.e., English & Urdu medium, rural and urbanized schools, and government and private sector schools. The patterns of teaching changed with the school and similarly, the learning pattern of learners also vary. In such circumstances, it seems challenging to achieve the desired goals without rational alignment of the subject matter with the appropriate teaching method. Many research studies assess the learners' academic achievements in the subject of mathematics at a different level in Pakistan and claim low performance of learners in 9th and 10th grades as compared to other subjects (Ali, T., 2011).

2.3 Theory of Educational Productivity

Educational productivity was always remained focused on by developed civilizations. Educationists did intensive research work in finding the factors influencing the achievements of students. A meta-analysis was done by Walberg and his companions in the 1980s to find the influential factors affecting the educational achievements of students. The focus was on nine factors that were considered to be the most influential in effect achievements in studies. Those nine factors were

1. Biological age and maturity level
2. Inner potential
3. Level of motivation
4. Amount of instruction
5. Instructional quality
6. Access to mass and media
7. Classroom environment: psychologically
8. Home Environment
9. Peers

Research provided clear evidence that these all factors are correlated to learning (Walberg, 1984).

In the current research, the researcher focused on instructional quality and classroom management that affects the learning achievements of students. Instructional quality is based on the teacher's professional competency and teacher's teaching skills. Teaching skill encompasses three comprehensive terms a) comprehensiveness b) Repertoire c) Matching (Saphier, Gower, & Haley-Speca, 1997).

2.4 Teaching and Learning Theories

The process of familiarizing with knowledge is categorized into two segments i.e., delivery and absorbing the knowledge. The former comes under the domain of teaching and the latter comes under the domain of learning. The teaching and learning trajectory are the two pivotal segments which transform, regulate, and share knowledge. This trajectory remained of utmost importance as it not only helps in transferring knowledge from generation to generation but also helps in developing civilization in a way to progress in science and technology. Philosophers and educationists did a lot of work to not only define the terms of teaching and learning but also present theories to make the process of teaching and learning the most effective.

The three basic theories of teaching and learning are Behaviourism, Cognitive Constructivism, and Social Constructivism.

2.4.1 Behaviourism

Behaviourist theory of learning was proposed by B.F. Skinner (1904-90). Skinner's learning environment is teacher controlled. Teacher is fully equipped with teaching material knowledge and teaching methodologies. The student of skinner is passive and absorbs knowledge according to the provided material. This theory proposes that learner has to learn in teacher controlled environment. To develop a desired behavior, the system may be based on reward and punishment. Skinner teacher is competent and active.

2.4.2 Cognitive Constructivism

Cognitive constructivism theory of learning was proposed by Piaget (1936). This theory describes learning as the mental process, that how a person receives a stimuli and built knowledge regarding that stimuli through mental process. The classroom environment of cognitive constructivism is fully equipped. Teacher is competent but serves as facilitator. Students learn things by doing. Problem-solving approach is basically used in cognitive constructivism classroom.

2.4.3 Social Constructivism

Social constructivism theory is presented by Bandura(1977). This theory is considered as a bridge between behaviorism and cognitivism theory. According to this theory learning is not only the mental cognitive process. Learning depends upon observation, imitation and modelling. Classroom environment of this theory is well organized regarding the topic of learning. And teacher is facilitator and model. Through positive reinforcement students learns and imitate,

2.5 Teaching of Mathematics

Education systems are established in each society to provide society with a product, which can not only face societal problems but also solve them intelligently (Chin & Chia, 2004; Walker & Lofton, 2003). So, the main thrust of every education system is to make the learners capable of solving their confronted problems in an effective way (Chin & Chia, 2004; Walker & Lofton, 2003).

2.5.1 Teacher knowledge

In Shuloman's taxonomy of teacher knowledge (1987), shuloman had divided teacher's knowledge into seven different categories. Among the most relevant to current study are as:

2.5.1.1 Content knowledge

The command of a teacher on the subject matter provides enough opportunities for teaching in novel and effective ways, which eventually enhances the academics of learners. According to Hanushek and Rivkin (2010); Todd and Wolpin, (2003), the performance of learners on standardized tests was affected because of the background of teachers and their command of subject matters.

2.5.1.2 General Pedagogical knowledge

The role of teachers in the development and promotion of the core academic skills of learners is predominantly emphasized by various theories of teaching and learning. For instance, the essence of high-quality teaching, an organizational technique, and a set of emotional considerations are important for learners in terms of teachers' instructional strategies (Pianta & Hamre, 2009). The research indicates that with the provision of stable emotional consideration and a conducive environment for the teaching and learning process, the learners are often equipped with self-confidence, a high degree of motivation towards the learning of new academic concepts, and can accommodate complex concepts. Furthermore, the learners can polish and refine their potentialities through the strong organizational structure provided by teachers.

In recent eras, two traditions in research are being employed i.e., firstly, focusing on observation of the classroom concerning the teaching approach of teachers during the classroom environment, which includes the teacher-learner academic interaction, organization, and management of the classroom, focusing teacher on the development of critical thinking, and development of learners' core academic skills (Blazar, Braslow, Charalambous, & Hill, 2015; Hamre et al., 2013). Secondly, assessing the contribution of teachers concerning outcomes of learners, primarily referred to as teacher's efficacy" (Chetty Friedman, & Rockoff, 2014; Hanushek & Rivkin, 2010). These studies reveal that the performance and capabilities of teachers vary with the performance of learners in their classroom and in addition, besides developing the academic perspective of learners, the focus of teachers was also crucial for the development of the social and emotional behaviour of learners (Backes & Hansen, 2015; Gershenson, 2016; Jackson, 2012; Jennings & DiPrete, 2010; Koedel, 2008; Kraft & Grace, 2016; Ladd & Sorensen, 2015;

Ruzek et al., 2015). Furthermore, a weak to moderate level of correlation between the effect and influence of teachers on learners' aptitude toward their learning outcomes were found, which indicated that the test scores of learners during their assessment may not explore the overall potential capabilities of teachers in the classroom.

The studies also reveal that certain factors are responsible for framing the learners' academic achievement, among which the dominant ones are the influence of emotions, personality, and thinking abilities of learners and the pattern of their learning (Duckworth, Quinn, & Tsukayama, 2012), self-control, emotional stability, consistency in nature, and high level of motivations (Borghans, Duckworth, Heckman, & Ter Weel, 2008; Chetty et al., 2011; Moffitt et. al., 2011). These factors are often considered strong predictors of outcomes with long terms status rather than scoring in tests (Chetty et al., 2011).

The research studies indicate that the teaching profession has multidimensional aspects. Good teachers are not merely a medium of getting high grades and scores on tests but are also supportive to stabilize the emotional aspect of learners, provision of a conducive environment to contribute to social and emotional development, the rectification of distorted classroom behaviour, teaching precise content and developing critical thinking abilities among learners (Cohen, 2011; Lampert, 2001; Pianta & Hamre, 2009).

2.5.1.3 Content pedagogical knowledge

The attitude of students can also improve by the teachers when they use or apply content-specific teaching strategies in the classroom to deliver the lectures. This may have a direct impact on getting high scores from the students on the topic at hand. The experts in the subject of mathematics suggest that the teachers are required to be engaged in pre-service training i.e., teaching practice for getting expertise in the pedagogical skills in their relevant subject. This could develop a platform for developing critical thinking and problem-solving approach among the students, which is the aim of the subject of mathematics (Lampert, 2001; National Council of Teachers of Mathematics [NCTM], 2014). While some research studies indicate the teacher plays an important role in developing a sense of self-efficacy among the students and also helps in reducing the anxiety often confronted by students in the subject of mathematics (Usher & Pajares, 2008). This anxiety develops due to a lack of conceptual understanding of the students getting command of the subject matter.

In the context of instructional approaches, an observational study was conducted to identify and highlight other competencies in teaching along with exploring the links and association with the behaviour and attitude of students towards their academics. For

instance, a study was conducted to explore the specific dimensions of teaching in the subject of mathematics through observing the classroom of teachers during teaching practice sessions. The study explored that the students engaged in teaching the concepts of mathematics through activities in the classroom developed strong cognitive approaches with precision and accuracy in solving the confronted problems (Blazar et al., 2015). The validity aspect of the study focused on the teaching and learning skills of students in terms of their academic scores in the subject of mathematics (Blazar, 2015; Kane & Staiger, 2012), which indicates strong theoretical associations between the content taught by the teachers, dissemination of content through various effective teaching methodologies or teaching strategies, and the understanding of students (Hill et al., 2008). In addition, the organization with a professional aptitude and researchers elicited theoretical associations between the pattern of teaching in the classroom and the learning outcomes obtained by the students after conducting the test on those topics which were taught to them (Lampert, 2001; NCTM, 2014; Usher & Pajares, 2008) that the knowledge of students has not been properly tested.

According to a study conducted by Jennings and DiPrete (2010), the correlation between the effects of teachers on social and behavioural outcomes of learners along with their performance in the subject of mathematics was 0.15. In another study, Kraft and Grace (2016) revealed the existence of a correlation i.e., 0.23, between the effect of a teacher on the academic achievement of learners and multiple social-emotional competencies. On the other hand, in a study by Gershenson (2016) and Jackson (2012), a weak relationship was found between the effects of a teacher on the academic performance of learners. In another study conducted by Ruzek et al. (2015), a positive correlation i.e., 0.50 was found between the effects of a teacher on the academic achievement of learners and their motivation in the grade of mathematics. According to Mihaly, McCaffrey, Staiger, and Lockwood (2013), a correlation coefficient of 0.57 between the effects of a teacher on learners' self-directed efforts and test scores in the subject of mathematics.

2.5.2 Effective Pedagogy for Teaching Mathematics

Learning theories proposed two basic approaches for effective pedagogy. Behaviorism proposes teacher-centered approach while constructivism and social constructivism enforce learner centered approach (Bandura, 1977).

In the learner-centered approach, the teacher solely acts as a facilitator and is responsible for providing a conducive milieu for active learning and providing opportunities to learners in resolving their confronted academic problems and absorb knowledge. Whilst, in teacher-centered learning, the teacher plays a pivotal role in delivering the knowledge to the

learners, where learners redeem the knowledge and process for inculcation as the part and parcel of their personality. The research proclaimed that in all active learning processes, the learners absorb knowledge according to their capacity, potential and needs (Orhan & Ruhan, 2007). In active learning processes, an environment is created for learners to provide them with opportunities to understand the problems and find their solutions. It trains the learner to self-regulation. The active learning process is personalized and not the standard process, it helps learners to able to face all multidimensional problems and resolve them intelligently in the light of their practical experience and to gain knowledge during the learning process.

2.5.2.1 Learner-Centered approach

It is imperative to mention that simply deliverance of knowledge is not teaching and similarly learning could not be possible solely by listening. Some learning processes revolve around the teacher; however, the learners solely act as passive learners. In the learners-centred approach, the teacher facilitates the learners in learning and provides individual attention whilst the learners are provided freedom in learning at their own pace and need (Orhan & Ruhan, 2007). The learners are given the opportunity in making decisions concerning various aspects of the learning process and to ensure self-regulation. Listening is not a fundamental parameter in learning in the case of active learning, but a personalized approach of an individual learner. Individuals face numerous problems in every run of life, and they solve those confronted problems based on their previous experiences and knowledge. The same applies to the learners and the need to prepare the learners to cope with academic problems, and real-life problems in future to make them stout personalities.

John Dewy emphasized learning by doing and was of the notion that learners learn most when they are involved in academic activities (Dewy, 1938) rather than by merely being passive learners. Through this approach, the teacher can achieve their set goals and aims for a particular grade in each time frame and even the confidence level of learners gets enhanced. In the current era, problem-based learning is widely employed in teaching different subjects and was first used in teaching medical subjects in the 1980s (Herrid, 2003). It is evident from the literature that problem-based learning was exponentially increased at the elementary, secondary, and higher secondary levels (Lambros, 2002).

Further studies reveal that the academic performance of learners at the secondary level is not much impressive and up to the mark and it has a low approach to problem-solving and other comprehensive competencies. For such outcomes, there are certain factors responsible i.e., the least exposure of teachers to new teaching methodologies and the attitude of learners towards learning the subject of mathematics. The development of

a curriculum framework for the subject of mathematics at the elementary and secondary levels determines the attitude of learners towards learning mathematics, which eventually shows the differences in achievement levels (Ma & Xu, 2004). It is evident from the research that the academic achievement of learners varies from school to school because of the variation in the implementation of different structured curricula. The research indicates that learners can qualify the items with a low level of precision and thus requires high learning intensity to refine their simple mathematical skills and knowledge. In addition, the gender aspect also plays a pivotal role in learning mathematics i.e., female learners learn the basic terminologies, symbolic forms, geometric properties, and numbers but with a low level of problem-solving skills. On the other hand, male learners have high abilities to cope with problem-solving skills (Mash'hadi & Shah, 2008) rather than cramming.

2.5.2.2 Teaching Mathematics Through Problem-Solving Approach

Nafees (2011) was of the notion that a problem-solving approach holds the attributes of developing high-order cognition, developing links among concepts, comprehension skills among learners, and developing logical skills and analytical abilities. Hence, the problem-based approach inspires the learners to promote and develop strategies through continuous practice and imitates to solve the problems for further reference (Weber, 2008). This ultimately helps the learners to develop their approach to developing and evaluating theoretical formulas to solve problems. It is imperative to mention that the teacher needs to be clear about the sequence and content of teaching and must carry an understanding of providing required skills to their learners for better results.

According to Tick (2007), in problem-based learning, the learners are considered the central point of interest and all the teaching strategies are learners oriented. In this approach, the concept of individualized learning and self-directed approach in learning the subject matter according to learners are emphasized, whilst the teachers are monitoring the learning aptitude where deemed necessary, it is tailored accordingly. The learning objectives designed by stakeholders are not solely for receiving the subject matter without learners' active involvement and merely the same reproduction of concepts without approaching innovative ideas.

The worth and importance of the subject of mathematics are well documented by educationists across time. History witnesses the efforts of renowned educationists in putting efforts into the subject of mathematics.

According to Yager (2000), the current era is dynamic and the social, political, and digital world; changing at an exponential pace. The learners of the current era are also

required to equip with modern trends concerning learning aspects. The stakeholders, on the other hand, are required to develop the curricula in line with the need and demands of the learners in particular and society in general. This ultimately ensures the resonance in the knowledge and skills of learners in an organized fashion and eventually prepares the learners to cope with the situation confronted in real life. The concept of learners' empowerment in academics needs to be highlighted to develop possible ways of solving any confronted problems.

Mathematics and learning through a problem-solving approach

In problem-based learning, the learners are considered the central point of interest. It aims to develop vigorous and motivated learning and enhances the skills of dealing with confronted problems (Major et al, 2000). In the classroom where the learners are engaged in learning through the application of problem-based learning, the learners are inquisitive and are much more responsible for learning at their own pace. This develops confidence among the learners and they can tackle the understanding of new and complex concepts on their own and hence enhance their learning.

According to Ahmed (2011), the performance of learners (measured via achievement and problem-solving ability test) taught through Problem Solving Approach is higher than the learners who are taught through the traditional teaching method. The study reveals that two groups i.e., Control and Experimental Groups were made and then the performance of learners regarding the problem-solving approach was observed. The results revealed that the performance of learners in experimental groups was high than that of learners placed in the control group. The study further claims that the learners of the experimental group performed well in achievement tests and problem-solving ability tests. Furthermore, the reasoning and critical thinking ability of learners taught through Problem Solving Approach was higher than the learners taught through the traditional teaching method.

Model of learning through a problem-solving approach

According to Yuzhi (2003) and Mangle (2008), in the model of problem-based learning, the learners' mode of passiveness is transformed into active, freedom in learning new concepts and resolving the confronted problems. Through this model, a paradigm shift from teaching to learning could be employed and also helps the learners in dealing with the confronted problems rather than feeling boredom. In this model, for instance, the learners are engaged in learning the techniques, which eventually accord them to deal with a diversified range of problems. The positive impacts of problem-based learning are on

the acquisition of information, information handling, leading the group, collaborative work, and flow of communication. Indeed, problem-based learning is a formal approach, which holds novel teaching methodologies, promoting the culture of creative thinking and an appropriate approach to achieving the desired goals in a given time frame. Besides these, the foremost aim of problem-based learning is to obtain information based on facts and figures.

Gallagher et. al, (1999) were of the notion that in problem-based learning, the learners are considered professionals and subjected to problems that require clear resolution in a structured manner, the establishment of hypotheses, analyzing the data collected from multiple sources, revamping the developed hypotheses based on collected data and proposing a tangible solution in this regard.

According to Eng (2001), problem-based education is considered the medium of emerging new innovative teaching method which helps learners in coping up with their academic needs. The teachers are employing problem-based learning methods in the classroom to modify the understanding of learners and equip the learners with practical experience, and develop capabilities among the learners to deal with problems and resolve the problems. In addition, the research indicates that problem-based learning provides a conducive milieu for the learners to focus on holistic development of learners.

