EFFECT OF COOPERATIVE LEARNING ON STUDENTS' MATHEMATICAL SKILLS AT ELEMENTARY LEVEL: A QUASI-EXPERIMENTAL STUDY

BY

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NATIONAL UNIVERSITY OF MODERN LANGUAGES ISLAMABAD

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THESIS AND DEFENSE APPROVAL FORM

The undersigned certify that they have read the following thesis, examined the defense, are satisfied with the overall exam performance, and recommend the thesis to the Faculty of Social Sciences for acceptance.

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AUTHOR'S DECLARATION

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Candidate of **Master of Philosophy** at the National University of Modern Languages hereby declares that the thesis **"Effect of Cooperative Learning on Students' Mathematical Skills at Elementary Level: A Quasi-Experimental Study"** submitted by me in partial fulfillment of MPhil degree, is my original work, and has not been submitted or published earlier. I also solemnly declare that it shall not, in the future, be submitted by me for obtaining any other degree from this or any other university or institution.

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ABSTRACT

Title: Effect of Cooperative Learning on Students' Mathematical Skills at Elementary Level: A Quasi-Experimental Study

This study aimed to explore the effect of cooperative learning on students' mathematical skills at the elementary level. The main objectives of the research study were to examine the effect of cooperative learning on students' mathematical skills, to assess the effect of cooperative learning on students' mathematical problem-solving skills, to examine the effect of cooperative learning on students' mathematical critical thinking skills, to determine the effect of cooperative learning on students' mathematical communication skills at the elementary level. A quasi-experimental research design was implemented through a quantitative research approach. The population of the study was 280 students out of which 30 students worked as a sample of 8th grade in a public school situated in the Nilore area of Islamabad. Pre-test, treatment, and post-test were taken from the students per week and the same procedure was repeated after every part of the study using the Student Teams- Achievements Division's method of cooperative learning. The dependent variables were measured through a self-constructed, 12-item test used both as a pretest as well as a post-test. The experimental group was exposed to the treatment of cooperative learning. The data were analyzed by using paired sample t-test. Validity of the instrument was checked by two experts having Ph.D., and reliability was checked through Cronbach's alpha. From the finding of the study, it was concluded that cooperative learning is highly effective for students' mathematical skills at the elementary level. It may be recommended that teachers may receive professional development and training on the principles and strategies of cooperative learning and how they specifically promote problem solving skills, critical thinking skills, and communication skills in mathematics.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATIONS

FDE	Federal Directorate of Education
IMSB	Islamabad Model School for Boys
STAD	Student Teams- Achievements Divisions
NCTM	National Council of Mathematics Teachers
ZPD	Zone of Proximal Development
TAI	Team Accelerated Instructions
PIES	Positive Interdependence, Simultaneous Interaction, and Equal Participation
STLM	Student Team Learning Methods
SPSS	Statistical Package for the Social Sciences
MST	Mathematical Skills Test
CL	Cooperative Learning
ES	Effect Size

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DEDICATION

This thesis is dedicated to my Mother, my eldest brother Gul Muhammad Brohi, and late Dad Lal Bux for their love, endless support, and encouragement. Dad, you are not away but in my every breath**.**

INTRODUCTION

1.1 Background of the Study

Cooperative learning is an approach to education that emphasizes student collaboration and active participation in the learning process. It is based on the idea that students learn better when they work together, and that social interaction can facilitate learning. Cooperative learning groups are formed to promote this type of learning environment. The characteristics of group formation based on cooperative learning are as follows:

The justification for forming cooperative learning groups based on these characteristics is that they promote a more engaging and effective learning environment. By working in groups, students are exposed to diverse perspectives and learn to work collaboratively. This can help improve communication skills, foster critical thinking, and enhance problem-solving abilities. Additionally, cooperative learning can increase student motivation and self-esteem by providing opportunities for success and positive social interaction. Overall, cooperative learning is a valuable approach to education that can benefit students in a variety of ways.

Lau et al. (2009), describe learning mathematics as "the mathematics skills required for the youth of today and adults of tomorrow to function in the workplace are different from that for youth and adults of yesterday." Therefore, now there is great requirement for new teaching strategies in which students' involvement should be emphasized toward understanding mathematics skills. The mathematical skills of students at the elementary level have not been satisfactory in our country. According to Posamentier et al. (2010), participation of pupils as actively engagement by collaboration and discussion in order to learn mathematics is the teaching of mathematics.

Education is essential to construct a productive nation. Many subjects play a vital role in education, but mathematics is the mother of all subjects. It covers a wide range of activities in every part of life. But there is a concern about the decline in mathematical skills and achievements among students and schools. According to students, the reason for the decline in mathematical skills is that it is a boring, tough, and difficult subject. Duran

et al. (2019), said that there is a need to overcome the frustration among students and teachers to have excellence in mathematics. According to researchers, students, or individual needs of the pupils should be taken as a center to make mathematics teaching effective. The lack of interest of students in science is the reason for weak mathematical skills.

Elementary-level students are needed to be given the opportunity to enhance their mathematical skills by the teacher or facilitator for problem-solving mathematically, reasoning mathematically, analyzing, and communicating mathematically (Duran et al., 2019). These approaches can be implemented by cooperative learning in mathematics. Breaking the classroom into minor batch to get common goals is called cooperative learning, which makes students learn using their social skills. Positive attitude toward mathematic skills and performance can be improved as shown by many studies. Students' lack of interest in mathematics at the secondary level can be reduced and they can be motivated if they are given the opportunity to collaborate, problem-solving, create solutions.