According to Khan, Akhter, and Hukamdad (2010), the performance of learners is affected by multiple confounding variables and factors. An experimental study was conducted in the Municipal Corporation Secondary School of District Rawalpindi, Punjab where two groups i.e., control and experimental groups were developed. The learners of the control groups were taught through Problem-Solving Approach whilst the learners of the experimental groups were taught through Traditional Teaching Method. The focus of the study was to assess the effectiveness of the problem-solving approach in the subject of mathematics on the performance of learners. The result of the study reveals that the learners taught through a problem-solving approach performed high in their academics; however, the performance level of the learners taught through the traditional method of teaching was low. This is because of the reason that in problem-based learning, the learners were actively involved in learning new and complex concepts and bestowed hands-on experience to the learners in learning mathematics. The result of the research indicated that the achievement scores of learners of the experimental group showed significant improvement as compared to the learners of the control group. The findings of the study conducted by Kousar (2009) that the academic achievement of learners taught through a problem-solving approach was high than those learners who were taught through the traditional method of teaching, were also in line with the findings yielded by

the study of Khan, Akhter, and Hukamdad (2010). Furthermore, the research conducted by Hsiao and Chang, (2003); Tang and Huang (2006) obtained the same results as that of Kousar (2009) and Khan, Akhter, and Hukamdad (2010). The study of Hsiao and Chang, (2003); Tang and Huang (2006) proclaimed that better performance of learners was due to an understanding of techniques for dealing with the problems they faced and promptly proposed viable solutions for the problem, learners with capabilities of making a decision, developed mathematical skills among learners, development, and enhancement of the learners' confidence level of the experimental group. In addition, the learners of the experimental group develop reflective thinking which enables them to think critically and logically. The result of the study reveals that the learners taught through a problem-solving approach have a command of the subject matter of mathematics and strong conceptual understanding as compared to the learners taught through the traditional method of teaching. Since the results also indicate that the learners taught through the problem-solving approaches dwell on high cognitive effective outcomes and deem helpful for high and low achievers learners in the subject of mathematics.

Understanding of learners via Problem-solving approach

According to Tick (2007), the learners in the problem-solving approach are subjected to new problems to give themselves opportunities to handle the situations, which strengthen their understanding regarding the application of knowledge and also develop their critical thinking abilities among them. The researcher was of the notion that in problem-based learning, the central point of interest is *the learner,* and focus is primarily given to the learning of learners rather than merely the reproduction and regurgitation of academic concepts without conceptual understanding. The research also stresses the developing and strengthening of critical thinking approach among learners and aimed to enhance self-directed learning among learners to exhibit academic freedom.

Role of Teachers in a Problem-solving approach

The significant achievement of teachers concerning their learners is the inculcation of aptitude for independent learning. In problem-based learning, the role of a teacher is just a facilitator or passive instead of becoming the central point of delivering learning. According to Roh (2003), the critical point in terms of teachers' teaching, the problem-based learning required more abilities and experiences in teaching as compared to the traditional method of teaching. The teachers are required to engage the learners in dealing with problems rather just merely presenting the mathematical skills. The ultimate aim of problem-based learning is to equip the learners with the subject matter and subject to the arena for exhibiting their application of knowledge in a real setting.

Pedagogical skills concerning the Problem-solving approach

Research indicates that the poor performance of learners in the subject of mathematics at the elementary school level reveals that the required technological and scientific application of mathematics knowledge could not be continued for a long period. This indicates that the appropriate method of teaching yields desired outcomes and it is imperative to sough content-specific teaching methodologies, where the involvement of learners is the high and practical implication of knowledge executed (Okigbo & Osuafor, 2008). Problem-solving method of teaching provides good results in the subject of mathematics, which primarily fulfils the teaching desire in terms of detailing the fundamental academic concepts and techniques of solving the problem. Since the problem-solving method has good outcomes, it is worthwhile to accommodate it in the subject of mathematics and the learners are engaged in their learning through this method to achieve the desired goals.

2.6 Learning Basics of Mathematics: Definition and Explanation

Learning mathematics is often segregated into two domains i.e., in-depth learning and cramming. These terminologies were widely explained by many researchers. In the former one, learning could be considered as academic achievement and often termed as the development of mathematical sense among learners, effective strategies employed for solving the confronted problems, concept development, and potentiality of argumentation in addition to the ideas built by conceptuality (Jenkins, 2010).

Some researchers proclaim different learning types concerning the subject of mathematics like relational and instrumental learning and the descriptions of views concerning relational and instrumental learning. Since, these are interconnected with the practice already implemented for the teaching of mathematics (Skemp, 1976). Reasoning in the context of concepts and the sense of learning can be developed with the help of relational learning whilst instrumental learning inclines towards the attainment of learning capacity despite involvement in logical reasoning (Mash'hadi & Shah, 2008).

2.6.1 Enhancing academic achievement through In-depth learning

The medium of enhancing learners' academics, as indicated by many researchers, is in-depth learning. On the other hand, the aptitude of cramming among learners in learning mathematics concepts leads to weak academic concepts and often the performance of learners is affected. That is why the teachers focus on in-depth learning and try to conceptualize the content knowledge.

The content knowledge contains information about the process and basic principles, which are revamped as per the need and demands of learners. Many studies indicate the benefit of in-depth learning of mathematics subject (Newton, 2000).

2.6.1.1 Advantages of In-depth Learning

There are many advantages of in-depth learning determined by the researchers. Firstly, learning can be influenced through in-depth learning. Mostly the learners' influencing points are the development of interest in learning the concepts and adopting a comprehensive approach to learning the basic principles. Hence, in-depth learning empowers the institution to bestow exposure opportunities to their learners in learning knowledge and gaining skills as per the need and demand.

Secondly, in-depth learning can be employed to expedite the learning capabilities of learners and pave way for innovative approaches to learning. In addition, the research also indicates that learners who have a stout understanding of academic concepts and have diversified knowledge of the subject matter can think logically and wisely. Furthermore, the learners could handle the confronted situation using innovative approaches (Newton, 2000). In-depth learning acts as a cognitive capability of an individual which enables the bearer to coordinate with the external world in an effective and precise way. Hence, a wisely developed mathematics curriculum for the secondary level promotes the learning sense.

This also develops the thinking potentialities and enhances the creativity factor among the learners (Newton, 2000). Despite the importance of in-depth learning in the subject of mathematics, it is placed at the secondary level, which shows injustice with the currently implemented curriculum of mathematics.

2.6.1.2 In-depth learning: Problems and Way forward

Even though the development of association among structural teaching and the implementation of mathematics curriculum in the classroom and the stances of teaching with the aim of in-depth learning is still shallow and dwells with certain concerns, it is clear that many academic institutions and professionals found it hard to teach for learning where the interest level of learners is not up to the mark. Mostly, the academic aptitude of learners is changing and often do not welcome learning opportunities (Government of Pakistan, 2009).

According to World Bank (2014), the problems often faced in understanding the concepts in the subject of mathematics are not solely related to Pakistan, but it is also a matter of concern for South Asian countries i.e., India, Sri Lanka, and India. in these

countries, the most prominent factors which affect learning in mathematics are the lack of trained and qualified teachers and also the lack of required resources. It is evident from the research that the due to non-availability of trained and qualified teachers, the learners are involved in learning the subject of mathematics without a clear understanding and are often involved in cramming. In other words, it is could be stated that the learners solely regurgitate the subject matter of mathematics and fail to conceptualize the concepts (Das & Barunah, 2010). Such academic behaviours are developed among the learners because of the lack of a formal assessment framework to measure the academic achievements of learners based on set parameters. Eventually, the learners of South Asia show the least interest in the subject of mathematics as compared to the learners of developed countries (Dundar et.al. 2014). The locality also acts as a confounding variable as the learners of urban areas have high mathematical skills than the learners of rural areas.

According to Das and Zajonc (2010), a test of the 9^{th} grade based on TIMSS mathematics items was executed among the learners of Odisha and Rajasthan: two states in India. The results revealed that the learners of these localities performed low scores and were placed at the bottom among 51 countries. For instance, the performance of learners (in the subject of mathematics) dwelling in Islamabad, Punjab and Khyber Pakhtunkhwa are higher in comparison with the learners of Baluchistan and Sindh (World Bank, 2013).

2.6.1.3 Effective Curriculum and factors affecting the In-depth Learning and Performance of learners

In-depth learning of the subject of mathematics not only requires the effective implementation of the curriculum but also solicits the cognitive aspect of learners to learn and apply the content in a practical arena. The instantaneous involvement of learners in employing the cognitive aspect in learning mathematics is difficult, which is deemed considered as the basic paradigm of learning, development of knowledge and skills in mathematics. This happens because of the differences in understandings, stances towards academics, potential skills, knowledge, and aptitude of learners, whilst it is difficult to seek uniformity in the deep engagement of learners in the classroom. Since there are numerous factors affecting the quality of mathematics curriculum to disseminate problem-solving skills among learners. These factors are:

i) External factors consist of a conducive milieu, parents, aspiration towards career, suitability, and absorption in the job market
ii) Internal factors constitute the motivation level of learners, commitment, and dedication of learners towards the learning of mathematics, aptitude of learners,

teachers' aptitude towards the teaching of mathematics and involvement of learners by respective teachers in the teaching-learning process

iii) Effectiveness and acquired knowledge in mathematics in the previous grades i.e., primary level. The learners' performance at the secondary level can get affected if they were not taught effectively in the previous grades because of the reason that the curriculum is linked logically from easy to complex. The learners who show low academic achievements in the previous grades may face difficulty in the higher grades which eventually leads to low academic performance in higher grades. Under such circumstances, the learners show boredom, a lack of interest in the subject and a low level of motivation towards academics.

According to Amirali and Halai (2010), the main factor in the education system which influences cognitive skills and is responsible for developing mathematical reasoning among learners is *the teacher*. The stance developed by the teachers among the learners regarding any subject is exponential in providing stout outcomes, therefore, the teachers' motivation level towards their assigned obligation matters a lot. If the motivation level of teachers towards their assigned duties and responsibilities is high, the performance of learners probably be high and often been observed by research that the teacher becomes a favourite among the learners. The teachers are in direct interaction with learners and work in alignment with the learning of learners using different effective teaching methods. Since the role of teachers is remarkable in the inculcation of knowledge rather than merely the transmission without developing learners' understanding. If, for instance, the conceptual understanding, command of the subject matter and skills of strong communication of teachers help them in making the subject interesting, which can be considered a great contribution. This eventually helps the learners in developing a strong conceptual understanding and take a keen interest in the subject at hand.

According to Physicist, Feynan (2002) the subject matter concepts are required to disseminate to the learners to stimulate their learning of previous accomplishments of various concepts and help to develop academic resonance. For the sake of generating a knowledge economy, the stakeholders are required to develop and implement concrete subject matter at the elementary and secondary levels, particularly in the subject of mathematics. As Wakefield (2001) was of the notion that the role of the mathematics teacher contributes significantly in terms of facilitating the learners in developing critical thinking and application of their learned knowledge.

The most significant and effective attributes of teaching mathematics are explored by the Education Alliance (2006) which are:

1 Application of previous knowledge gained and experience in teaching help in developing new knowledge
2 Employability of cooperative learning approaches and developing linkage with real-life situations
3 Development of linkages with academic concepts, processes and understanding
4 Questions with an analytical nature require proper justification from learners in giving their responses
5 Focus on teaching activities which are learner-centered and emphasized a problem-solving approach
6 Focus on the development of fundamental skills in the subject of mathematics
7 Focus on topics, based on developing certain skills according to the standards
8 Development of flexible groups to specify the effective teaching strategies
9 Adjusting the lesson according to the assignments and nature of questions deemed to develop required skills.

2.7 Synthesis of Literature Review

Since the literature review has been critically assessed for checking any potential gaps. It was evident that multiple studies were conducted on teaching methods and styles of teaching the subject of mathematics. The literature review indicates that the teachers often teach the surface content rather than focusing on in-depth studies, due to which the students often fail to absorb the crux of the content and subject. This leads to boredom among the students and the potential capabilities of students lack refinements. For the development of mathematical skills as per the National Curriculum of Mathematics, a comparative study addressing the main component of mathematics i.e., numbers, algebra, geometry, and data handling was not found particularly in Pakistan. This study aims to fill this gap and provide tangible recommendations to seek improvement in the performance of students.

CHAPTER 3

RESEARCH METHODOLOGY

3.1 Introduction

This chapter comprises all the important details regarding a plan for conducting a research study. It explains the research design which was selected for the current study. Further, it provides information about the population and sampling technique. It also provides details about the design of research tools and their verification. it provides a complete review of how data was collected and analyzed with strict care of research ethics.

3.2 Research Design

Research design convergent parallel-mixed method was selected to get meticulously detail about the subject researched. Symbolically it could be represented as qualitative and quantitative (QUAL+QUAN; Morse 1991, Creswell & Pablo-Clark, 2011). In this study design qualitative and qualitative data were collected simultaneously and then analyzed separately. After analysis, both qualitative and quantitative data were triangulated and then interpreted.

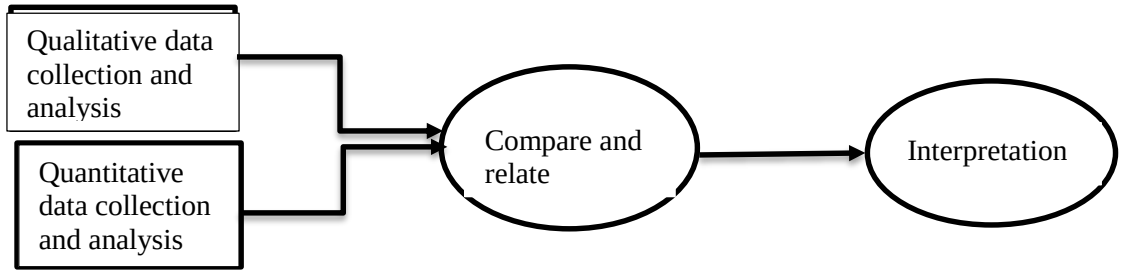


Fig. 3.1.

Convergent Parallel mixed method design (Morse 1991, Creswell & Pablo-Clark, 2011).

3.3 Population

The population of the study constitutes 5 registered private and 12 public schools of elementary level located in I-9, and I-10 sectors. The population of 7th-grade students was as under:

Table 3.1

Population

S. No.	School	Teacher	Students
1.	Public School	30	600
2.	Private School	10	450
	Total	40	1050

Source: Pakistan Education Statistics (2017)

3.4 Sample Technique

3.4.1 Quantitative sampling technique

From the population of the study, a total of 06 schools were selected. Among these schools, 03 public sector, and 03 private sector schools were selected. The students who constituted the sample of the study were $278 \approx 300$ using sample size determination techniques (Krejcie & Morgan, 1970). From each school, 50 students were selected through Stratified Random Sampling using equal allocation technique (Alvi, 2016).

Table 3.2

Sample distribution

S. No.	School	Teacher	Students
1.	Public School	3	150
2.	Private School	3	150
	Total	6	300

3.4.2 Qualitative sample Technique

From the population of the study, a total of 06 schools were selected. 06 teachers were selected from the selected schools i.e., 03 teachers from the public sector and 03 teachers from the private sector respectively, who were teaching mathematics to grade 7^{th} using purposive sampling technique.

3.5 Research Instrument

For data collection, three research instruments were used in the current study. Two for qualitative study and one for quantitative study

1. Semi-structured interview (Appendix-I)
2. Observation checklist(Appendix-II),
3. Test (Appendix- III, Appendix -IV, Appendix -V, Appendix -VI), encompassing five basic standards of mathematics subject.

3.5.1 Development of Research Instruments

The semi-structured interview and observational checklist were developed in light of the empirical approach, literature review, and reflection on the research objectives. The semi-structured interview was designed on the selected theme of Professional Competencies of math teachers (which includes the qualification, teaching experiences in years, area of specialization, understanding of the subject matter of mathematics, availability of syllabus according to the guidelines of National Curriculum of Mathematics, and pedagogical skills). The observational checklist's main theme was Classroom observation (which includes a lesson plan and its application, clear dissemination of objectives, and intended outcomes);

According to the National Curriculum of Mathematics (2006), there are five standards required to be inculcated in the students of grade 7th, which include numbers and operations, algebra, measurement and geometry, and information and data handling. These standards are underpinned by the common standard of reasoning and logical thinking.

The tests were developed accordingly from the textbook of mathematics subject of grade 7th. The first test of standard one (Appendix-III) i.e., numbers and operation contain 14 items (which were further divided into parts to cover all the important concepts) related to sets (02 items, the first one was easy and the second one was complex), rational numbers (02 items), decimals (02 items), exponents (02 items), square root of positive numbers (02 items), direct and indirect variation (02 items), and financial arithmetic (02 items). Due to the length test, it was divided into two segments to avoid the anxiety of students during data collection. In the second test of standard two (Appendix-IV) i.e., algebra contains four items related to algebraic expressions (02 items), and linear equations (02 items). Similarly, the third test of standard three (Appendix-V) i.e., measurement and geometry include six items related to the fundamental of geometry (02 items), practical geometry (02 items), and circumference area and volume (02 items). The fourth test of standard four (Appendix VI) contains two items related to information and data handling (02 items). All these four tests were underpinned by a common standard of reasoning and logical thinking abilities among the students.

3.6 Verification of Tool

3.6.1 Validity of the Qualitative Research Instrument

The validity of the interview and observational checklists were checked through Focus Group Design, where the instruments were provided to at least two experts in the relevant field, whose suggestions were incorporated into the qualitative research instrument.

3.6.2 Validity of the Quantitative Research Instrument

The validity of the tests were checked through Focus Group Design (giving tests to experts in the relevant field) emphasizing face, content, and construct validity.

3.6.2.1 Face Validity of the tests

The face validity was checked by the experts (who have PhD in the relevant field with a specialization in mathematical education) in terms of assessing the nature, type, and degree of complexity of the items in the tests. Furthermore, the data was checked with the assumption of the Cohen Kappa Index (CKI), which was not fulfilled and hence the CKI was not applied to the collected data.

3.6.2.2 Content Validity of the tests

For checking the content validity of the tests, Q-sorting techniques were used to check the logical sequence of items in the tests. It was the initial stage of checking the content validity of any research instruments.

3.6.2.3 Construct Validity

The construct validity of tests was checked through Q-sorting techniques at an initial stage. Through Q-sorting techniques, the items were properly prepared and have a reflection of the standards set for teaching and learning the subject of mathematics. Later on, Average Variance Extracts (AVE) were measured which was 0.67 with composite reliability of 0.51, thus indicating that the constructs were properly designed and validated.

3.6.3 Pilot Study

Prior to proceeding to the actual study, a pilot study was conducted using Adaptive Trail Design. The sample (30 students and 2 teachers) for the pilot study was selected through a rule of thumb from the population, which was not part of the actual study.

The aim of conducting the pilot study was to check the difficulty level of tests and observational checklists and amend, edit, or modify the tests and observational checklists for a better understanding of respondents. Necessary amendments and revisions were made to the observational checklist in consultation with the supervisor

3.6.4 Reliability of the Research Instrument

After the pilot study phase, the reliability of the tests was checked through the Coefficient of Cronbach Alpha. The Cronbach Alpha was checked for all four standards and mentioned against each. In addition, the first test was lengthy and hence divided into two parts in the longer interest of the student

The Coefficient of Cronbach Alpha for tests were as given below:

Table 3.2

Coefficient of Reliability

S. No.	Test	Cronbach Alpha
1.	Numbers	0.817
2.	Algebra	0.729
3.	Measurement and geometry	0.782
4.	Information and data handling	0.791

If the values fall in the range of 0.4 to 0.9, then the questionnaire will be considered reliable for data collection. Table 3.2 reveals that the coefficient of Cronbach alpha for numbers, algebra, geometry, and data was 0.817, 0.729, 0.782, and 0.791 respectively. This means that the tests were suitable for data collection from the respondents as the Coefficient of Cronbach alpha falls in the acceptable range.

3.6.5 Triangulation of the research instruments

The process of triangulations helps in refining and facilitates in validating the findings of the study. It helps in identifying the consistency of findings obtained from different types of research instruments to confirm and explain the phenomenon (Denzin, & Norman K., 1973). Since this study was mixed method in nature, the instrument triangulation was done in which initially semi-structured interview and the observational checklist were developed and accordingly observations were made in the classroom. Tests were developed from the textbook to check and assess the learning skills of students. So, the tests have the reflection of the observation made in the observational checklist and hence triangulate the research instruments.

This triangulation helps in validating the responses collected from the teachers during the observational checklist and the test scores of students after teaching the topics at hand. In the results and discussion chapter, results triangulation was also done to draw a stout conclusion based on qualitative and quantitative results.

3.7 Data Collection

After administering the research instruments in their particular settings, the data collection was initiated by the respondents. The data collection process started on 15 November 2021 to 10 February 2022 (almost 03 months). The data collection phases of all three research instruments were mentioned as given below:

3.7.1 Semi-structured interview

The researcher actively conducted semi-structured interviews with 7th-grade math teachers. A total of six interviews were conducted. The basic theme of the interviews was "Professional Competencies of Math Teachers". A total of eight questions were asked regarding the teacher's academic qualification, personal qualification, experience, subject specialization, awareness about the national curriculum of Pakistan 2006, following the curriculum, a subject breakdown according to the curriculum, and using a teacher guide. And their responses were noted down by the scholar on paper. These interviews were conducted before classroom observation.

3.7.2 Observational Checklist

The researcher actively completed the observational checklists through classroom observations of selected 7^{th}-grade math grades. Five observational checklists regarding each standard (Numbers and Operations, Algebra, Measurements and geometry, Data handling) were completed from each of six selected 7^{th}-grade math grades of six selected schools, so a total of 30 observations were made. Those observational checklists were based on the theme "Classroom Observation Compact". Under the theme of "Classroom observation compact" focus was on eight basic points that were lesson plan, the objective of the lesson, delivery of topic, audio/visual aids, teaching methodology, teaching strategy, teaching technique, and time management. The classroom observation was made during the teaching-learning process.

3.7.3 Tests

After validation and checking the reliability of the tests, these tests (each on number and operations, algebra, measurements and geometry, and data) were distributed among the students in the specified time frame. The students were engaged in the test at the end of the session when the teacher had completed the topic with the students. These tests were

administered to the students during grade time and the environment was comfortable, so, to avoid any sort of anxiety among the students. These tests were marked by the researcher (as the researcher has studied mathematics subject in the bachelor level) after completion of the test and preparing an award list of the tests. These marked tests were entered in the SPSS for analysis.

3.8 Data Analysis

Preceding data collection from the respondents (semi-structured interviews from teachers, observation checklist by grade observation, and tests from students), the qualitative and quantitative data were prepared for analysis. Detail process regarding data mining was as follows:

3.8.1 Qualitative Data Analysis (Semi-structured Interview & Observational Checklist)

The data collected through semi-structured interviews and observational checklists were analyzed through thematic analysis. The respondents were assigned pseudo-names (i.e., R1, R2, R3 and so on) to maintain the anonymity of respondents and observe ethics in research. The basic theme of the semi-structured interview was "Professional Competencies of 7^{th}-grade math teacher "and the basic theme of the observational checklist was "Classroom Observation Compact" while the trailing items in each theme were analyzed after recording. At the end of each segment, a conclusion was also drawn to highlight the prevailing issues and problems and also address the potential reasons which affect the academics of students.

3.8.2 Quantitative Data Analysis (Tests)

The administered tests on students regarding specified mathematics standards were collected and prepared for data analysis. These tests were marked by the researcher who prepared an award list of each test. These marks were fed into the SPSS and prepared for analyzing the data. The variable learning skills of mathematics at the elementary level were dependent, which were assessed by using tests.

The data was analyzed using One-Way-MANOVA. Prior to applying the test for analysis, assumptions of One Way MANOVA were checked and then run the test.

3.9 Research Ethics

Ethical consideration was given due consideration. The following points were given due consideration:

3.9.1 Informed Consent

Prior to data collection, formal informed consent from teachers and principals was taken by showing the Departmental permission letter. Those teachers who withdrew themselves from participation in the study were not included in the research process.

3.9.2 Observing Grade Timing and School Norms

During the data collection phase, the grade timing and schedule were recorded and visited at the time of the mathematics grade. The researcher completely obeyed the norms and culture of schools.

3.9.3 Ensuring the Anonymity of the respondents

The researcher ensures the anonymity of schools, teachers, and students. The schools were given abbreviations and pseudo-names for teachers and students. It was made sure that the data was only used for research purposes and not to be shared with any third party without their informed consent. In addition, the data collected from the teachers and students were kept confidential by all means.

3.9.4 Avoid Wastage of Students' and Teachers' Time

The researcher was mindful of time management and avoided the wastage of time of the students and the teachers. The momentum of teachers was not disturbed in teaching the subject.

CHAPTER 4

ANALYSIS AND INTERPRETATION OF DATA

This chapter provides complete detail about how the collected data was analyzed through standardized approaches. Since the research approach was a mixed method, two types of data were collected. The qualitative data was collected through the semi-structured interview with 7th-grade math teachers before the observation of their sessions in the classroom and the observation checklist was filled during the math period of the 7th grade after the session tests were taken from the students of the same grade to collect quantitative data. Qualitative data were analyzed using a thematic approach while the quantitative data collected through tests were analyzed through One-Way-MANOVA. The chapter is divided into three main sections as given below:

4.1 Qualitative Data Analysis

4.2 Quantitative Data Analysis

4.3 Triangulation of Results

Section I

4.1 Qualitative Data Analysis

The data was collected from the six mathematics teachers, who were teaching grade 7th at selected schools. With each teacher, a semi-structured interview was conducted and five observational checklists were completed during the 7th-grade maths classroom on four standards i.e., numbers and operations, algebra, measurements and geometry, and data handling during the teaching process. Hence, a total of six semi-structured interviews were conducted and 30 observational checklists were completed by the six teachers. These teachers were given pseudo-names (i.e., R1, R2 and R5 are public sector schools' teacher and R3, R4, and R6 are private sector schools) aiming to ensure the anonymity of respondents as per research ethics. The theme of the Semi-structured interview was the Professional Competencies of Math Teachers. The observational checklist was designed on the themed classroom Observation Compact In each theme, the observations recorded were transcribed and mentioning all the essential information addressed by each respondent.

4.1.1 Professional Competencies of Math teacher

Objective 1: Find the similarities of teachers' teaching mathematical skills in different schools in the public and private sectors at the elementary level.

Objective 2: Find the differences in teachers' teaching mathematical skills at different schools in the public and private sectors at the elementary level.

Research Questions:

1. Does the teacher's academic and professional qualifications according to Federal Board?
2. Does teachers have math teaching experience?
3. Is the teacher's subject specialized?
4. Does the teacher have complete awareness regarding Pakistan National Curriculum 2006 of Math for grade 7th?
5. Does the teacher follow the Pakistan National Curriculum 2006 of Math for grade 7th?
6. Is the teacher provided with a Teacher Guide?

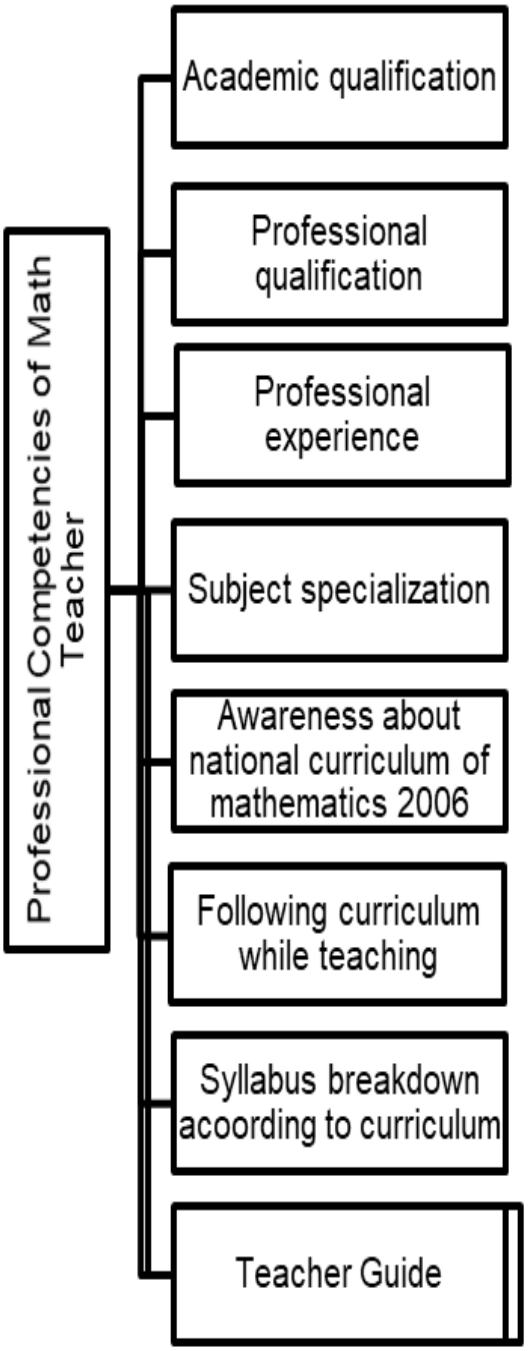


Figure 4.1

Professional Competencies of Math Teacher(sholoman,1987)

Table 4.1

Professional Competencies of Math Teacher

Themes	Codes	Responses
Professional Competencies of Math Teacher	Academic qualification	R1was M.Sc. Economics.R2, bachelor in social sciences. R3 was bachelor in sciences,R4 was F.SC R5 and R6 were M.sc in Mathematics
	Professional qualification	R1. R5 and R6 was M.ed,R2 and R3 were B.Ed,
	Professional Experience	R2, R3, R4, and R5 had 1 year of experience. R6 had 2-year experience
	Subject Specialization	R5, and R6 were subject specialized
	Awareness about the National Curriculum of Mathematics 2006 of Pakistan	R1, R2, R3, R5, and R6 had awareness about the National Curriculum of mathematics 2006 of Pakistan.
	Observing Curriculum in teaching	R 5, and R6 were observing the curriculum while teaching.
	Syllabus breakdown according to curriculum	R 5, and R 6 had syllabus break down according to the curriculum.
	Teacher Guide	Not available with anyone

Note. R represents the respondent teacher.

In a semi-structured interview, information regarding the teacher's academic and professional qualification, experience, subject specialization, awareness about the National Curriculum of Mathematics for 7th grade 2006 of Pakistan, following the National Curriculum of Mathematics for 7th grade 2006 of Pakistan while teaching, syllabus breakdown according to the National Curriculum of Mathematics for 7th grade 2006 of Pakistan, and use of teacher guide were collected.

4.1.1.1 Academic qualification

The teachers who were engaged in semi-structured interviews have obtained the qualification i.e., R1 had a Master's in Economics, R2 had a bachelor's in social sciences, R3 had a bachelor's in science, R4 had F.Sc, while R5 and R6 had a master in Mathematics.

4.1.1.2 Professional qualification

Besides the core discipline, the R1, R5, and R6 had obtained professional qualifications i.e., M.Ed. while the R2 and R3 had B.Ed.

4.1.1.3 Professional Experience

The teaching experience varies among the respondents as R2, R3, R4, and R5 had 1-year experience while R6 had 2 years of teaching experience.

4.1.1.4 Subject Specialization

R5 and R6 are subject-specialized. While the rest of the teachers were not subject-specialized.

4.1.1.5 Awareness of National Curriculum of Mathematics of 7th Grade 2006 of Pakistan

Respondents R1, R2, R3, R5, and R6 had an idea about the National Curriculum for Mathematics 2006 of 7th grade only, while R4 was unaware of it.

4.1.1.6 Teaching as per National Curriculum Framework

Since the R1, R2, R3, R5, and R6 had an idea about the National Curriculum for Mathematics of 7th grade while the teaching was not made as per the given framework. However, the R5 and R6 were following the curriculum framework during their teaching.

4.1.1.7 Syllabus breakdown according to curriculum

R5 and R6 had an understanding of the subject breakdown according to the National Curriculum while R1, R2, R3, and R4 were not acquainted with the syllabus by the school principal but merely engaged in teaching the mathematics subject from the given textbooks.

4.1.1.8 Teacher Guide

Teaching module for each subject is mandatory. As per all the respondents, the teacher guide was not provided for teaching.

During the completion of semi structured interviews, it had been observed that R1 was M.Sc. Economics with M.Ed. and R2 had obtained Bachelor in Social Sciences with B.Ed. while without having mathematics as an optional subject in their terminal degrees and having no experience in teaching the subject of mathematics. The qualification of R3 was a Bachelor of Science with a specialization in statistics with a B.Ed. and one-year teaching experience in the subject of mathematics. The R4 has obtained only qualification of Higher Secondary School Certificates (HSSC) with one year of teaching experience in the subject of mathematics. The R5 and R6 did their Master of Science in mathematics.6 had 2 years of teaching experience and had sound knowledge about teaching their specialized subject.

In nutshell, R5 and R6 were doing their job according to the instruction of the National Curriculum of Mathematics.

4.1.2 Classroom Observation Compact

Objective 2: Evaluate the teaching-learning environment of math classrooms at the elementary level in public and private sector schools.

Research Questions:

7. Does the teacher have a prepared lesson plan?
8. Does the teacher deliver the clear objectives of the lesson in the classroom?
9. Does the teacher have started the topic actively?
10. Are audio/visual aids present in the grade according to the topic?
11. Does the teacher select the proper method of teaching related to the topic?
12. Does the teacher adopt a clear teaching strategy in the classroom environment?
13. Does the teacher use suitable teaching techniques?