The way to exchange thoughts and ideas and present tables, objects, graphs, and symbols are known as mathematical communication (NCTM, 2000). Ifamuyiwa, S. A., and Kinsola (2008) found that in the exploratory study, the experimental batch students displayed a great tendency in learning mathematics. Student Teams-Achievement Divisions (STAD) is keeping the desire to research cooperative learning effect in teaching, especially mathematics, and improving pupils' attitudes towards mathematical skills. It is essential to create motivation in the students regarding mathematical skills in order to get desired goals in life in different fields. The determination of the design is to examine the perceived and expected outcome of cooperating learning on students' mathematical skills at the lower secondary level. Due to rapid changes in the education domain, it is dynamic, not static. The effect of cooperative, competitive, peer, and individualistic learning were examined by 117 meta-analyses on multiple social variables, personal and educational institutions found that cooperative learning performed better as compared to competitive, individualistic, and lecture learning since it creates a positive attitude towards motivation, develops productive social skills and achievements.

When students change their minds with ideas and cognizance, then they get achievements in their studies and these accomplishments depict their cognitive behavior. Mathematics is the foundation for all education in every part of the world, which shows the value of learning it. Therefore, it can be said that if one learner gets excellence in mathematics, he can excel in other domains of education as well. Cognizing mathematics is the competence of the other subject of the curriculum and can construct and develop society genuinely (Johnson & Johnson, 2008).

According to Shimazoe, and Aldrich (2010), cooperative learning produces many good results like deep learning in reasoning mathematics, problem-solving, analyzing in mathematics, social skills, and values, critical thinking skills, and developing positive attitudes toward mathematic skills. According to the Self-Determination Theory that autonomy (feeling a level of control), association and capability are psychological needs of individuals. When students learn through cooperating groups, then these needs can be met. It also contributes to learners' happiness and motivation by meeting their psychological needs. There are three groups of cooperative learning: informal group, formal group, and long-term base groups. Breaking class into minor sets for a short while to help out each other during the teaching, to think about the going on topics and particular quarries, and to sum-up on the important points in the lesson is known as informal cooperative strategy. Learners contribute their propositions, support each other to get required goal, and reflect together on their progress.

The importance and value of mathematics subject are on the gradual rise in schools and colleges not only in Pakistan but also all over the world. Learning mathematics is mandatory at the elementary level in Pakistan therefore, learning with enthusiasm is essential. Mathematics learners need to have a strong grip on mathematical communication skills, including reading, writing, listening, and speaking. There are very important skills in mathematics like mathematical presenting, brainstorming, and problem solving skills that required to be conceived by the learners. Pakistani schools, colleges, and universities are the traditional methodology centers for the pupils. There is a need for new motivational, active learning and stimulated methods required for achieving the objectives and goals of mathematic learners (Roselainy et al., 2010). Students are formed on a long-term based through permanent membership is called long-term cooperative learning. In the meantime,

Awful et al. (2013) concluded from research studies that TAI is able to give a productive effect on pupils' attitudes and students' mathematical skills by applying a learner-centered cooperative learning strategy.

Cooperative learning can improve academic achievement and real growth because it creates an interactive learning strategy for educators (Tran, V. D., 2013). Albee et al. (2015) found a conclusion taken by their research study that during the practical, the learners gave importance and took interest in the cooperative learning strategy (model) as compared teacher-centered learning model. According to Arsheen (2015), there is a need of eliminating the fears of students, motivating the learners, developing their cognitive behaviors, constructing relationships between daily life and mathematics, and developing critical thinking skills. Mathematical and reasoning skills can be improved by implementing learning models and learners-centered strategies. There are multiple reasons and drawbacks; coming from instructors and learners that caused low learning outcomes in mathematics. Traditional pedagogical strategies still used by instructors and they require the learners to be passive during classroom learning. Implementing any strategy is the responsibility of the educator since it is a display of competence and commitment to the nation and education. Planning and implementing multiple learning methods in order to develop the interest and talent of the learners using versatile teaching resources and multimedia is the capability of the instructor (Sanjaya, 2016).

It was also concluded in his research that learners were actively involved during the cooperative learning process and the teacher was also satisfied and effectively focused during the teaching-learning process. Veloo et al. (2016) claimed that learners' learning and their communication skills in mathematics can be enhanced through implementing a cooperative learning strategy because many studies reported that students can be more active and increase their outcomes through cooperative learning. Higher outcomes and problem-solving ability in mathematics increase the communication skills of pupils in the classroom and the claim has been validated by several studies. Students are more motivated in activity-based learning. Students' learning outcomes may interrelate to the teaching strategy model for student-centered teaching technique attracts the intention of learners. In order to foster the learners' achievement, the teaching strategies could be TAI, students' mathematical skills, and STAD.

Alzahrani (2017), concluded that dividing students into groups assists in developing motivation, fostering cooperation, and increasing achievement and communication. Further, Johnson and Johnson (2005) and Buchs et al. (2017), claimed that students are able to improve their achievement more if cooperative learning strategies are implemented as compared to when traditional approaches to learning are utilized.

Chain, L. L., and Noraini, E. (2017), conducted research and concluded that the performance of learners of cooperative learning was better than traditional strategies implemented by the educators. Further, they observed that the motivational level, participation, and involvement of the pupil also enhanced during the getting education when traditional teaching methods were exchanged fora cooperative learning strategy. Education is varying continuously from traditional strategies to modern strategies, and lecture methods to activity methods. Therefore, the internationally immerging teaching strategy among researchers and getting value in education, especially social science studies, is a cooperative learning strategy. One of the learners-centered approaches is a cooperative learning strategy dissimilar from the traditional method (Al-Ahram, 2017).

Research conducted by Ling, S. C., and Fung (2017), concluded that they found no difference in the learning attitude of the learners in cooperative learning. Accordingly, it was deduced that the cooperative learning strategy is able to be implemented for students' mathematical achievements and they also suggested reforms in education by applying cooperative learning.

According to Ahuja, R. (2018), Mathematics is all around us, in every aspect of life, all things we do, that deal with reasoning, critical thinking, problem-solving, and shaping things are mathematically carried out. According to Oxford Language, in social or daily life mathematics is used at every step of life. Without it, we cannot survive like money, art, architecture, mobile devices, and sport. But here most students feel mathematics is a tedious, tough, difficult, and uninteresting subject. They are inattentive during mathematics learning in the classroom and they are unable to memorize and recall what they are taught in school. Poor instructional methods, approaches, and strategies are the cause of the substandard presentation in mathematic assessments of the learners and reason the reducing motivational levels. Many studies show that conventional instructional

methods of mathematics are not very successful as compared to new pedagogical strategies. There is a need for effective instructional techniques, strategies, or methods for teaching mathematics. In these innovative teaching strategies, one very important is cooperative learning. Cooperative learning is directly opposite to the conventional teaching strategies where a win-win situation is applied.