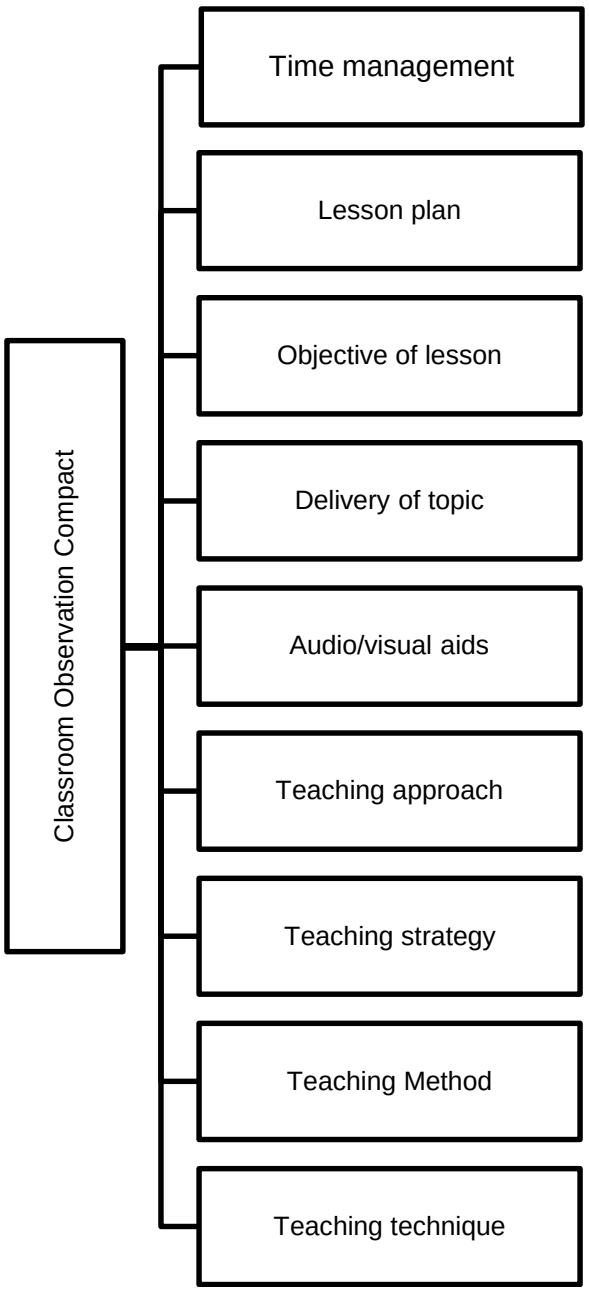


Figure 4.2

Classroom Observation Compact(shuloman,1987)

Table 4.2

Classroom Observation Compact Codes

Theme	Code	Responses
Classroom Observation Compact	Time Management	R5 and R6 did time management
	Lesson plan	R5 and R6 planned lesson
	Objective of lesson	R5 and R6 mentioned the objective of the lesson
	Delivery of topic	R5 and R6 actively delivered the topic
	Audio/ visual aids	Only board
	Teaching methodology	The lecture method was used by all Respondents
	Teaching strategy	Reciprocal teaching
	Teaching technique	Nil

The second theme of the study was the Classroom observation compact. The codes duly focused were time management, lesson plan, objectives of the lesson, Delivery of topic, audio/visual aids, teaching methodology, teaching strategies, and teaching techniques.

4.1.2.1 Time management

R5 and R6 had designed their proper lesson plan with proper time management for each segment from communicating the objectives to the assessment.

4. 1.2.2 Lesson plan

During the classroom observation, it was observed that R1, R2, R3, and R4 did not develop their lesson plan. Due to this the teacher's classroom management approach was not effective. The grade was comparatively disorganized and there was difficulty to follow sequential steps. Whereas R5 and R6 had designed their proper lesson plan with proper time management for each segment from communicating the objectives to the assessment.

4.1.2.3 Objective of the lesson

The objectives for topics (i.e., numbers and operations, algebra, measurements and geometry, and data handling) to be taught were not properly communicated to students, although the objectives for each topic were given in the textbook so, both students and teacher were unaware of the topic's importance for learning.

4.1.2.4 Delivery of topic

R5 and R6 had a proper lesson plan. They actively and effectively delivered the topic.

4.1.2.5 Audio/visual aids

Necessary Audio Visual aids unavailability made the concept difficult to understand and the learner felt anxiety. . For each topic (numbers and operations, algebra, measurements and geometry, and data handling), R5 and R6 used Audio-Visual aids properly and effectively in a timely fashion, which enhance the motivation level of students in the class.

4.1.2.6 Teaching methodology

The teaching methodologies used by teachers in all the classes were chalk and talk method with no defined teaching strategies, and techniques. Since due to the least effective teaching methodologies, the holistic academic development of students was not possible, and they were engaged merely in copying the subject matter from the whiteboard with the least understanding and lacked the skills to integrate theory into practice. It was also observed during the teaching-learning process that the teachers were just engaged in completing the important subject matters despite checking the understanding level of students. Some topics (e.g., word problems, logical reasoning) were skipped with the notion that the topic was of the least importance while these topics were of great importance in a real-life situation.

The dimension of teaching, alignment of goal and content, teaching arrangement and preparation, effective learner-teacher interaction with a clear set of outcomes, and mentorship, were the standards of effective teaching. In the classroom observation, R1, R2, R3, and R4 were at entry level while for R5 and R6, the dimensions of effective teaching were at basic to professional level.

4.1.2.7. Teaching strategy

In the grade observations of R1, R2, R3, and R4, it was observed that the teachers sometimes invite the learner to the whiteboard and subject a question for a solution. Since this method is often called reciprocal teaching and is effective for learning when there might be a sense of competition among the students. However, during observation, when one learner was engaged in reciprocal teaching, the rest of the students were not concentrating on the topic or subject matter and engaged in playing.

In the topic of numbers, students were facing difficulty in recalling the meaning of symbols (∪, ∩) and solving the numerical step-wise. Similarly, the performance of learners in algebra was also not satisfactory. In the topic of geometry, the teacher only focuses on memorizing the preparation of diagrams but fails to conceptualize the essence of geometry. The conceptual understanding of students in geometry was weak and they couldn't implement the learned concepts in their daily life situations. As per the National Curriculum of Mathematics 2006 of Pakistan, the highest weightage is given to numbers (45%), then geometry (35%), then algebra (15%), and lastly to data handling (5%). Contrary to that, the teachers focus on data handling while skipping all the remaining essential topics.

While in the case of R5 and R6 the performance of students was enhanced due to actively engaging in the learning process. The students were feeling comfortable and the environment for learning was conducive. The attitude of students was inquisitive in grade and asked critical questions which showed their degree of interest in the subject of mathematics. R5 and R6 integrated the theory into practice as most of the topics were linked with the mathematical concepts used in daily life, which enhance the capacity of students in learning mathematics. It was observed that students were learning new concepts enthusiastically. After the completion of the session, a group task was given to students to assess their learning in a friendly environment.

Section II

4.2 Quantitative Data Analysis

Objective No. 4: Compare teachers' teaching mathematics skills regarding their effect on students' learning mathematics skills at the elementary level in public and private sector schools.

Null Hypotheses

Ho1 There was no significant difference in learning the concept of numbers in public and private sector schools.

Ho2 There was no significant difference in learning the concept of algebra in public and private sector schools.

Ho3 There was no significant difference in learning the concept of geometry in public and private sector schools.

Ho4 There was no significant difference in learning the concept of data in the public and private sectors in schools.

The quantitative data analysis was analyzed using One-Way MANOVA. Prior to applying the One-Way MANOVA test, the assumptions were checked. The reason for using One-Way MANOVA was that there was one independent variable with three dependent variables. The assumptions of One Way MANOVA were given as under:

a) Regression

b) Checking Outliers in the data

c) Absence of Multicollinearity

d) Normality of data, and

e) Homogeneity of data

For checking each assumption, certain statistical tests were used, which are as under:

a) Linear Regression

The One-Way MANOVA test is sensitive to outliers and their results are often affected by the presence of outliers in the data. For checking the outliners in the data, it is imperative to apply linear regression analysis to the data. The model summary from table 4.3 can be revealed as under:

Table 4.3

Linear Regression

Model	R	R Square	Adjusted R Square	SE
1	.938	.880	.879	.59554

Table 4.3 reveals that the value of r is 0.938 indicating strong relation thus fulfilling the assumption of One Way MANOVA. This indicates that a high degree of correlation exists, hence the assumption for One Way MANOVA was fulfilled.

b) Checking the Outliers in the data

Further to check for outliers in the data, Mahalanobis distance was assessed form table 4.4 of residual statistics.

Table 4.4

Residual Statistics

	Minimum	**Maximum**	**Mean**	**SD**	**N**
Predicted Value	.7273	6.1705	3.5000	1.60515	300
Std. Predicted Value	-1.727	1.664	.000	1.000	300
Standard Error of Predicted Value	.039	.154	.074	.019	300
Adjusted Predicted Value	.7129	6.1911	3.5003	1.60534	300
Residual	-2.09588	1.45538	.00000	.59154	300
Std. Residual	-3.519	2.444	.000	.993	300
Stud. Residual	-3.531	2.462	.000	1.001	300
Deleted Residual	-2.10962	1.47669	-.00026	.60071	300
Stud. Deleted Residual	-3.602	2.483	.000	1.005	300
Mahal. Distance	.269	18.941	3.987	2.833	300
Cook's Distance	.000	.031	.003	.005	300
Cantered Leverage Value	.001	.063	.013	.009	300

The Mahalanobis distance was measured in the unit of Standard Deviation. For calculating the outliers in the data, first, the p-value was calculated from the results of Mahalanobis distance using computing variable command in SPSS. The p-values were obtained for each data and checked that if the p-value was less than 0.001 then that was the outlier of the data. For verification of outliers, the data was checked for any wrong entries. Those p-values, which were less the 0.001, and the entry of data was correct, it was removed from the analysis part to not affect the overall results of One Way MANOVA.

c) Absence of Multicollinearity

The Multicollinearity in data was checked through the Pearson test mentioned in table 4.5

Table 4.5

Pearson Correlation

		Number	Algebra	Geometry	Data
Numbers	Pearson Correlation	1	.859	.662	.380
	Sig. (2-tailed)		.000	.000	.000
	N	300	300	300	300
Algebra	Pearson Correlation	.859	1	.685	.424
	Sig. (2-tailed)	.000		.000	.000
	N	300	300	300	300
Geometry	Pearson Correlation	.662	.685	1	.411
	Sig. (2-tailed)	.000	.000		.000
	N	300	300	300	300
Data	Pearson Correlation	.380	.424	.411	1
	Sig. (2-tailed)	.000	.000	.000	
	N	300	300	300	300

Table 4.5 reveals a strong Pearson correlation of numbers to algebra and geometry was 0.859 and 0.662 respectively, while for data, the value was 0.380 showing a weak correlation with the number. Similarly, a strong correlation between algebra to number (0.859) and geometry (0.685) was found while a medium correlation for data (0.424) was found. In addition, a strong correlation of geometry to number (0.662), algebra (0.685), and medium correlation for data (0.411) was found. On the other hand, the correlation of data to numbers (0.380) was weak while for algebra (0.424) and geometry (0.411) was found medium.

The important component in table 4.5 was the checking of the Multicollinearity of the data. It was observed from table 4.5 that the Pearson Correlation value in almost all the cases was less than 0.9 while higher than 0.2, thus fulfilling the assumptions of One Way MANOVA.

d) Normality of Data

The normality of data was checked using the Shapiro-Wilk test.

Table 4.6

Test of Normality

	Kolmogorov-Smirnov			**Shapiro-Wilk**		
	Statistics	**Df**	**Sig.**	**Statistics**	**Df**	**Sig**
Number	.116	300	.000	.921	300	.000
Algebra	.166	300	.000	.875	300	.000
Geometry	.184	300	.000	.875	300	.000
Data	.325	300	.000	.753	300	.000

Table 4.6 reveals that the Shapiro-Wilk test was significant, and the p-values were less than 0.05, which shows that the data was not normal. However, the Komogorove-Smirnov and Shapiro-Wilk tests are sensitive to the large sample size (as the sample size of the research was 300), for which the results of Shapiro-Wilk were not reliable.

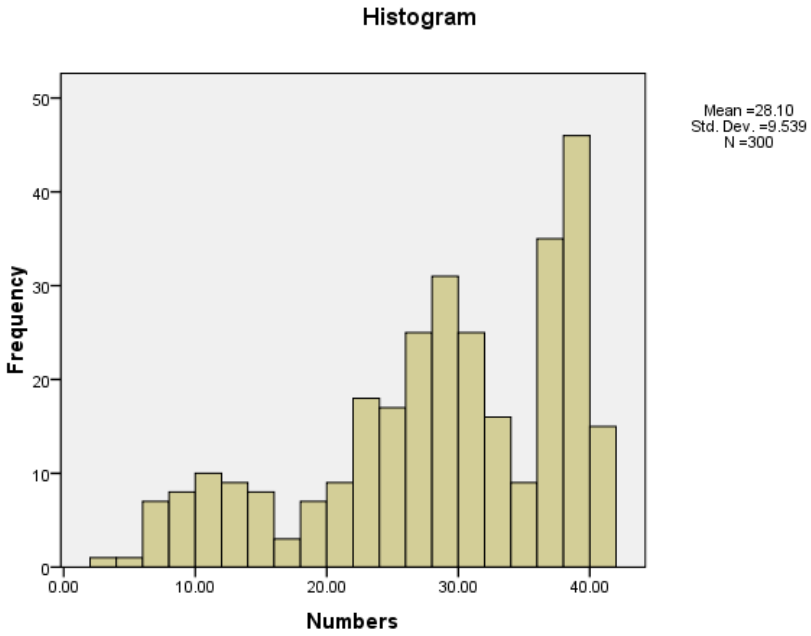
Table 4.7

Descriptive Statistics

	Mean	Median	Variance	SD	Range	IR	Skewness	Kurtosis
Number	28.103	29	30.996	9.539	38	14	-0.720	-0.368
			Std. Error				0.141	0.281
Algebra	8.093	9	28.152	5.305	15	11	-0.334	-1.422
			Std. Error				.141	.281
Geometry	17.290	13.50	89.906	9.481	32	19	0.387	.141
			Std. Error				-1.361	.281
Data	4.296	5	.818	.904	4	1	-1.167	.824
			Std. Error				.141	.281

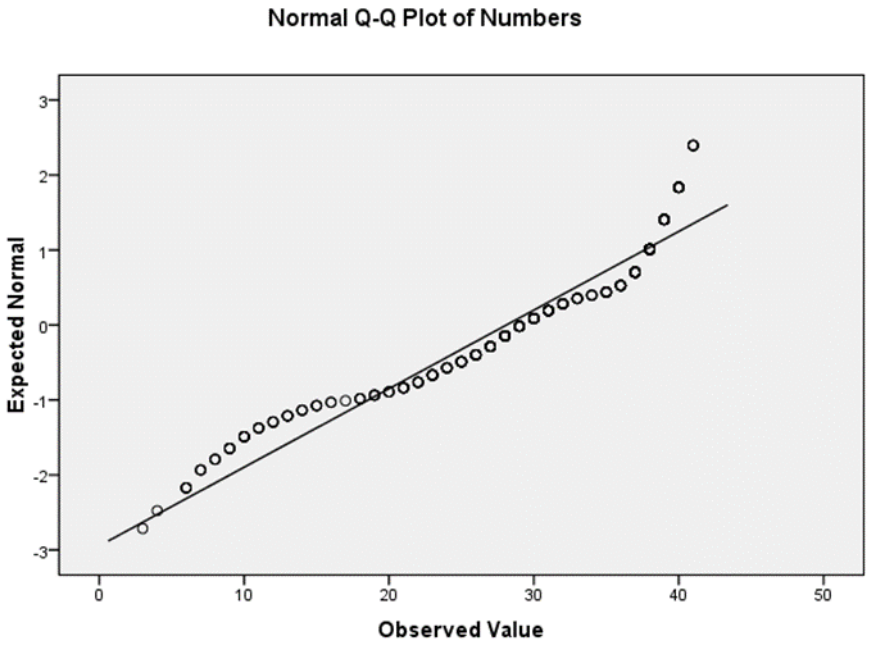
Another way of checking the normality of data was the Skewness and Kurtosis of the data. Since the sample size was 300 for which the Shapiro Wilk was not reliable, so Skewness and Kurtosis were used to check the normality of the data. For a large sample size (300 or more), the absolute Skewness values exist between -2 and +2 and the absolute Kurtosis values exist between -7 and +7. So, the Skewness and Kurtosis of each variable were divided by their respective standard errors. For the number variable, the Skewness (-5.10 showing slight deviation from the absolute Skewness value range) and Kurtosis (-1.31 fall in the absolute Kurtosis value range), while for the algebra variable, the Skewness (-2.368 falls in the absolute Skewness value range) and Kurtosis (-5.06 fall in the absolute Kurtosis value range). In addition, for the geometry variables, the Skewness (-.28 falls in the absolute Skewness value range) and Kurtosis (-1.31 fall in the absolute Kurtosis value range), and for the data variable, the Skewness (-8.27 showing slight deviation from the absolute Skewness value range) and Kurtosis (2.93 falls in the absolute Kurtosis value range). So, from the Skewness and Kurtosis, the normality of data was checked, and concluded that the Skewness for number and data variables do not fall in the normal range while the Skewness for algebra, geometry and Kurtosis for numbers, algebra, geometry, and data falls in the acceptable range of normality.

Graph 4.1. ***Histogram for Number***



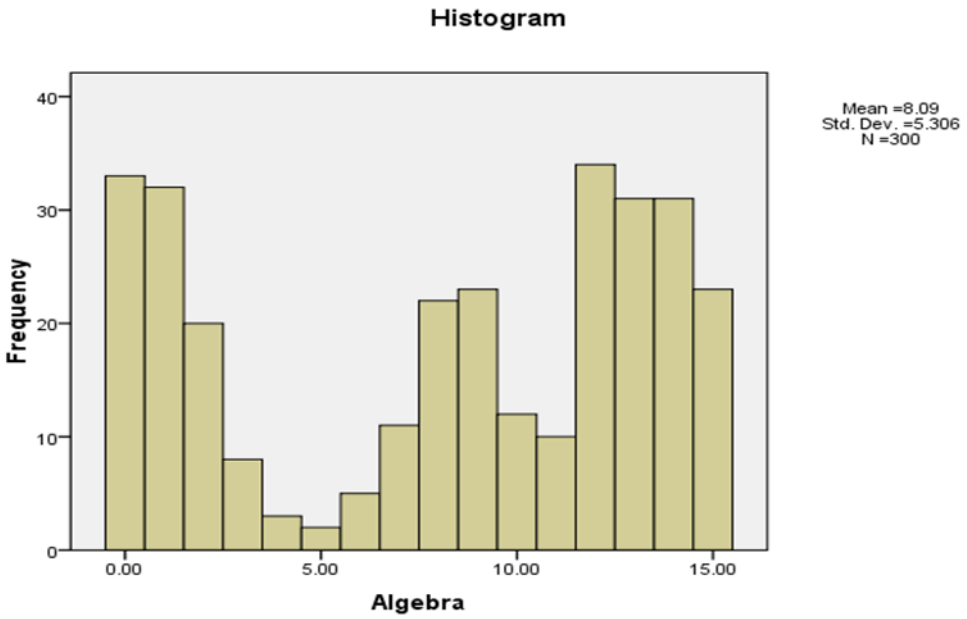
Graph 4.1 shows negative Skewness with dominancy of leptokurtic. In other words, it can be stated that the data (for numbers) in the graph4.1 is not normally distributed as revealed in the table 4.7.

Graph 4.2. ***Normal Q-Q Plot of Numbers***



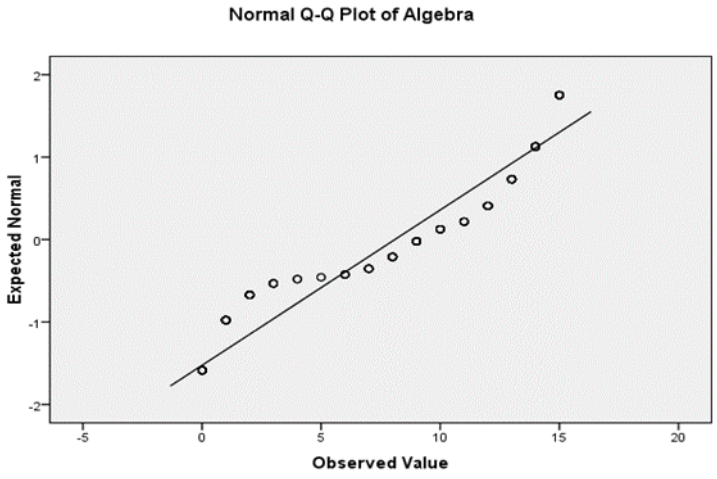
Graph 4.2 of Normal Q-Q plots shows most of the data lies on normal distribution while slight deviations were recorded on the extreme anterior and posterior segments. . The decision of the normality of data (in case of plot) are made on the basis of showing no deviation of data from the 45-degree reference line and vice versa.

Graph 4.3. ***Histogram for Algebra***



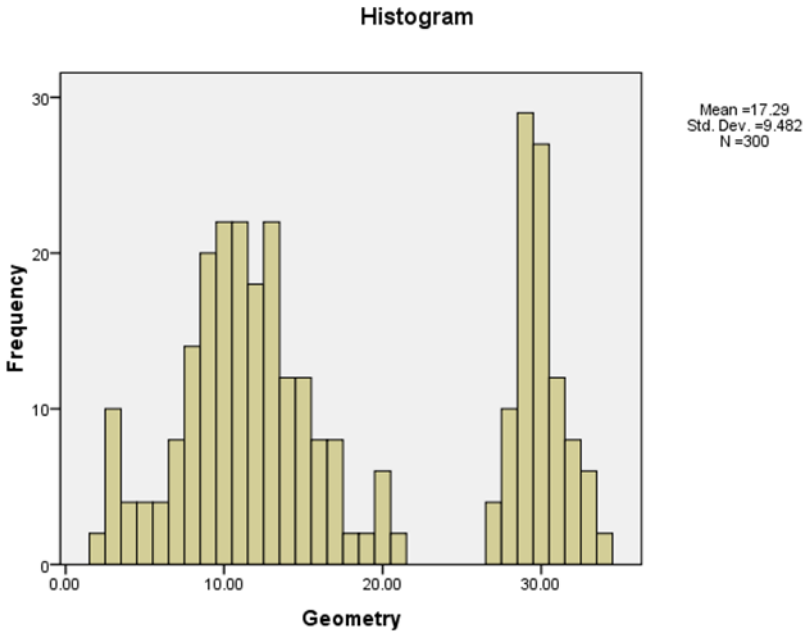
Graph 4.3 shows leptokurtic with negative Skewness for the algebra variable. In other words, it can be state that the data (for algebra) in the graph is not normally distributed as revealed in the table 4.7.

Graph 4.4 ***Normal Q-Q Plot of Algebra***



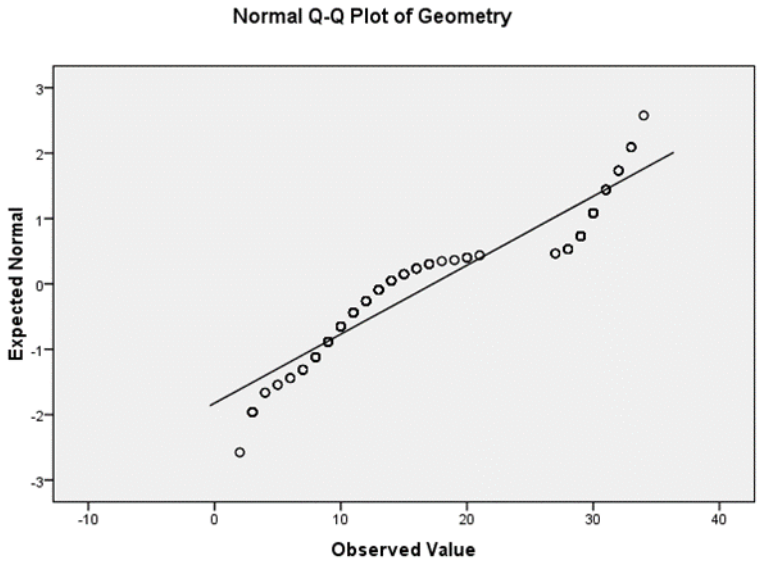
Graph 4.4 shows the distribution of data approximately near to normal distribution, while the alight deviation of data was recorded from the normal distribution at the anterior and posterior segments. The decision of the normality of data (in case of plot) are made on the basis of showing no deviation of data from the 45-degree reference line and vice versa.

Graph 4.5. ***Histogram for Geometry***



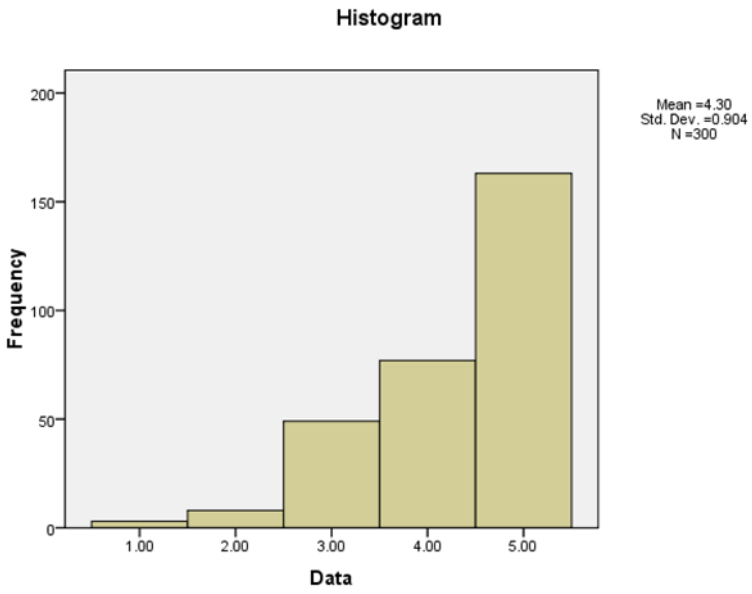
Graph 4.5 shows segmentation and interruption in data while revealing mesokurtic with normal destruction of data for the geometry variable. . In other words, it can be state that the data (for geometry) in the graph is normally distributed as also revealed in the table 4.7.

Graph 4.6 ***Normal Q-Q Plot of Geometry***



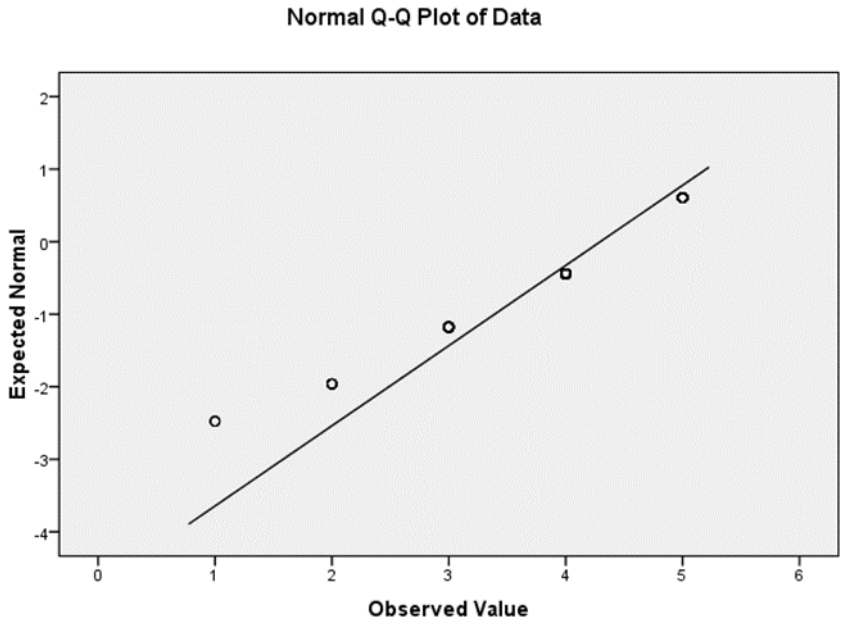
Graph 4.6 shows the distribution of data on a normal distribution with a slight deviation in the middle and posterior segments. The decision of the normality of data (in case of plot) are made on the basis of showing no deviation of data from the 45-degree reference line and vice versa.

Graph 4.7. ***Histogram for Data***



Graph 4.7 reveals that the distribution of data leptokurtic with negatively skewed. In other words, it can be state that the data (for data) in the graph is not normally distributed as also revealed in the table 4.7

Graph 4.8 ***Normal Q-Q Plot of Data***



Graph 4.8 reveals the normal distribution of data while slight deviation at the anterior segment. The decision of the normality of data (in case of plot) are made on the basis of showing no deviation of data from the 45-degree reference line and vice versa.

e) Homogeneity of data

The homogeneity of data was checked through Levene's test of equality of variance. Table 4.8 reveals that the $p<0.00001$ for numbers, algebra, geometry, and data was less than $p<0.05$, thus violating the assumption of One Way MANOVA.

Table 4.8

Levene's test of Equality of Variance

	F	df1	df2	Sig.
Numbers	9.320	5	294	.000
Algebra	10.109	5	294	.000
Geometry	9.741	5	294	.000
Data	18.371	5	294	.000

a) Box's M test

In addition, the Box's M test was done, which aims to compare the variation in multivariate samples. Table 4.9 shows that the p-value for Box's M Test was less than 0.005 i.e., $p=0.000$, thus violating the assumption of One Way MANOVA.

Table 4.9

Box's Test of Equality of Covariance Matrices

Box's M	295.159
F	5.700
df1	50
df2	158641.70
Sig.	0.000

1. One Way MANOVA

After checking the assumption for One Way MANOVA, the test was applied to the data and mentioned in table 4.10 Since, the assumptions i.e., homogeneity of data, and Box M Test for One Way MANOVA were not fulfilled, then the Pillai's Trace was opted instead of Wilks' Lambda. For determining the statistical significance of OneWay MANOVA, the sig. value was p = 0.0001, which means that the p-value is less than 0.05. Therefore, it can be concluded that the performance of students in terms of learning mathematical skills (conceptual understanding, procedural knowledge, and problem-solving) were checked through the topics (numbers, algebra, geometry, and data-handling) and hence, found significantly dependent on the schools.

Table 4.10

Multivariate Test

	Effect	Value	F	Hypothesis df	Error df	Sig.	Partial Eta Squared
Intercept	Pillai's Trace	.992	9257.385[a]	4.000	291.000	.000	.992
	Wilks' Lambda	.008	9257.385[a]	4.000	291.000	.000	.992
	Hotelling's Trace	127.249	9257.385[a]	4.000	291.000	.000	.992
	Roy's Largest Root	127.249	9257.385[a]	4.000	291.000	.000	.992
SSchool	Pillai's Trace	2.178	70.272	20.000	1176.000	.000	.544
	Wilks' Lambda	.007	165.997	20.000	966.088	.000	.709
	Hotelling's Trace	22.146	320.565	20.000	1158.000	.000	.847
	Roy's Largest Root	18.642	1096.158[c]	5.000	294.000	.000	.949

There was a statistically significant difference in the learning skills of mathematics (numbers, algebra, geometry, and data) in schools, F (20, 1176) = 70.272, p<0.0005; Pillai's Trace = 2.178, partial η^2 = 0.544.

1. Univariate ANOVA

For determining the degree of variance or difference of Independent variables from the dependent variables, the tests of between-subjects effects were required.

Table 4.11

Univariate ANOVA

Source	Dependent Variable	Type III Sum of Squares	Df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	Numbers	24122.617[a]	5	4824.523	459.749	.000	.887
	Algebra	7801.187[c]	5	1560.237	744.417	.000	.927
	Geometry	23377.390[d]	5	4675.478	392.249	.000	.870
	Data	54.577[e]	5	10.915	16.888	.000	.223
Intercept	Numbers	236939.203	1	236939.203	22578.950	.000	.987
	Algebra	19650.613	1	19650.613	9375.658	.000	.970
	Geometry	89683.230	1	89683.230	7523.976	.000	.962
	Data	5538.403	1	5538.403	8569.048	.000	.967
School	Numbers	24122.617	5	4824.523	459.749	.000	.887
	Algebra	7801.187	5	1560.237	744.417	.000	.927
	Geometry	23377.390	5	4675.478	392.249	.000	.870
	Data	54.577	5	10.915	16.888	.000	.223
Error	Numbers	3085.180	294	10.494			
	Algebra	616.200	294	2.096			
	Geometry	3504.380	294	11.920			
	Data	190.020	294	.646			
Total	Numbers	264147.000	300				
	Algebra	28068.000	300				
	Geometry	116565.000	300				
	Data	5783.000	300				
Corrected Total	Numbers	27207.797	299				
	Algebra	8417.387	299				
	Geometry	26881.770	299				
	Data	244.597	299				

In table 4.11, the results reveal that schools have a statistically significant effect on the learning of students (regarding conceptual understanding, procedural knowledge, and problem-solving approach) in mathematics i.e., Number, (F (5, 294) = 459.749; p<.05; partial η^2 = 0.887); Algebra, (F (5, 294) = 744.417; p<.05; partial η^2 = 0.927); Geometry (F (5, 294) = 392.249; p<.05; partial η^2 = 0.870); and Data (F (5, 294) = 16.888; p<.05; partial η^2 = 0.223). Hence, it is evident from the results that there was statistical significance at p<.05.

Section III

4.3 Triangulation of Results

There are four types of triangulation i.e., instrument, data, methodology, and results triangulation, which often validates the research in different ways. In the current study, the triangulation of results was employed to draw relevant findings.

Table 4.12

Similarities in public and private sector schools' teachers (Qualitative Analysis)

Codes	Similarities in Public and Private Sector Schools' Teachers
Academic qualification	Both in public and private schools' teachers had a different qualifications.
Observing Curriculum in Teaching	One teacher each from the public and private sector with M.Ed. were observing the curriculum
Syllabus breakdown according to curriculum	One teacher each from the public and private sector with M.Ed. were following the breakdown of the subject according to the curriculum
Teacher Guide	Both public and private sector schools' teachers did not have teacher guides.
Audio/visual Aids	Black board
Teaching Methodology	Lecture method
Teaching Strategy	Reciprocal Technique
Teaching Technique	Nil

Table 4.13

Differences in Public and Private Sector Schools' Teachers (Qualitative analysis)

Code	Differences in Public and Private Sector Schools' Teachers
Professional qualification	All the teachers of public sector schools were professionally qualified.
Professional Experience	Only one private-sector teacher did not have professional experience.
Subject Specialization	One teacher each from the public and private sector was subject specialized
Awareness about the National Curriculum of Mathematics 2006 of Pakistan	Only one teacher from the private sector schools was unaware of the national curriculum of Mathematics 2006 of Pakistan
Time management	One teacher each from the public and private sector did time management
Lesson Plan	One teacher each from the public and private sector had prepared lesson plan
Objective of Lesson	One teacher each from the public and private sector presented objectives of the lesson
Delivery of Topic	One teacher each from the public and private sector delivered lesson actively.

Table 4.14

Quantitative Data Analysis

Schools	Respondent	Mean	Std. Deviation
School GA	R1	9.97	2.43
School GB	R2	13.00	2.60
School PA	R3	13.16	2.61
School PB	R4	13.59	2.54
School GC	R5	21.46	1.21
School PC	R6	21.48	1.40

The result of qualitative research reveals that the qualification, experience, understaffing of subject matter, and pedagogical skills of teachers were not suitable and appropriate for teaching the subject of mathematics, which affect the performance of students in their academics. Since the teaching skills of most teachers were not effective and standardized for teaching the subject of mathematics, eventually the students' academics were compromised. In connection with these factors, the pedagogical skills of teachers were recorded through the observational checklist. Since the results of qualitative research concluded that most teacher does not have prerequisite qualification for teaching the subject of mathematics and have no extensive teaching experience, which results in providing low performance for students. Furthermore, being inexperienced teachers, the teaching methodologies, strategies, and techniques used in teaching the subject of mathematics were not standardized, which affect the learning skills of students in the subject.