Developing social skills, building the confidence of the learner, constructing wisdom and intelligence, enhancing the motivation of the pupils, promoting high self-steaming, producing self-actualization, and joint-working with learners is the purpose of cooperative teaching. Learning mathematics is a demanding task observed by multiple learners. Therefore, there is a need for effective learning strategies to make learners understand mathematics effectively. The main element of education in Pakistan's curriculum is mathematics as it is adopted as a compulsory subject at secondary level education. The key to success in multiple areas of life is mathematics considered by many researchers. According to Hu et al. (2018), achievement in mathematics is dependent upon true teaching strategies. There is one kind of cooperative learning strategy, the Jigsaw strategy, in which the classrooms are divided into small groups in order to achieve shared goals, joint learning, and improvement of each other’s cognizance. Every small group comprises learners having multiple skills, giving occasion and facilitating them, to build and understand the assigned goals in order to be “Experts” regarding content and to transform the knowledge to other members of the group. The researchers realized the actualization of the Jigsaw style on the student’s mathematical understandings.

Hobri et al. (2018) concluded from the research study that there is a productive effect of cooperative learning strategy on learners’ mathematical skills especially communication skills, pupils’ study outcomes, and their attitudes. It is concluded from the above description that implementation of student-centered strategies; like cooperative learning, can enhance through the motivation or interest of the learner. During the class, improve the students’ mathematical skills and understanding, and attitudes because teacher-centered methods like traditional methods cause low learning achievements, especially in mathematics, examining the relationship of critical thinking, reasoning, and problem-solving skills toward cooperative learning.

Filling the academic needs of students, appropriate teaching strategies should be designed by the twenty-first century preceptors, especially in the growth of skills like logic, investigative, analyzing, and critical thinking. According to Calkins et al. (2020), all levels of the cognitive domain (Bloom's Taxonomy) are the intellectual processes that need to reflect skillfully and actively. For the growth of brainstorming skills, instructors need to implement cooperating learning methods in order to achieve the desired objectives. Role of teachers in school is cooperative in structuring schooling situations and fixing the lesson objectives during the teaching methods. Educators have to place the students in constructive groups, describe and construct goals of cooperative learning regarding productive learning, keeping eye on the students in order to monitor the learners and evaluate the results of instructing strategies. The researcher has got ambition to observe the trueness of cooperative learning being an English instructor on the skills of critical thinking development. The researcher made small groups of classroom students in order to get the same goals of critical thinking skills in mathematics subject.

There is a need for modern teaching models to enhance pupils' MCT skills at the elementary level. Therefore, it was concluded through the experimenter to explore "effect of cooperative learning strategy on students' mathematical skills at primary level".

1.2 Rationale of the Study

Trueness and genuineness of cooperative learning on multiple grounds have been discussed and supportive arguments are provided by the researcher rather than emphasizing individualistic and competitive learning strategies which have been overly used for the last 90 years. Cooperative learning technique implementation is genuine for all subject domains, problem-solving, categorization, remembering, retention judging, and for completing assigned tasks. Some recent studies regarding cooperative learning are being discussed as follows:

Khan A. (2015), in Pakistan, argued the cooperative learning's outcome on writing skills of pupils in easy at the university level. The article's conclusions were supported in favor of implementing the said learning strategies. They established a productive outcome from implementing cooperative instructional strategy on pupils' success and attitude at

graduation degree. Accordingly, it became researcher's aim to examine results of CL on students' mathematical skills at lower secondary level.

Quines E. (2016), examined a study on the cooperative learning's results approach in assimilating brainstorming skills for secondary-level learners mentioned in the book "Empowering 21st Century Learners Through Holistic and Enterprising Learning" (Theh, G. B., & Choy, S. C. 2016). It is found that cooperative learning on students' critical thinking skills is proved to be beneficial compared to traditional teaching. Consequently, it was desired to assess the outcome of cooperative learning on pupils' mathematical critical thinking skills at elementary level by the researcher in Pakistan.

In Pakistan Munawar, and Chaudhary, A. H. (2019), determined in their study that it was instituted in the subject English writing implementing cooperative learning that scores of experimental group was higher than the scores of control group of seventh-grade school students. The duration of the study consisting of two weeks was found to yield highly positive effect on science students' achievements. Accordingly, the outcome of cooperative learning was significantly productive after experimental study. Accordingly, the researcher aimed at study the cooperative learning's effect on learners' mathematical skills at lower secondary level.

Abed et al. (2019), who worked on the "Predicting Effect Implementing the Jigsaw Strategy on the Academic Achievement of Students in Mathematics Classes "found good results regarding the study. Murtiyasa, B., and Hapsari, N. S. (2020), in Indonesia examined the effect of student team achievement divisions and team-accelerated instruction learning methods on education and reviewed outcomes from mathematical communication skills. They introduced a positive effect of cooperative leaning on students' mathematical communication skills in Indonesia. Therefore, it was determined to determine the outcome of learning by implementing cooperative method based on the line of STAD method on students' mathematical communication skills at elementary level by the researcher in Pakistan**.**

Kwame et al. (2020), Department of Mathematics and ICT, St. Francis College of Education conducted a research study on Cooperative Learning Strategy and Students 'Performance in Mathematics in Junior High Schools in Hoopoe Municipality. They introduced favorable effect of cooperative learning on the performance in mathematics at

secondary level. Accordingly, the researcher intended to determine the outcome of learning by implementing cooperative method on students' mathematical skills at elementary level.