Based on these results, the tests developed for checking the learning skills of students were developed each from simple to complex. There were five standards of learning as per the National Curriculum of Mathematics, which includes numbers, algebra, geometry, data handling, and logical reasoning. The logical reasoning standard was primarily involved in all the tests, so only four tests were developed to check the learning skills of students. The result of quantitative research reveals that there was a significant difference in learning skills (number, algebra, geometry, and data) of mathematics. So, from the triangulation of results, it was evident that the developing learning skills of students in the subject of mathematics are directly related to the commitment, dedication, and command of the subject matter, suitable qualifications, and rich teaching experience of teachers.

CHAPTER 5

SUMMARY, FINDINGS, DISCUSSION, CONCLUSION AND RECOMMENDATIONS

In light of quantitative and qualitative results obtained after analysis, the findings, conclusion, and recommendation were formulated. The findings of the study were followed by the conclusion of the research and assessed the degree of achievement of objectives. In light of the conclusion, tangible recommendations were drawn.

5.1 Summary

Teachers play a significant role in the building of nations on defined parameters. Teachers often extend their role in an exponential way to get a chance of providing required services in the best way. The teachers refine the skills of their students to make them suitable for society. It is imperative to train students in the right direction in the current era. For the students of elementary level, it is important to equip them with modern skills in the subject of mathematics to define their career choices in future. Effective teaching skills by the teachers develop effective learning efficacy among the students and the students become motivated towards their academics. It has been observed that the students are often reluctant to learn the concept in the subject of mathematics because of its dryness, whilst, if the teachers taught the subject effectively and efficiently, the performance of students could be increased exponentially. Hence, it is evident that the effective teaching of teachers is important for the development of a strong conceptual understanding among students.

Teachers are considered the backbone and central component of any academic process. The fundamental role of teachers in any academic setting aims to provide mentoring and engage the students in the teaching and learning process. Since the students only rely on the concepts taught to them in the classroom and then apply their learned concepts in the practical field. This conceptual understanding of students primarily depends upon the effective teaching of teachers and techniques employed during the teaching-learning process. In the current study, the teaching strategies of teachers were explored to check the students' conceptual understanding of the subject of mathematics at the elementary school level. In this regard, a mixed method study was adopted concerning sequential design (where the qualitative research was conducted prior to quantitative research), which helps in the assessment of students learning and teachers' professional capabilities. In the qualitative study, six teachers from

different types of schools at the elementary level were selected and data were collected through observational checklists. This checklist was analyzed and in the light of qualitative results, tests (duly validated and checked for reliability) were developed for collecting data from the students of grade seven from three different types of schools. The results were triangulated, and conclusions were drawn reveling the degree of achievement of objectives.

5.2 Findings

As the current study was mixed method in nature, the findings drawn from qualitative and quantitative aspects were separately mentioned:

5.2.1 Findings of Qualitative Research

After analysing the semi-structured interview and observational checklists, the following findings were drawn:

5.2.1.1. Findings of a semi-structured interview, Table 4.1

1. The qualification of teachers was not standardized as some of the teachers were underqualified.
2. The teachers teaching the subject of mathematics do not have relevant teaching experience in the subject of mathematics.
3. The subject of mathematics was not allocated to specialized teachers.
4. The teachers of mathematics were not provided with the National Curriculum of Mathematics for teaching the subject to achieve the desired goals.
5. In the National Curriculum of Mathematics, each content was given a specific weightage, while the teachers do not follow the specified weightage during the teaching of mathematics subject.
6. The teacher's guide or teaching module for the subject of mathematics was not provided to the teachers which cause a zigzag teaching pattern, and the entire course content are not covered in time.

5.2.1.2 Findings of observation checklist, Table 4.2

1. The lesson plans for each topic were not properly developed to ensure an effective teaching-learning process.
2. The objectives of each topic were not clear to teachers as well as to the students, due to which the teachers and students were not aware of the gravity of the topics.
3. The content-specific Audio Visual Aids for teaching the subject of mathematics were

missing.

4. Classroom time management was not followed by the teacher to teach the specific content.
5. In the curriculum and textbooks, each topic was given a specific duration for completion of course content, which pattern was from easy to complex. The teachers in the classroom often did not follow that given duration to complete the course content. Due to this, the essence of the topic was skipped, and the learning skills of students remain undeveloped.
6. The teaching methodologies of teachers were not effective to enhance the learning of students in both public and private sector schools where competent teachers were not hired. Where the teacher was competent the effectiveness of teaching was recorded in both public and private sector schools.
7. Only competent teachers were able to integrate the theory into practice in both public and private sector schools.

5.2.2. Findings of Quantitative Research

The data collected through tests from students of grade 7th was analyzed using One-Way MANOVA. Table 4.11 results revealed that schools have a statically significant effect on the learning of students (with reference to conceptual understanding, procedural knowledge and problem-solving). Following Table 4.11 findings were drawn from the analysis:

1. There was a significant difference in learning the concept of numbers (0.0001) in schools.
2. There was a significant difference in learning the concept of algebra (0.0001) in schools.
3. There was a significant difference in learning the concept of geometry (0.0001) in schools.
4. There was a significant difference in learning the concept of data (0.0001) in schools.

5.3 Discussions

The current research aims to explore the comparison of teacher teaching skills on the conceptual understanding of the subject of mathematics among students. In this regard, the qualitative data were collected from the teachers through semi-structured interviews and observational checklists from public and private school teachers and quantitative data were collected through designed tests from students after the observed session. The purpose was to compare the teachers' professional competencies and teaching skills of mathematics in the classroom.to check the student achievements in public and private schools, tests were given to the students and then their results were compared. The study's first research question was focused on finding the similarities and differences in teachers' professional competencies (academic qualification, professional qualification, subject specialization, experience, and awareness about the National Curriculum of Mathematics for grade 7^{th}. use of teacher guide) in the public and private sector schools. The study findings indicated that the qualification and experience of teachers were not fulfilling the pre-requisite criteria. In addition, some of the teachers hold irrelevant qualifications i.e., Bachelor of Social Science and were engaged in teaching the subject of mathematics. Furthermore, the teacher may have awareness of the curriculum but they are not using it. These all factors badly effect the teacher's performance in the classroom, which results in the compromised achievement of students in mathematics (Kunter, M., Klusmann, U., Baumert, J., Richter, D., Voss, T., & Hachfeld, A. (2013)). The current research indicates that due to the non-effective teaching methodologies used by the teachers during the delivery of subject matter, the academic achievement of students is not good and often students try hard to learn the concepts to qualify for their exams. Underqualified teachers may never be able to teach the concept in a well-organized way but fail to provide conceptual understanding to the students. These findings are supported by the studies of Galadima & Yusha'u 2007; Mazana, Montero, and Casmir, 2019. Subject command builds confidence and the teacher never gets confused if some question is asked related to the topic while the under-qualified teacher either gets confused, provides a wrong understanding or may pretend to be in an aggressive mood.

Furthermore, effective teaching has certain attributes which include subject specialization, competencies of teachers, teaching the relevant subject rather than assigning the subject with no specialization, a sense of responsibility and commitment, and awareness about the National Curriculum. These attributes make the teaching effective and have the potential to achieve the desired outcomes. For instance, subject specialization is important in academics. The lack of command of teachers on the subject matter and weak teaching strategies affects the academics of students in terms of their test scores. Concerning the

teaching of mathematics subjects, the current study claims that the competencies of teachers could be better assessed when they are assigned the subject of their area of specialization. Effective learning of students can only be possible when the specialized teacher is assigned the appropriate subjects as they have an understanding of content-specific teaching methodologies. The teaching of mathematics subject can be made effective by using the Audio Visual Aids in the classroom but due to the non-availability of the relevant and required Audio Visual Aids, the students face problems in understanding the subject matter. Thus, such factors develop anxiety among students towards the subject of mathematics.

A curriculum is a set of instructions, which provides the foundation for all essential steps of delivering knowledge from objectives to assessment. Therefore, the curriculum could be considered a complete guideline for teachers. It not only provides guidance and understanding regarding the logical sequence of the subject matter but also allocates time for each topic. In addition, it provides the weightage for each topic according to the degree of its importance and the percentage of topics to be covered for assessing the performance of students in the examination. Having prime importance, the teachers are required to know the curriculum. But unfortunately, during the observation, it was found that the teacher only knew the name of the curriculum while having no knowledge about the curriculum and its implementation. Similarly, the textbooks are developed in the light of the National Curriculum, but it was found that the local private sector schools have a one to two-page subject breakdown, which was made by their teachers according to their understanding. This creates problems in the delivery of uniform education and comprises the national aim in this regard. The study thus concludes that the proper monitoring of the public and private schools needs to be done and also proceeded through an academic audit so that to seek harmony in the course content.

Besides these, a sense of responsibility was lacking among the teachers. For instance, if a teacher may have a specialized degree in the relevant subject, teaching experience, and complete information regarding curriculum but has no sense of responsibility, then he or she is unable to contribute effectively to the national cause of promoting quality education. It was observed that the teachers are merely engaged in completing the subject matter irrespective of developing conceptual understanding of students but rather maintain the notebooks of students. The conceptual understanding of students was explored by their performance in the tests prepared from the textbooks even at the end of the academic year. The students were not able to attempt the simple arithmetic items and the items developed to identify their reasoning and problem-solving skills were completely blank or not solved. Hence, an enthusiastic and committed teacher can stimulate students' energy and make them able to grasp knowledge. Standard private school students show comparatively better results.

In public and private sector schools, the teacher only prefers chalk and talk methods for teaching. If any learner has any problem regarding the understanding of a concept, the teacher repeats the solution of that varied concept, and this was the sole method of teaching adopted by the teacher. When the teachers were asked about any other strategy or methodology for teaching, it was observed that the only response from the teacher was the usability of the chalk-and-talk method for teaching the subject of mathematics.

On the other hand, the topic needs to be started with an attractive idea, that links the topic with daily life. The lesson was planned and adopted a suitable teaching strategy and technique to make the concept easy for the students. The teacher has adopted pair work, and group work, in geometry a simple project was given so the learner can understand the concept easily, but the learning skills of students were hard to develop.

Concerning the conceptual understanding of the students regarding the subject matter of mathematics, the results of students in the test taken indicated their scores and performance. If the students have a clear understanding regarding the subject matter, they will perform better and obtain high marks on the test, while the test was taken almost at end of the academic session. This means that the students do not have command of the subject matter of previous grades.

The knowledge of the practicability of learned concepts is procedural knowledge, which was the prime objective of the National Curriculum of Mathematics. It was observed that in schools either public or private sectors, the focus was only to solve the question of exercise given in the textbook. These lessons were not integrated with daily life problems. Even students are unable to follow the steps to solve a simple question in algebra. The topic in numbers standard is to build stepwise knowledge from simple to complex. But the way topics are selected and taught in schools without consulting the curriculum, makes the task more complex for the students. During observation, it was found that the whole learning was only to pass paper not to acquire knowledge. As a result, procedural knowledge during the session and after the session was eliminated. Students were unable to solve the questions, which they did not note down in their grades, which seems like a failure of implementing the curriculum and achieving the desired outcomes of mathematics.

5.4 Conclusions

The current research was designed to compare the teaching and learning mathematical skills of different school teachers and their students respectively at the elementary level. Through semi-structured interviews and observation checklist information was gathered regarding teacher professional competencies and methodologies and strategies used in the grade-room while teaching their students mathematics. To find the outcomes of their teaching, tests were designed for the students on relevant topics. Based on the research objectives and findings of the study, the following conclusions were drawn.

The difference between public and private school teachers is their professional credentials. In public sector schools, all teachers have B.Ed. or M.Ed. degrees whereas in private sector schools, some teachers were highly qualified and some were underqualified and did not have a professional degree in education. So, often face problems in teaching the content. Teachers with professional degrees may have knowledge about the curriculum up to some extent but the implementation of the curriculum is not even on the basic level. Only qualified and experienced teachers provided good results in terms of good students performance in their academics, irrespective of the school (public or private sector)

It was observed in some cases that school administration did not assign the subjects according to the qualification and expertise of teachers, which affected the performance of teachers as well as was a serious compromise on the academics of learners. . It is imperative to teach the learners through diversified teaching methodologies and strategies to enhance their academic performance. But it was found that the Lecture method was used only as a methodology. Since the teacher did not get any training regarding the teaching of the subject of mathematics, made the situation more vulnerable to the low academic performance of learners. To get the desired plethora of outcomes from learners, the proper training of teachers is required in the application of different teaching methodologies, strategies and techniques.

Mathematics is considered the mother of sciences. Cramming in mathematics means zero understanding of the concept. It is mentioned in the Pakistan National Curriculum of Mathematics for 7^{th} grade 2016 that while teaching mathematics focus must be on conceptual understanding, procedural knowledge, and problem-solving. It was observed during the classroom sessions that the teacher's focus was only on the completion of the topic. And students were busy only copying the answers from the whiteboard in their notebooks.

After the session when tests were taken, the results of tests were clear evidence that students were unable to grasp knowledge. Only those students were able to solve the problems that were taught by the subject specialist.

5.5 Recommendations

1n the light of results and conclusion, the following recommendations were drawn:

1. The subject of mathematics needs to be assigned to the relevant teacher with sufficient qualifications and rich teaching experience. If for instance, the teachers are not experienced and the subject is assigned, the teachers are to be monitored by the experienced teachers and if any discrepancies are found in the teaching methodology then they need to be rectified on the spot.

2. The study recommends that Fedral Directorate of Education needs to recruit teachers with professional degrees i.e., Associate Diploma in Education (ADE), Bachelor of Education (B.Ed.), Master of Education (M.Ed.), Master of Education (M.A.) and other certificate in accordance with their terminal degrees. For instance, if a pre-service or in-service teacher has done his / her terminal degree in science subjects, then in B.Ed., the specialization courses in pedagogy need to be given in the science subjects rather than arts subjects.

3. Most of the teachers were unaware of the National Curriculum and they were only relying on the textbooks. Hence, the unawareness of teachers about the curriculum makes it difficult to implement it in its real essence. The study recommends that the teachers need to be provided with the curriculum developed by the National Curriculum Wing.

4. The teacher may be provided training regarding the implementation of the curriculum, and this could only be done when the teacher has strong pedagogical skills. The training scope may not be limited to the public sector schools, but the private sector schools may also be provided with the chance to participate. Proper pieces of training (curriculum development and implementation, pedagogy, teaching methodology, assessment) and workshops are required to be provided by trained and experienced resource persons, this will help teachers to easily articulate the philosophy of teaching and setting the targets while implementing the curriculum.

5. The study recommends that at the end of each academic year, the teaching methodology and the implementation of the curriculum by the teachers need to be evaluated in terms of learners' academic achievements.

For any discrepancies or shortcomings, corrective measures need to be taken for the next

academic year.

6. The study recommends that teachers need to be engaged in training regarding pedagogical skills. Training modules need also to include the development of lesson plans, classroom time management, teaching through content-oriented activities in the classrooms, and summative assessment of the students in each grade. The scope of such training needs also to be expanded to include the pre-service and newly appointed teachers.

7. The study recommends that teachers need to focus on the development of conceptual understanding and ability of reasoning and problem-solving approach among students rather than solely teaching to complete the course without developing their conceptual understanding, which leads to the habit of cramming or rote memorization among students.

5.6 Limitations

The current study faced many limitations which include the limited time frame from the Department, the least cooperation of school teachers in research activity because of the fear that the findings may be reported to the school's administration, lack of learner motivation towards the subject of mathematics, frequent closure of schools due to COVID-19 and a new variant of COVID-19 i.e., Omicron, teaching the required subject matter at the mid of February 2022. But all these constraints were discussed with the supervisor for guidance.

REFERENCES

Ahmad, M. (2011). Effect of problem-solving teaching strategy on the performance of 8th-grade students. PhD unpublished thesis, University of the Punjab, Lahore.

Akinoglu, O., and Ruhan, O. T. (2007). The Effect of Problem-Based active Learning in Science Education on Students' Academic Achievement, Attitude and Concept Learning. *Eurasia Journal of Mathematics, Science & Technology Education*, 3(*1*), 71-81

Alvi, M. (2016). A manual for selecting sampling techniques in research. University of Karachi. http://mpra.ub.uni-muenchen.de/70218/

Backes, B. Hansen, M.. (2015). Working Paper 146. Washington, D C: National Center for Analysis of Longitudinal in Education Research. Teach for America impact estimates on non-tested student outcomes. Retrieved on 28 Feb 2022 from http://www.caldercenter.org/sites/default/files/WP&%20146.pdf

Blazar, D. (2015). Effective teaching in elementary mathematics: Identifying classroom practices that support student achievement. *Economics of Education Review*. 48:16–29

Blazar, D., Braslow, D., Charalambous, C. Y., Hill, H. C. (2015). Working Paper. Cambridge, MA: National Center for Teacher Effectiveness. Attending to general and content-specific dimensions of teaching: Exploring factors across two observation instruments. Retrieved from http://scholar.harvard.edu/files/david_blazar/files/blazar_et_al_attending_to_general_and_content_specific_dimensions_of_teaching.pdf

Bloom, B. S. (1969). Taxonomy of educational objectives*: The classification of educational goals : Handbook I, Cognitive domain*. New York: McKay

Boote, D. N., Beile, P. (2006). On "literature reviews of, and for, educational research": a response to the critique by Joseph Maxwell. *Educ. Res.* 35(*9*):32-35.