Azmin (2016),Karacop, and Doymus (2013) worked and mentioned multiple research studies that cooperative learning results in achievements, analytical skills, and reasoning skills in the form of cognitive perspectives. It was found that development of cooperative learning is higher than the individual and competitive learning. Consequently, it was desired by the researcher to determine the outcome of learning by implementing cooperative method on students' mathematical skills at elementary level.

Johnson et al. (1998) stated that the research studies more than a hundred correlation studies and over six hundred experimental studies were conducted on individualistic, competitive, and cooperative learning. It is observed that by cooperative learning strategies, learners get the higher ability and greater productivity, enhancing intrinsic and extrinsic motivation, encouragement to present the assigned material. From the above studies, it was observed that there is a need of research studies in Pakistan regarding the cooperative learning consequences on students' mathematical skills at the middle level.

Therefore, to fill the gap this study "Effect of cooperative learning on students' mathematics skills" is selected. Teaching mathematics at the elementary level is typical, boring, and tough compared to other subjects, and students get bored and face difficulties in mathematics subjects; therefore, teachers need to be sensitized about different teaching methods to make students interested in mathematics and stimulate modern skills development to facilitate and impart quality education. Accordingly, it has been essential for mathematics teachers to use modern teaching rather than drill teaching methods: hence, cooperative learning can be a significant addition to teaching mathematics.

1.3 Statement of the Problem

Mathematics is a subject that requires a strong foundation of basic skills, critical thinking, and problem-solving abilities. Most of the students struggle to develop these skills through lecture method often termed as traditional method of teaching. Previous results of eighth class students in the public schools of rural area (in Islamabad) showed poor performance in the subject of mathematics taught to them through lecture method. This indicates that students face difficulties in conceptualizing mathematics taught to them through lecture method. Therefore, it was imperative to conduct an experimental research

to explore the effectiveness of the viable teaching method, which could develop the understanding of students in the subject of mathematics.

It is generally believed that cooperative learning enhances academic understandings of students at elementary level. Therefore, an experimental study conducted to explore the effectiveness of cooperative learning at secondary level in the subject of mathematics. Therefore, the researcher conducted a study on effect of cooperative learning on students' mathematical skills focusing on students' mathematical problem-solving skills, critical thinking skills, and communication skills.

1.4 Study Objectives

The study objectives were as following:

i. To assess the effect of cooperative learning on students' mathematical problem-solving skills at the elementary level.
ii. To examine the effect of cooperative learning on students' mathematical critical thinking skills at the elementary level.
iii. To determine the effect of cooperative learning on students' mathematical communication skills at the lower secondary level.

1.5 Null Hypotheses of the Study

i. **Ho1**: There is no effect of cooperative learning on students' mathematical problem-Solving skills at the elementary level.
ii. **Ho2**: There is no effect of cooperative learning on learners' mathematical Critical Thinking skills at the elementary level.
iii. **Ho3**: There is no effect of cooperative learning on pupils' mathematical Communication skills at elementary level.

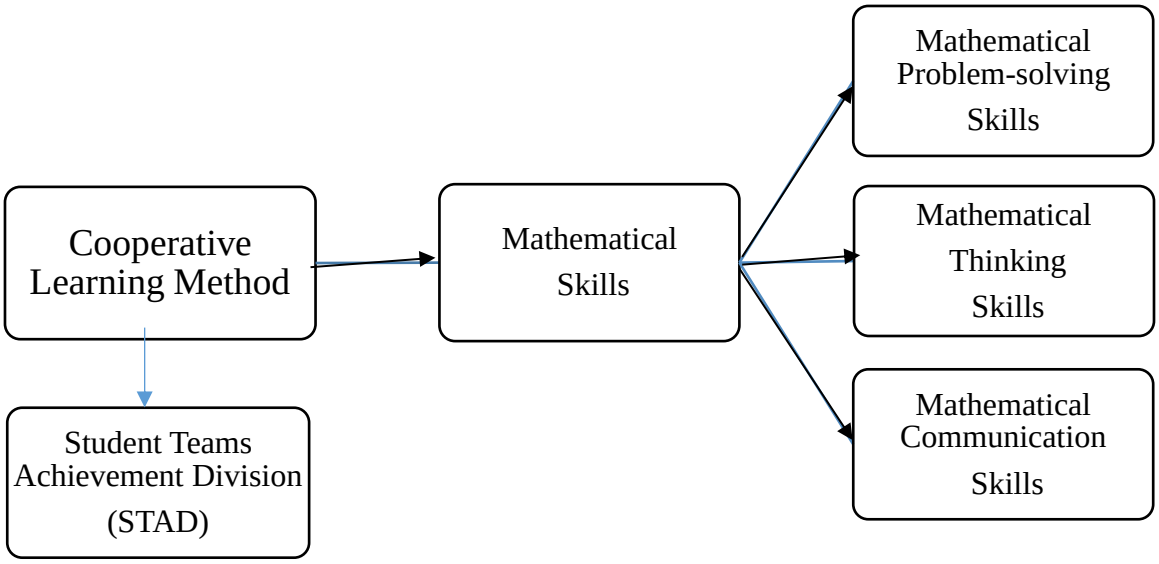
1.6 Conceptual Framework of the Study

This research is a cause-and-effect study research in which two variables independent and dependent examined. The independent variable is cooperative learning and the dependent variable is students' mathematical skills. Mathematical skills have further three components: problem-solving skills, critical thinking skills, and communication skills. Cooperative learning has three groups: formal group, informal

group, and cooperative base group. Methods of making groups are four; STAD, STLM, TAI, and Jigsaw. Buchs et al. (2017), worked on challenges for Cooperative learning, and implementation among elementary school teachers. There are multiple definitions of cooperative learning strategies in the eyes of different researchers. "Cooperative learning is group learning activity organized in such a way that learning is based on the socially structured change of information between learners in groups in which each learner is held accountable for his learning and is motivated to increase the learning of others."