Borghans, L., Duckworth, A. L., Heckman, J. J., Ter Weel, B. (2008). The economics and psychology of personality traits. *Journal of Human Resources*. 43(*4*):972–1059.

Chetty, R., Friedman, J. N., Hilger, N., Saez, E., Schanzenbach, D., Yagan, D. (2011) How does your kindergarten classroom affect your earnings? Evidence from Project STAR. *Quarterly Journal of Economics*. 126(*4*):1593–1660.

Chetty, R., Friedman, J. N., Rockoff, J. E. (2014). Measuring the impacts of teachers I: Evaluating Bias in Teacher Value-Added Estimates. *American Economic Review*. 104(*9*):2593–2632.

Chin, C. & Chia, L. G. (2004). Problem-Based Learning using Students' Questions to drive Knowledge Construction, *Science Education*, 88(*5*), 707-727.

Chiu, M. S. (2007). Mathematics as mother/basis of science in affect: Analysis of TIMSS 2003 data. In *Proceedings of the 31st Conference of the International Group for the Psychology of Mathematics Education, Taiwan* (Vol. 2, pp. 145-152).

Cohen, D. K. (2011). Teaching and its predicaments. Cambridge, MA: Harvard University Press.

Denzin, Norman, K. (1973). The research act: A theoretical introduction to sociological methods. New Jersey: Transaction Publishers.

Duckworth, A. L., Quinn, P.D., Tsukayama, E. (2012). What No Child Left Behind leaves behind: The roles of IQ and self-control in predicting standardized achievement test scores and report card grades. *Journal of Educational Psychology*. 104(*2*):439–451.

Eng, C. S. (2001). Problem-Based Learning-Educational Tool or Philosophy. University of Newcastle, Australia. http://edweb.sdsu.edu/clrit/learning tree/PBL/PBLadvantages.html

Feynan, R. (2003). The new book of popular science. Danbury, Conn: Grolier.p.328.

Frenzel, A. C., Goetz, T., Lüdtke, O., Pekrun, R., & Sutton, R. E. (2009). Emotional transmission in the classroom: exploring the relationship between teacher and student enjoyment. *Journal of Educational Psychology*, 101(*3*), 705. https://doi.org/10.1037/a0014695

Galadima, I., & Yusha'u, M. A. (2007). An Investigation into Mathematics Performance of Senior Secondary School Students in Sokoto State. *ABACUS*, 32(*1*), 24-33. Retrieved from http://www.academia.edu/download/31508658/ABACUS_AN_INVESTIGATION_INTO.pdf

Gallagher, S. A., Stephien, W. J., Sher, B. T. &Workman, D. (1999). Implementing Problem-Based Learning in Science Classrooms. *School Science and Mathematics*, 95(*3*), 136-146.

Gershenson, S. (2016). Linking teacher quality, student attendance, and student achievement. *Education Finance and Policy*. 11(*2*):125–149.

Government of Pakistan (2006). National curriculum for mathematics 2006. Islamabad: Federal Ministry of Education.

HakiElimu. (2012). School Children and National Examinations: Who Fails Who? A Research Report on the Relationship between Examination Practice and Curriculum Objectives in Tanzania. Dar es Salaam:HakiElimu. Retrieved from http://hakielimu.org/files/publications/Relationship%20between%20examination%20Low%20res.pdf

Hamre, B. K., Pianta, R. C., Downer, J.T., DeCoster, J., Mashburn, A. J., Jones, S.M., Brackett, M. A. (2013). Teaching through interactions: Testing a developmental framework of teacher effectiveness in over 4,000 classrooms. *The Elementary School Journal*. 113(*4*):461–487.

Hanushek, E. A., Rivkin, S. G. (2010). Generalizations about using value-added measures of teacher quality. *American Economic Review*. 100(*2*):267–271.

Herreid, C. F. (2003). The Death of Problem-Based Learning. *Journal of College Science Teaching*, 32, 6.

Hill, H. C., Blunk, M. L., Charalambous, C. Y., Lewis, J. M., Phelps, G. C., Sleep, L., Ball, D. L. (2008). Mathematical knowledge for teaching and the mathematical quality of instruction: An exploratory study. *Cognition and Instruction*. 26(*4*):430–511.

Hsiao, H. C., & Chang, J. C. (2003). A quasi-experimental study researching how a problem-solving teaching strategy impacts on learning outcomes for engineering students. World Transactions on

Engineering & Technology Education. *UICEE* 2(*3*). Retrieved from http://www.eng.monash.edu.au.

Iqbal, M. (2004). Effect of cooperative learning in academic achievement of secondary school students in mathematics (Unpublished P.hD. thesis) University of Arid Agriculture Rawalpindi (Pakistan) PP. 2-3.

ISR. (2011). Industry and Enterprise Development in the Far East. An international survey of modernization and development. Industrial System Research. Industrial System Research.

Jackson, C. K. (2012). NBER Working Paper No. 18624. Cambridge, MA: National Bureau for Economic Research. Non-cognitive ability, test scores, and teacher quality: Evidence from ninth grade teachers in North Carolina.

Jennings, J. L., DiPrete, T. A. (2010). Teacher effects on social and behavioral skills in early elementary school. *Sociology of Education*. 83(*2*):135–159.

Kafyulilo, A., Fisser, P., Pieters, J., & Voogt, J. (2015). ICT Use in Science and Mathematics Teacher Education in Tanzania: Developing Technological Pedagogical Content Knowledge. *Australasian Journal of Educational Technology*, 31(*4*), 381-399. https://doi.org/10.14742/ajet.1240

Kane, T. J., Staiger, D. O. (2012). Gathering feedback for teaching. Seattle, WA: Measures of Effective Teaching Project, Bill and Melinda Gates Foundation.

Ker, H. W. (2013). Trend Analysis on Mathematics Achievements: A Comparative Study Using TIMSS Data. *Universal Journal of Educational Research*, 1(*3*), 200-203. https://doi.org/10.13189/ujer.2013.010309

Khan, A., Akhter, A, & Hukmadad. (2010). Asian school science. Effect of using problem solving method in teaching mathematics on the achievement of mathematics Students. 6.

Kilasi, D. V. (2017). Characteristics and Development of Students' Mathematical Identities. The Case of a Tanzanian Classroom. Helsinki: Yliopistopaino Unigrafia. Retrieved from https://helda.helsinki.fi/ bitstream/handle/10138/187058/Characte.pdf?sequence=1

Kitta, S. (2004). Enhancing mathematics teachers' pedagogical content knowledge and skills in Tanzania (PhD thesis), University of Twente. Retrieved from http://eduka.no/onewebmedia/thesis_Kitta.pdf

Klusmann, U., Kunter, M., Trautwein, U., Lüdtke, O., & Baumert, J. (2008). Teachers' occupational well-being and quality of instruction: The important role of self-regulatory patterns. *Journal of Educational Psychology*, 100(*3*), 702. https://doi.org/10.1037/0022-0663.100.3.702

Koedel, C. (2008). Teacher quality and dropout outcomes in a large, urban school district. *Journal of Urban Economics*. 64(*3*):560–572.

Kousar, P. (2010). Effect of the problem-solving approach on academic achievement of students In mathematics at the secondary level. Contemporary Issues in Education Research.

Kraft, M. A.., Grace, S. (2016). Working Paper. Providence, RI: Brown University. Teaching for

tomorrow's economy? Teacher effects on complex cognitive skills and social-emotional competencies. Retrieved from http://scholar.harvard.edu/files/mkraft/files/ teaching_for_tomorrows_economy_-_final_public.pdf

Kunter, M., Klusmann, U., Baumert, J., Richter, D., Voss, T., & Hachfeld, A. (2013). Professional competence of teachers: Effects on instructional quality and student development. *Journal of educational psychology*, *105*(3), 805.

Ladd, H. F., Sorensen, L. C. (2015). Working Paper No. 112. Washington, D C: National Center for Analysis of Longitudinal in Education Research. Returns to teacher experience: Student achievement and motivation in middle school. Retrieved from http://www.caldercenter.org/sites/default/ files/WP%20112%20Update_0.pdf

Lambros, A. (2002). Problem Based Learning in K-8 Classroom: A Teacher Guide to Implementation. California. Corvine Press Inc.

Lampert, M. (2001). Teaching problems and the problems of teaching. Yale University Press

Major, C. H., Baden, M. S. & Mackinnon, M. (2000). Issues in Problem Based leaning: A Message from Guest Editors. *Journal on Excellence in College Teaching*, USA, Web Edition, 11, 3.

Maliki, A. E., Ngban, A. N., & Ibu, J. E. (2009). Analysis of Students' Performance in Junior Secondary School Mathematics Examination in Bayelsa State of Nigeria. *Stud Home Comm Sci*, 3(*2*), 131-134. https://doi.org/10.1080/09737189.2009.11885288

Mangle, S. K. (2008). Advanced Educational Psychology (2 nd Edi). Prentice Hall of India New Delhi. Pp.378-380.

Mazana, M. Y., Montero, C. S., & Casmir, R. O. (2019). Investigating Students' Attitude towards Learning Mathematics. *International Electronic Journal of Mathematics Education*, 14(*1*), 207-231. https://doi.org/10.29333/iejme/3997

Mbugua, Z. K. (2012). Factors contributing to students' poor performance in mathematics at Kenya certificate of secondary education in Kenya: A case of Baringo county, Kenya. *American International Journal of Contemporary Research*, 2(*6*), 87-91. Retrieved from http://www.aijcrnet.com

Michael, I. (2015). Factors Leading to Poor Performance in Mathematics Subject in Kibaha Secondary Schools (Masters dissertation), The Open University Of Tanzania. Retrieved from http://repository.out.ac.tz/ 1293/1/Dissertation_-_Isack%27s_Final. pdf

Mihaly, K., McCaffrey, D. F., Staiger, D. O., Lockwood, J. R. (2013). A composite estimator of effective teaching. Seattle, WA: Measures of Effective Teaching Project, Bill and Melinda Gates Foundation.

Ministry of Education, (2006). National curriculum for mathematics grades I-XII,2006. Government of Pakistan Ministry of Education, Islamabad.

Moffitt, T. E., Arseneault, L., Belsky, D., Dickson, N., Hancox, R. J., Harrington, H., Houts, R., Poulton, R., Roberts, B. W., Ross, S. (2011). A gradient of childhood self-control predicts health, wealth,

and public safety. *Proceedings of the National Academy of Sciences*. 108(*7*):2693–2698.

Nafees, M. (2011). An experimental study on the effectiveness of problem-based versus lecture-based instructional strategy on achievement, retention and problem solving capabilities in secondary school general science students. PhD unpublished thesis, International Islamic University, Islamabad.

Leinwarnd, S. E. (2014). National Council of Teachers of Mathematics. *Principles to actions: Ensuring mathematical success for all. Reston*: *VA: Author*.

Okereke, S. C. (2006). Effect of prior knowledge of implementing of mathematical tasks/concepts to career types and gender on students' achievement, interest and retention. In U. Nzewi (Ed) STAN Proceedings of the 47 th Annual Conference, 253-259.

Okigbo, E. C. & Osuafor, A. M. (2008). Effect of using Mathematics Laboratory in Teaching Mathematics on the Achievement of Mathematics Students. *Educational Research and Review*, 3(*8*), 257-261.

Pianta, R. C., Hamre, B. K. (2009). Conceptualization, measurement, and improvement of classroom processes: Standardized observation can leverage capacity. *Educational Researcher*. 38(*2*):109–119.

Ponera, G., Mhonyiwa, J., & Mrutu, A. (2011). Quality of Primary School Inputs in Tanzania Mainland. policy brief Number 2, SACMEQ. Retrieved from www.sacmeq.org

Roh, K. H. (2003). Problem Based learning in Mathematics. ERIC Clearing house for Science Mathematics and Environmental Education. http://www.ericdigests.org/2004-3/math.html

Ruzek, E. A., Domina, T., Conley, A. M., Duncan, G. J., Karabenick, S. A. (2015). Using value-added models to measure teacher effects on students' motivation and achievement. *The Journal of Early Adolescence*. 35(*5*–*6*):852–882.

Sa'ad, T. U., Adamu, A., & Sadiq, A. M. (2014). The Causes of Poor Performance in Mathematics among Public Senior Secondary School Students in Azare Metropolis of Bauchi State, *Nigeria. Journal of Research & Method in Education (IOSR-JRME),* 4(*6*), 32-40. https://doi.org/10.9790/7388-04633240

Saphier, J., Gower, R. R., & Haley-Speca, M. A. (1997). *The skillful teacher: Building your teaching skills* (p. 544). Carlisle, MA: Research for Better Teaching.

Sharan & Sharama. (2008). Teaching of Mathematics. New Delhi: Sterling Publications.

Shulman, L. (1987), "Knowledge and teaching: Foundations of the new reform*", Harvard Educational Review*, Vol. 57/1, pp. 1-23, https://doi.org/10.17763/haer.57.1.j463w79r56455411.

Sumra, S., & Katabaro, J. K. (2014). Declining quality of education: Suggestions for arresting and reversing the trend. Special THDR Issue: ESRF Discussion Paper, 63. Retrieved from http://www.thdr.or.tz/docs/THDR-BP-9.pdf

Tang, j., & Huang, M. (2006). An application of problem-solving model on the auto mechanics practicum course in vocational high schools *Journal of Education and Psychology*, 29(*3*).

Retrived from www.fed.cuhk.edu.hk\en\jep.

Tick, A. (2007). Application of Problem-Based learning in classrooms activities and multimedia. In 5th Slovakian Hungarian Joint Symposium on Applied Machine Intelligence and Informatics. Poprad, Slovakia. http://bmf.hu/conferences/sami2007/36_Andrea.pdf

TIMSS. (2011). Results History . Retrieved March 19, 2022, from http://theuseducationjoke.com/TIMSS_history.html

Todd, P. E., Wolpin, K. I. (2003). On the specification and estimation of the production function for cognitive achievement. *The Economic Journal*. 113(*485*):F3–F33

Topçu, M. S., Erbilgin, E., & Arikan, S. (2016). Factors predicting Turkish and Korean students' science and mathematics achievement in TIMSS 2011. *Eurasia Journal of Mathematics, Science and Technology Education*, 12(*7*), 1711-1737. https://doi.org/10.12973/eurasia.2016.1530a

Tshabalala, T., & Ncube, A. C. (2016). Causes of poor performance of ordinary level pupils in mathematics in rural secondary schools in Nkayi district: Learner's attributions. *Nova Journal of Medical and Biological Sciences*, 1(*1*), 1-6. https://doi.org/10.20286/nova-jmbs-010113

Tu, R., & Shen, W. (2010). Fundamental focuses of Chinese mathematics education: Characteristics of mathematics teaching in China. *Journal of Mathematics Education*, 3(*2*), 160-169.

Usher EL, Pajares F. (2008). Sources of self-efficacy in school: Critical review of the literature and future directions. *Review of Educational Research*. 78(*4*):751–796.

Uysal, S., & Banoglum, K. (2018). Hogging the middle lane: How student performance heterogeneity leads Turkish schools to fail in PISA? *Cypriot Journal of Educational Science*, 13(*2*), 448-460. https://doi.org/10.18844/cjes.v13i2.3196

Wakefield, A. P. (2001). Teaching young children to think about math. Principal, 80(*5*), 26-29.

Walberg, H. J. (1984). Improving the productivity of America's schools. Educational leadership, 41(8), 19-27.

Walker, J. T. & Lofton S. P. (2003). Effect of Problem Based Learning Curriculum on Students' Perceptions of Self-directed Learning. Issues in Educational Research, 13, University of Mississipi Medical Centre.

Weber, K. (2008). Mathematicians validation of proofs. *Journal for Research in Mathematics Education*. 39(*4*), p.432.

Yager, R. E. (2000). A vision for what science education should be like for the first 25 years of a new millennium. *School Science and Mathematics*, 100, 327-341.

Younger M, Brindley S, Pedder D, Hagger, H. (2004). Staring points; teachers' reasons for becoming teachers and their perceptions of what this will mean. *In Eur. J. Teacher Educ.* 27(*3*):245-264.

Yuzhi, W. (2003). Using Problem Based Learning and Teaching Analytical Chemistry. *The China*

APPENDIX – I

Observational checklist on

COMPARATIVE ANALYSIS OF TEACHING AND LEARNING MATHEMATICs AT THE ELEMENTARY LEVEL

I, Kalsoom Riaz, M.Phil Education Scholar conducting research under the supervision of Dr, Hukam Dad Malik, Associate Professor, Department of Education, National University of Modern Languages, Islamabad. Your responses, and cooperation in response to this study will be considered a noble cause of promoting knowledge. Your responses will be used only for research purposes, and they will not be shared with any third party without your formal consent.

Kalsoom Riaz

M.Phil Education Scholar

Biographical Information

1. Teacher Name (Optional):_____
2. Teacher Qualification:

a. Academic Qualification

F.A/F.Sc	B.A/B.Sc	M.A/M.Sc	M.Phil	PhD.

b. Professional Qualification

CT/ PTC	B.Ed	M.Ed	M.Phil	PhD.

3. The subject of Specialization:

Mathematics	

4. Professional Training in Mathematics Teaching:

a. More than 5 but less than 10	b. More than 10 but less than 15	c. More than 15 but less than 20	d. More than 20

5. Experience teaching Mathematics (In years)

a. 1 – 4	b. 5 – 8	c. 9 – 12	d. 13 – 16

6. School Name:

	Statements	Yes	No
7	Is the teacher provided with a copy of the National Maths Curriculum 2006 of Pakistan for grade 7th by the school administration?		
8	Does the teacher follow the syllabus breakdown which is given in the National Maths Curriculum 2006 of Pakistan for grade 7th?		
9	Does the teacher design paper according to the unit-wise weightage given in National Math Curriculum 2006 of Pakistan for grade 7th?		
10	Is the teacher provided with a Teacher guide? If yes, then Name with publisher and year, please.		

APPENDIX – II

Research Tool #2

Observational checklist on

COMPARATIVE ANALYSIS OF MATHEMATICAL SKILLS OF ELEMENTARY SCHOOL TEACHERS

I, Kalsoom Riaz, M.Phil Education Scholar conducting research under the supervision of Dr, Hukam Dad Malik, Associate Professor, Department of Education, National University of Modern Languages, Islamabad. Your responses, and cooperation in response to this study will be considered a noble cause of promoting knowledge. Your responses will be used only for research purposes, and they will not be shared with any third party without your formal consent.

Kalsoom Riaz
M.Phil
Education Scholar

Biographical Information

7. Teacher Name (Optional):_____
8. School Nam (optional): _____

Classroom Observation compact (Per Grade per session)

Topic	Numbers and operations	Algebra	Measurements and Geometry	Information handling	Reasoning and logical thinking

Areas	Domains	Yes	No
General	Time management		
	Lesson plan		
	Objectives of topic		
	Delivery of the objectives		
	Use of audio/visual aids		
Methods	Lecture Method		
	Inductive Method		
	Deductive Method		
	Heuristic Method		
	Analytic Method		
	Synthetic Method		
	Problem Solving Method		
	Laboratory Method		
	Project Method		
Teaching Strategy	Brain Storming		
	Small group Discussion		
	Demonstration		
	Games		
	Independent Study		

APPENDIX – III

Test 1 for Students

STANDARD 01

Name: _____ Father Name: _____Roll no. : _____

Grade: 7th Section: _____ Period: _____ Time:_____

Subject: Mathematics Topic: 1a. SETS

1b. RATIONAL NUMBERS

1c. DECIMALS

1d. EXPONENTS

1e. SQUARE ROOTS OF POSITIVE NUMBERS

1f. DIRECT AND INDIRECT VARIATION

1g. FINANCIAL ARITHMETIC

School: _____

_____

_

1a.SETS

Q1. If A = {*a, c, e, g*}, B = {*x/x is first of letters of English Alphabets in small case*} and C = {*b, d, f, h*}, then verify that:

(i) AU (B UC) = (AUB) UC (ii) A∩ (B∩C) = (A∩B) ∩ C

i._____

_____

_____

_____

_____

_____

_____

ii.

_____

_____

_____

_____

_____

_____

Q2. If U = {*a, b, c, d, e*}, A = {*a, b, c*} and B = {*b, d, e*}, then show through Venn diagram

(i) A'	(ii) B'
(iii) A U B	(iv) A ∩ B

1b. RATIONAL NUMBERS

Represent each rational number on the number line.

(i) $1\frac{1}{2}$ (ii) $3\frac{1}{3}$ (iii) $-\frac{1}{4}$ (iv) $-1\frac{4}{5}$

Solution:

Prove that:

(i) $\frac{1}{5}\times\left(\frac{9}{10}+\frac{1}{2}\right)=\left(\frac{1}{5}\times\frac{9}{10}\right)+\left(\frac{1}{5}\times\frac{1}{2}\right)$

Solution:

1c. DECIMALS

Q1a. Without actual division, separate the terminating and non-terminating decimals.

(i) $\frac{13}{8}$ (ii) $\frac{7}{25}$ (iii) $\frac{8}{3}$ (iv) $\frac{5}{11}$

(v) $\frac{9}{6}$ (vi) $\frac{20}{15}$ (vii) $\frac{22}{7}$ (viii) $\frac{4}{9}$

Terminating decimal fraction:_____

Non-terminating decimals fraction:_____

Q1b. Round off the following decimals up to three decimal places.

i. 5.41679	Solution:	ii. 11.10365	Solution:
iii. 0.92517	Solution:	iv. 3.10351	Solution:
v. 0.74206	Solution:	vi. 23.15147	Solution:

1d. EXPONENTS

Q1d.i. Use the laws of exponents to find the value of ***x*.**

$[(-7)^3]^6 = 7^x$

Solution:

Q1d.ii. Simplify and write the answer in a simple form. $[(18/11)^3 \div (18/11)^2]^5 \div [(18/11)^2]^2$

Solution:

1e. SQUARE ROOTS OF POSITIVE NUMBERS

Q1e.i Find the square root of the following.

i. 020225	ii. 1* 539/1225

iii. 3.0625	iv. 196/49

Q1e.iiA teacher drew a triangle of 8cm height and 18cm base. Now he wants to draw a square whose area must be twice that of the triangle. Calculate the length of each side of the square that he has to draw.

Solution:

1f. DIRECT AND INDIRECT VARIATION

Q1f.i. Find the value of m in the following proportion.

:m = 0.55:0.27	21 =5 :m

Q1f.ii. 12 water pumps can make a water tank empty in 20 minutes. But 2 pumps are out of order. How long will the remaining pumps take to make the tank empty?

Solution:

1g. FINANCIAL ARITHMETIC

Q1g. i. Zia bought an out-of-order clock for Rs.750 and got it repaired for Rs.425. What should be the selling price of the clock if Zia wants to earn 25% profit?

Solution:

Q2g. ii. Calculate the amount payable as Ushr by a farmer who earned Rs.88,460. Find the actual amount, if rate of Ushr is 5%.

Solution:

APPENDIX – IV

Test 2 for Students

STANDARD 02

Name: _____ Father Name: _____Roll no. : _____

Grade: 7th Section: _____ Period: _____ Time: _____

Subject: Math Topic: 2a. ALGEBRAIC EXPRESSION

2b. LINEAR EQUATIONS

School: _____

Q2ai. ALGEBRAIC EXPRESSION

If $A = 2(x^2 + y^2 + z^2)$, $B = -x + 3y^2 - 2z^2$ and $C = x^2 – y^2 - 3z^2$, then

Find:

i. $A + B + C$	ii. $B + C – A$

Q2aii. Simplify the following polynomials

i. $(a^2 – b^2)(a^2 + b^2)$	ii. $A^2(b^2 – c^2) + b^2(c^2 – a^2) + c^2(a^2 – b^2)$

Q2a. Simplify the following by using the identity.

$49(x + y)^2 - 16(x - y)^2$

Q2b. solve the linear equation

1/3 (x- 3)+ 2/3= 1/3 (4x- 3)+ 7/2 Solution:

Q2b. Find the number.

i. A number divided by 5 gives 9 less than twice the number.	ii. The sum of three consecutive numbers is 45.

APPENDIX – V

Test 3 for students

Standard 03

Name: _____ Father Name: _____Roll no. : _____

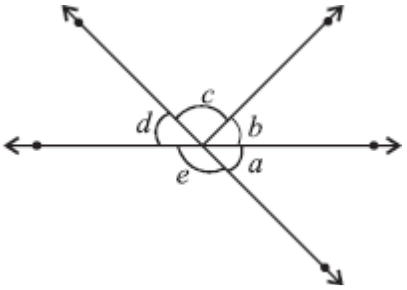
Grade: 7th Section: _____ Period: _____ Time: _____

Subject: Mathematics Topic: 3a. FUNDAMENTALS OF GEOMETRY
3b. PRACTICAL GEOMETRY
3c. CIRCUMFERENCE, AREA AND VOLUME

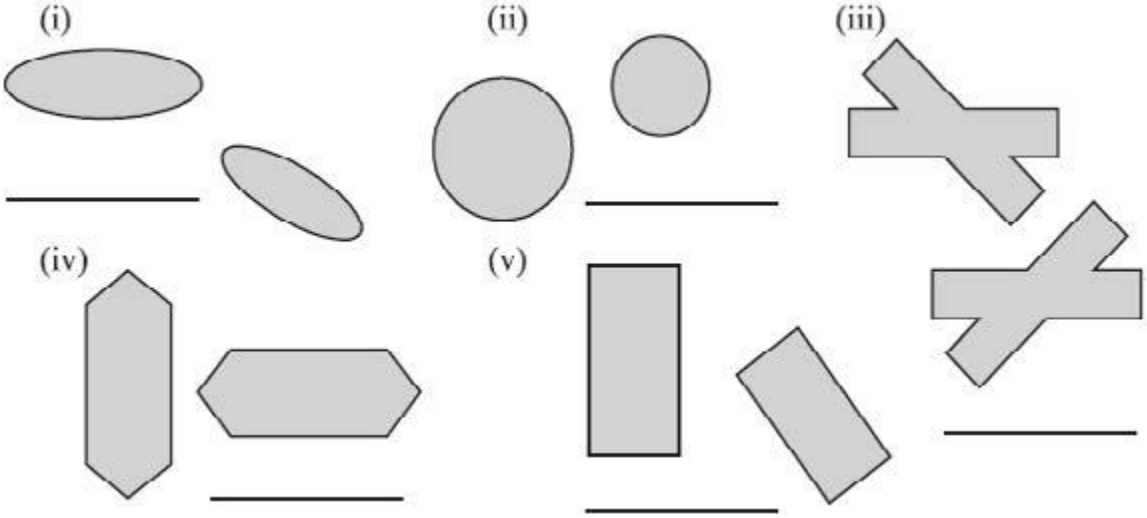
School:_____

_____

Q3a.i. Name all the angles in the figure which are adjacent.



Q3a.ii. Identify congruent and similar pairs of figures.



Q3a.iii. Draw a circle and cut it into two segments. Construct two inscribed angles in each of the segments and measure them.

Q3b.i. Construct an isosceles triangle with altitude = 3.5cm and vertex angle = 50^o

Q3b.ii. Construct the parallelogram *DBAS* where *mBA* = 9*cm mAS* = 2.8*cm m∠DBA* = 400

Q3c.i . The radius of the wheel of Aslam's cycle is 35cm. To reach school from the house, the wheel completes 1200 rounds. Find the distance from the house to the school (when $\approx$ 22/7).

Q3c.The internal diameter of a round mosque is 31.5m and the height of the walls is 7m. Find the cost of cementing the round wall of the mosque at the rate of Rs.l9/m2.

APPENDIX – VI

Test 4 for Students

Standard 04

Name: _____ Father Name: _____Roll no. : _____

Grade: 7th Section: _____ Period: _____ Time: _____

Subject: Mathematics Topic: INFORMATION HANDLING

School:_____

Q1a. The following data is showing the sale of a bike company during the last few months. 571, 692, 700, 533, 832, 744, 649, 584, 613, 735, 872, 900, 512, 864, 654, 782, 777, 555, 632, 880, 628, 529, 680, 756, 567, 548, 824, 719, 678, 721. Make a frequency table by taking 100 as a grade interval. And also state what the upper-grade limit, lower-grade limit and grade interval are.

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

Lower Grade limit:_____, Upper Grade limit: _____, Grade Interval : _____

Q1b. A media reporter surveyed persons visiting the market during the two hours. He found that 720 persons visited the market out of which 320 were women, 220 men and 180 children. Draw a pie graph.

_____

_____

_____

_____

_____

APPENDIX – VII

Certificate of Validity

COMPARATIVE ANALYSIS OF TEACHING AND LEARNING MATHEMATICAL AT THE ELEMENTARY LEVEL

By Kalsoom Riaz

M.Phil. Scholar, Department of Education
Faculty of Social Sciences, University of Modern Languages, H-9, Islamabad,
Pakistan.

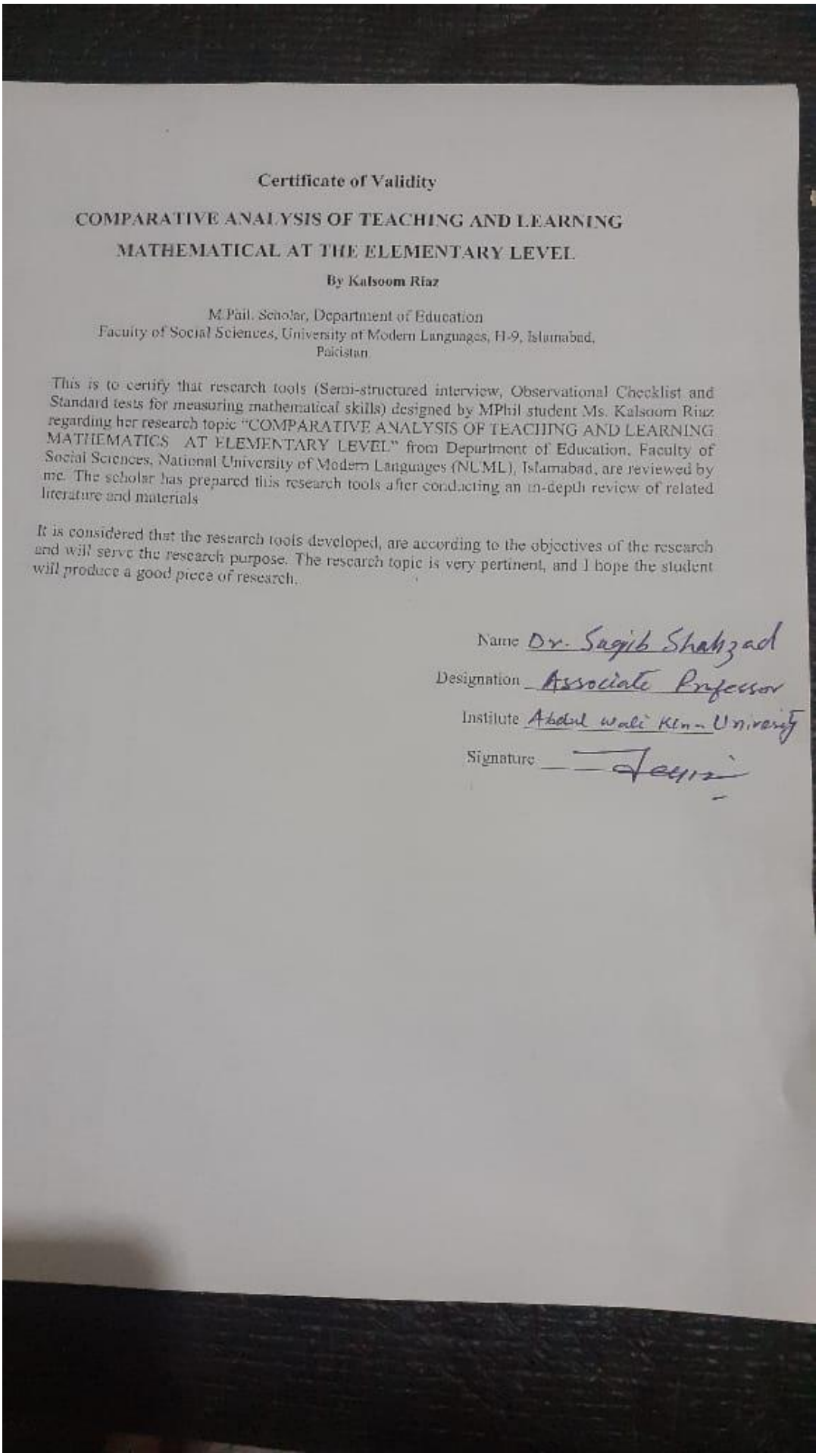
This is to certify that research tools (Semi-structured interview, Observational Checklist and Standard tests for measuring mathematical skills) designed by MPhil student Ms. Kalsoom Riaz regarding her research topic "COMPARATIVE ANALYSIS OF TEACHING AND LEARNING MATHEMATICS AT ELEMENTARY LEVEL" from Department of Education, Faculty of Social Sciences, National University of Modern Languages (NUML), Islamabad, are reviewed by me. The scholar has prepared this research tools after conducting an in-depth review of related literature and materials

It is considered that the research tools developed, are according to the objectives of the research and will serve the research purpose. The research topic is very pertinent, and I hope the student will produce a good piece of research.

Name Dr. Muhammad Iqbal Majoka
Designation Prof.
Institute Hazara University
Signature

Dr. Muhammad Iqbal
Professor
Department of Education
Hazara University Mansehra

APPENDIX VIII



Certificate of Validity

COMPARATIVE ANALYSIS OF TEACHING AND LEARNING MATHEMATICAL AT THE ELEMENTARY LEVEL

By Kalsoom Riaz

M.Phil. Scholar, Department of Education
Faculty of Social Sciences, University of Modern Languages, H-9, Islamabad,
Pakistan

This is to certify that research tools (Semi-structured interview, Observational Checklist and Standard tests for measuring mathematical skills) designed by MPhil student Ms. Kalsoom Riaz regarding her research topic "COMPARATIVE ANALYSIS OF TEACHING AND LEARNING MATHEMATICS AT ELEMENTARY LEVEL" from Department of Education, Faculty of Social Sciences, National University of Modern Languages (NUML), Islamabad, are reviewed by me. The scholar has prepared this research tools after conducting an in-depth review of related literature and materials

It is considered that the research tools developed, are according to the objectives of the research and will serve the research purpose. The research topic is very pertinent, and I hope the student will produce a good piece of research.

Name Dr. Saqib Shahzad
Designation Associate Professor
Institute Abdul Wali Khan University
Signature

kalsoom Riaz
Turnitin report.pdf