Cooperative strategy is a strategy in which students of small groups in a classroom environment interact with each other to get common goals together and help each other (Tran, & Lewis 2012). Cooperating or collaborating means, from different dictionaries, working jointly for producing an accomplishment. In the social sciences, the use of conceptual frameworks may be traced back to the early 20th century and even earlier. Scholars like Emile Durkheim, Max Weber, and Karl Marx employed conceptual frameworks to understand social phenomena and develop theories in sociology and related fields. Cooperation can be said to be a joint operation for a biological, economic, and social explanation. Cooperation in education or learning is a twentieth-century development by Kurt Lewin first time introduced in 1940. Its roots found in many ancient societies but the establishment of the important approaches and strategies relating to cooperation belongs to the twentieth century through scientific research.

It aimed to study the effect of cooperative learning on students' mathematical skills at elementary levels. The main independent variable was students' mathematical skills, which further classified into three parts: mathematical problem solving skills, mathematical critical thinking skills, and mathematical communication skills. Teaching experience, teacher qualification, and learning environment were the moderating variables. Thomas Kuhn: Kuhn, an American philosopher of science, introduced the concept of "paradigm shifts" in his book "The Structure of Scientific Revolutions" (1962). He emphasized the importance of conceptual frameworks in shaping scientific inquiry and understanding (Reis, H. T., Collins, W. A., & Berscheid, E. 2000).



(Reis, H. T. 2000; Mughal, S. H. 2020).

***Fig.1.1*. Conceptual Framework**

1.7 Operational Definitions

1.7.1 Cooperative learning

The process in which students actively engage in collaborative activities or group work, where they work together to achieve common learning goals. It involves interactions, discussions, and shared responsibilities among students, fostering teamwork and mutual support.

1.7.2 Mathematical skills

The proficiency in understanding, applying, and manipulating mathematical concepts and operations. It includes abilities such as numerical computation, problem-solving, geometric reasoning, algebraic manipulation, data analysis, and logical reasoning within the context of mathematics.

1.7.3 Problem-solving skills

The ability to identify, analyze, and resolve problems effectively and efficiently. It involves the application of logical thinking, critical

analysis, and decision-making techniques to overcome obstacles, make decisions, and find innovative solutions in various contexts.

1.7.4 Critical thinking skills

The capacity to evaluate, analyze, and interpret information and arguments in a logical and systematic manner. It encompasses skills such as logical reasoning, evidence-based decision-making, and evaluation of different perspectives, identification of assumptions and biases, and the ability to construct and present coherent arguments.

1.7.5 Communication skills

The proficiency in effectively transmitting and receiving information, ideas, and messages using verbal, non-verbal, and written forms. It includes the ability to articulate thoughts clearly, listen actively, use appropriate language and tone, and engage in respectful and empathetic dialogue and present information in a coherent and organized manner.

1.7.6 Students team achievements divisions

The grouping of student team achievements according to the specific skills demonstrated or utilized by the teams. This could involve categorizing achievements into divisions such as problem-solving-focused teams, communication-focused teams, or critical-thinking-focused teams, based on the dominant skills exhibited in their accomplishments.

1.7.7 Positive Interdependence

This refers to the level of interdependence among group members. When group members work together and rely on each other to achieve a common goal, it indicates positive interdependence.

1.7.8. Equal Participation

This refers to the level of participation of each member of the group. When all members of the group have an equal opportunity to contribute to the group's work, it indicates equal participation.

1.7.9. Social Skills

This refers to the level of social skills that group member’s exhibit during cooperative learning. When group members exhibit good

communication skills, active listening, and respect for others' opinions, it indicates good social skills.

1.7.10. Group Processing

This refers to the level of group processing that occurs during cooperative learning. When group members reflect on their own and others learning and performance, it indicates good group processing.

1.7.11 Individual Accountability

This refers to the level of accountability that each member of the group has for their own learning and performance. When each group member takes responsibility for his own learning, it indicates individual accountability.

1.8 Significance of the Study

Cooperative learning strategy is significantly related to mathematics skills like critical thinking skills, communication skills, and problem-solving skills rather than traditional or drill methods of mathematics teaching. Therefore, the importance of cooperative learning significantly exists in education. The findings of this research study on the effects of cooperative learning strategies on students' mathematical skills may implement in the Federal Directorate of Education (FDE), Islamabad, may utilize by various stakeholders.

Education administrators and policymakers can use the results to emphasize the importance of cooperative learning strategies in enhancing students' mathematics skills and problem-solving abilities. Educationists can apply the study's findings in designing effective teaching methods that incorporate cooperative learning strategies for mathematics education at the elementary level.

Government officials and curriculum planners can also use the study’s results. To make evidence-based decisions in creating policies and curricula that promote the development of mathematics skills among students. Ultimately, this study can contribute to the improvement of mathematics education in Pakistan by providing valuable insights on the effectiveness of cooperative learning strategies in enhancing students' mathematical skills and problem-solving abilities at the elementary level. The multiple strategies of

cooperative learning is to be employed by teachers to enhance the instruction-learning process, communication skills, problem-solving skills, and critical thinking, especially in mathematics, and reduce the impact of mathematic phobia on learners' minds by using this study.

This study may use by other of my research fellows as a reference for future integrated studies. In comparison, this study strategy with other strategies is to improve or develop communication, critical thinking, and problem skills. This study can provide an effective way of teaching mathematics, and cooperative learning, to improve or develop students' mathematics skills: therefore, mathematics teachers may use cooperative learning strategies while teaching mathematics.

1.9 Delimitations

This study was delimited in the following ways.

Schools" The focus of the study was limited to the public schools of the federal (Rural) area in Nilore.

Population The population of the study was limited to students in elementary level up to 8th grade.

Content The study content was limited to specific topics within the 8^{th}-grade mathematical curriculum, including Sets, Algebra-1, Algebra-2, Basic Statistics, and Geometry.

Mode of communication was limited to written communication.

1.10 Methodology

The research study was based on a quantitative approach using a quasi-experimental research design method. The cooperative learning method was used as a treatment for measuring the students' mathematical skills, skills of critical thinking in mathematics, skills of communication in mathematics, and problem solving skills in mathematics. Three types of tests; pilot tests; pre–tests before treatment, Post-tests after treatment, were developed by this experimenter. The purpose of the pilot test was to verify reliability of the developed test. Aim of pre-tests and post-tests of the four topics, Sets, Algebra, Basic Statistics, and Geometry was to assess the outcome of CL on pupils'

mathematical skills at the lower secondary level. The doctoral committee and mathematics subject specialists verified the content validity of tests. 13 multiple-choice test items were developed for every test of selected four topics -- Sets, Algebra, Basic Statistics, and Geometry. A mathematics teacher from Islamabad Model School for Boys Herno Thanda Pani, Nilor Sector Islamabad was selected for the implementation of cooperative learning. Before starting the experiment, the mathematics teacher was provided 4-day training to implement cooperative learning in the classroom. This chapter has included the nature of experimental research; enclose population, sampling technique, sample, and development of instrument, research validity, pilot study of research, and study reliability.

CHAPTER 2

REVIEW OF THE RELATED LITERATURE

2.1 Effect of Cooperative Learning

Studying the effect of cooperative learning strategies on students' mathematical skills in the classroom at the elementary level was the aim of the study at hand. Therefore, the below-mentioned subheadings are included for the review of related literature, which is very important for critically examining the said topic: definitions, theoretical bases of the effect of cooperative learning and mathematical skills, the rationale for cooperative learning strategies at the elementary level, the specialty of cooperative learning strategies, and elements of cooperative learning strategies. Furthermore, the kinds of cooperative learning groups, cooperative learning strategies and their methods, weakness of cooperative learning, instructions regarding implementing instructional cooperative learning, mathematical skills, and cooperative learning strategies, research studies, and cooperative learning strategies have been addressed. According to Venlo et al. (2016), learners' learning mathematical communication skills can improve their performance by implementing cooperative learning strategies because many studies reported that students can be more active and increase their outcomes through cooperative learning.

2.2 Elements of Cooperative Learning

According to Kegan (1994), there are four basic elements of cooperative learning designed for cooperative instructional strategy: individual accountability, positive interdependence, simultaneous interaction, and equal participation (PIES). There are many elements of cooperative learning used in the classroom but according to Kegan, these elements are significant. In 1999, Johnson and Johnson agreed with Kegan's four principles, and the acronym PIES was generated.

According to Tran, & Lewis (2012), the performance of the students was increased by using cooperative learning in the classroom as an instructional strategy. They studied that various teachers teach students through the group-work method, which is a cooperative learning method, but they miss the concept of the strategy of cooperative learning; therefore, they introduced more elements of cooperative learning.

2.2.1 Positive Interdependence

According to Duran et al. (2019), community-based interdependence is classified in two kinds: productive (cooperation), and pessimistic (competition) interdependence. Positive Interdependence: Positive interdependence exists when peculiars think that learners can achieve their common tasks only the remaining group associates of the same group integrate cooperatively achieving their tasks. Pessimistic Interdependence: it exists when peculiars think that it could be theirs if the remaining batch representative of the same batch integrating cooperatively do not achieve their common tasks. No interdependence exists where individuals do not think about the success or failure of their group members.

Effendi-Hsb, et al. (2019), states that the measurable indicators of positive interdependence are promoting, encouraging, facilitating, supporting, and assisting each other to achieve the assigned goal by employing sharing, problem-solving, communicating, and critical thinking. On the other hand, the negative interdependence variables are to oppose and only think of one's own success in the group. There is no thought without thinking about other group members. It means the gain of one student is the gain of another student in the group or class.

Positive interdependence means that the achievement or outcome of one student is the success of all group members, and similarly, the failure of one group member is for all groups. "Every team member's success is the success of all members and failure of one team member is the failure of the whole group" (Gok, T., & Sylay, I. 2010). The effect of problem-solving strategies on students 'achievement, attitude, and motivation, group members are taught to "sink or swim together". Beyond individual success, team members have to accomplish their work together in positive interdependence. Creation of the sensitization in group members should learn and make sure that others learn too. Learning from other group members is the main function of positive interdependence. It is because of positive interdependence that every team member gets insights that not only his accomplishment but also others' accomplishment of assigned work can get successful results. Therefore, there is no free rider due to positive interdependence, and they are away from individualistic and competitive learning.

2.2.2 Equal Participation

Equal participation means every student should explain the work individually; no one can get dominant or boss in the group. It should be self-explanatory which means that everyone must present the assigned work and answer the quizzes on his turn showing the equal participation of each group member. According to Botha, W. (2021), in cooperative learning for building collaboration skills and cognitive exercise in students, one is allowed to dominate the group and it is also the teacher's responsibility to make sure no "free rider" occupies the place. A student has to speak, contribute and participate in the presentation equally on his turn and it is mandatory for each learner.

2.2.3 Individual Accountability

Individual accountability means there is a performance assessment for every member of the group that can be compared against the standard performance. After assessing the overall performance of the group, group accountability is achieved. After assessing each member, individual accountability is obtained. The individual and group result is given back to them for comparison against the standard performance. Every group member is responsible for the group's success or failure on account of his performance (Tran, & Lewis,2012).

2.2.4 Simultaneous Interaction

Simultaneous interaction means to work face to face, and facilitate each other, encourage and facilitate each other's effort while learning in groups. It is the discipline of cooperative learning that completing work, sharing it with other group mates, encouraging others, and promoting group mates' work ensure success and the achievement of the assigned work together. It follows various effective processes: first, to meet the group members, secondly, to work together and thirdly, to achieve individual and group targets. These are the main functions of cooperative learning strategies. There are some more elements of cooperative learning other than Kegan's (1994) four basic principles (PIES) mentioned as the following:

2.2.5 Interpersonal Social Skills

Interpersonal social skills mean how to interact with each other effectively and

productively. Learners should be highly motivated and knowing high–quality collaboration makes team assignments productive. Due to task work and group work simultaneously, individualistic, and competitive learning is easy as compared to cooperative learning. Getting common-goal learners. It is essential for students to trust each other, interact unambiguously, promote, encourage each other, and solve problems with each other (Johnson & Johnson, 1999).

2.2.6 Group Processing

Group processing means working and functioning step by step for effective group work. For an effective product, there should be a scientific order of events and group work taking place. It can be described as the helpful and unhelpful actions, and which actions change or continue for the effective outcome. For achieving the group goals, group processing must be structured and effective (Zakaria et al., 2010). It may be said that competitive or individualistic learnedness is easier rather than cooperative learnedness as, in cooperative learnedness; students are simultaneously engaged in teamwork and target work. Cooperative knowledge involves a- Communicate Effectively, b- Getting Knowledge and Reliability, c- Honors and Facilitate each other, and Solving Conflicts Positively. Five effective steps are involved in structuring group processing for continuously improving the quality of the teamwork and the group's target work (Johnson & Johnson, 1999).

i. Conducting small-group and class celebration

ii. Functioning of the whole class efficiently

iii. Setting goals to improve their effectiveness.

iv. giving feedback to every group learning

v. assessing the quality and maximizing the outcome of each group

2.2.7 Responsibility Sharing

Every member of the group is to play a role of leadership during the learning activities while sharing their ideas in the learning cluster during classroom instruction.

2.2.8 Maximizing and Maintaining Good Working Relationship

According to Zakaria et al. (2010), the success of each class cluster is based on not only a good working relationship but is also dependent upon maintaining and maximizing it. All members of each group not only stimulate but also help and back each other in the class cluster while learning. They interact, debate, and argue with each other while sharing mathematical ideas. Learners not only facilitate each other but also aid each other within the group to achieve the assigned goal and task by the educator. Therefore, they maintain their good group working relationship and maximize achievement.

2.2.9 Social Collaborative and Interpersonal Skills

Camacho-Minuche et al. (2021), suggest that cooperative language learning elements enhance social skills in English classrooms are very important elements of cooperative learning promoting social collaboration and interpersonal skills where pupils lead, communicate, and trust each other while solving their issues using critical thinking skills. The class educator also must create a conducive to learning environment for the pupils where they understand the diversity of culture, language, age, gender, and ethnicity. Learners may develop their decision–making skills to care for and respect each other. They could resolve their disagreements, dislikes, and diversities and develop the learning environment. This learning environment works out mathematical communication and critical thinking skills while learning mathematics. Some educators view that there are some disadvantages of cooperative learning strategy using different techniques that support collaborative learning and some strategies don’t allow the learner to be flexible as other structures.

2.3 Types of Cooperative Learning Teams

Kagan (1985), and Johnson and Johnson (1994, 2002) divided cooperative learning into three strategies based on different grasp between the elements appropriated for temporary and short-term use: formal group, informal cooperative learning group, and cooperative base groups: A comparative effect of three cooperative learning strategies is as following:

2.3.1 Formal Cooperative Learning groups

This group is designed for lasting from one class period to many periods or maybe multiple weeks. The formal group is the typical, and core of cooperative learnedness. The purpose of this group is to involve students' wisdom in their content work, explaining, integrating, interdepending, and summarizing it into functioning abstract composition. It is the heart of the learning strategies. This group is highly structured, scientifically selected, and responsible for every action and outcome of the assigned work (Johnson & Johnson, 2002).

2.3.2 Informal group of cooperative learning

This group is designed for lasting from some moments to one class period. The purpose of this group is to gain the motivation of students during the instructional methods. An informal group is also called an ad–hoc group because it can be made any time during the period or before the starting period on emergency basis. The purpose of this group is to motivate the learners and make learning more effective and productive (Kagan, 1985).

` 2.3.3 Cooperative Base Groups

This group is designed for long-lasting, from one year to many years. The purpose of this group for members is to assist, encourage, facilitate, and value each other (Johnson & Johnson 1994). This group is also called a long-lasting group. It can be made when starting the session for the whole year to communicate, resolve the problems among the group, promote each other for a long time, and encourage each other. It can be called a subclass team as well.

2.4 Cooperative Learning and Mathematics

Mathematics is the mother of all subjects because it integrates medicine, engineering, computer, industry, construction, commerce, transportation, construction field, traffic patterns, and manufacturing in the present day. No field or subject can be completed without mathematics use. Therefore, the understanding and concept of mathematics is highly valued in education. Since the birth of children, mathematics has played important role in every domain of their life. Understanding mathematics is mandatory for secondary-level students to be good citizens and compete at the international

level. It is therefore essential for learners to become sufficient, competent, efficient, and effective in mathematics (Slavin, R. E., 2011). Medical, nuclear science, environmental protection, engineering, computer, and astrology are incomplete without mathematics and mathematical skills. Generally, mathematics is a boring and tough subject in the eyes of students and therefore they have a lack of interest in mathematics. It is, therefore, needed to reconsider traditional mathematics instructional strategies. Motivational and incentive instructional learning methods are required to be implemented for mathematics.

Research studies indicate that cooperative learning strategies are better to understand and learn mathematics as compared to traditional teaching procedures (Johnson & Johnson, 1991). All forms of cooperative learning make students involved in learning and facilitating each other working jointly which creates motivation among students regarding mathematic skills (Slavin, R. E., 2011). In cooperative learning, learners work in a group and support each other like family members. They collaborate, interact confidently and share ideas constructively and wisely. Students reduce their anxiety, depression, confusion, and fear of mathematics through cooperative learning because it is a student-centered learning approach. Everyone gets an equal opportunity to have the knowledge, solve problems, fulfill his target assigned homework in the group, understand mathematics, and communicate his views regarding mathematics. Therefore, the implementation of a formative group of cooperative learning in mathematics may be effective and fruitful.

2.5 Theoretical Bases of Cooperative Learning Strategies

For the last 50 years, cooperative learning has been nurturing, exploring, and getting importance in the domain of education. It is the cause of the lying deep roots of cooperative instructional strategy in learning theories. There is a conceptual base and sound theoretical framework regarding related literature for cooperative learning. Multiple researchers worked on cooperative learning but the pivotal contribution comes from Johnson and Johnson (1999), Slavin (1996), and Vygotsky (1978). There are five broad aspects of cooperative learning as following:

2.5.1 Motivational Perspectives

According to Slavin (1984), an individual gives more outcomes where his interest exists because motivation compels the person to work hard and achieve his goal. Motivation can be increased through group activities by applying cooperative learning strategies. Slavin (1987a), and Johnson and Johnson (1999) both embraced motivational interests of cooperative learning from humanistic and behavioral learnedness theories. Creating a situation in which individual group members achieve goals and each team member is successful. To help team members meet their own goals can be done by motivated members only.

The motivation of teammates and encouragement from the instructor is the reason for willingly doing work. It is the key to the success of the group. Motivational concerns have been adopted by Johnson and Johnson (1999) and Slavin (1987b) for cooperative learning from humanistic and behavioral learning theories. Team members' achievement is directly co-relational with classmates' behavior and response. Motivating other learners help in the achievement of the goals. (Slavin, 1995)

According to Bandura (1965), the other behaviorist concept plays a very important role in getting reinforcement and motivating the individual member to get reinforcement to achieve his or her goal as a member or as a group. By seeing getting rewards and punishment, one can be motivated to get a target. Only intrinsic and internal motivation keep the learner's whole day busy in his work is unrealistic because motivational theorists are of the view that group reward also play important role in cooperative learning methods.

According to Slavin (1995), rewards, recognitions, and certificates can make students exceed as compared to previous achievement rewards. If the student, that helps learners to encourage and motivate one another to get too much as, values the success of the group compared to the competitive and traditional situation.

2.5.2 Social Cohesion Perspectives

According to Johnson, D. W., and Johnson, R. T. (2018) helping, interacting, and sharing with students in the group is known as the social interaction of the learners. These activities increase learning and interest in social interaction. The social interdependence

theory, which determines the way of interacting among students', boosts learning interests. Social interaction develops the quality education outcome for the learners. Therefore, cooperative learning strategies are important for the social interaction that affects the understanding of the students. The cohesiveness of the group's effect on cooperative learning and the success of the learner are strongly dependent upon the group or social interaction among the team members. This perspective is more motivational than cognitive learning. Due to social cohesiveness, students take an interest and start working hard to get their goals. This aspect encourages learners to share their problems without any hesitation and take participation in the presentation and quizzes. This is the way positive interdependence became the main element of cooperative learning. Social cohesiveness is responsible for motivating, encouraging, promoting, and boosting the learners without any incentive.

2.5.3 Cognitive Perspectives

Boston et al. (2015), state that cognition is the activity of the mind in which humans remember, understand, apply, analyze, and synthesize the process of learning. Interaction and facilitation among the students increase their understanding of knowledge or learning. Therefore, cooperative learning develops cognitive learning.

According to cognitive theorists, neither social interaction nor motivational perspective helps the cooperative development methods. Cognitive theorists developed cooperative learning strategies or methods only from a cognitive perspective, and they ignored the social interaction and motivational perspectives.

The perspectives, as argued above, have sound rationale and empirical support for their existence, and use in cooperative learning strategies. However, as for their practical implication is concerned, there is a need for favorable conditions for them. Ahangari, S., and Samadian, Z. (2014) studied on cooperative learning outcomes through exercising on Iranian's EFL pupils' skills of writing. The first two perspectives require inherent and extraneous incentive in the classroom for the learners. As far as cognitive, developmental, and elaboration perspectives are concerned, they require interaction, discussion, communication, and elaboration in the classroom. It is said that if motivation is seen in the children, they enhance their participation, argument, and presentation and improve their

high-level thinking skills.

2.5.4 Developmental Perspectives

The socio-cognitive dissents come into existence when peculiars work successional. This results in cognitive disequilibrium, and therefore, discussion and reasoning start from here. Jean Piaget and Lev Vygotsky ground this. This is the reason that these perspectives refer to Piagetian perspectives as well. According to Johnson and Johnson (1999), the societal product is presented knowledge by Vygotsky's theories. Developmental perspectives, the work of Piaget and Vygotsky assumed that social-brain disagreement occurs and develops cognitive abilities is developed when peculiars work jointly, and simultaneously. It produces the skills of reasoning, communication, and problem solving.

2.5.5 Cognitive Elaboration Perspectives

O'Donnell, A.M., and J. O'Kelly (1994) described elaboration that cognitive restructuring and rehearsing is known as an elaboration of cognition which enhances and develops learnedness. The elaboration role involves the assimilation and accommodation of schema that can be possible using a cooperative learning strategy. Explaining and demonstrating learning material to someone else within the group is the best way of elaboration. Changing existing schema and developing new knowledge through physical activities and storing concrete in the place of abstract or existing knowledge is known as accommodation. Both processes can be better through cooperative learning rather than traditional learning. There is empirical support and sound rationale for all the above five cooperative learning perspectives for their integrity, but for their implications, there is a need for favorable conditions.

The explanation and presentation of the assigned materials are the best processes for learning. Students only learn when they explain the problem solutions in front of their teammates and the presence of their facilitator. Using elaboration to boost the learning ability of students, cooperative learning methods are used. It discovers that children who provided elaborated explanations to others were the best academic achievers, and it made possible by cooperative learning (Webb, & Farivar, 1999). The children who got elaborate

explanations achieved more than those who just work and did not take part in giving presentations. Questions asked to promote the elaboration process.

2.6 Rationale for Cooperative Learning

Cooperative learning is a popular teaching approach that has been extensively studied and found to be effective in promoting student learning outcomes in various academic fields, including mathematics. Here are some references that highlight the rationale for cooperative learning. Cooperative learning encourages active learning among students by promoting group discussion, peer teaching, and collaboration. According to a study by Slavin (2015), cooperative learning has been found to be one of the most effective teaching methods for promoting active learning in the classroom.

Promoting Social and Interpersonal Skills Cooperative learning promotes social and interpersonal skills by providing opportunities for students to work together and communicate effectively with each other. A study by Johnson and Johnson (2014) found that cooperative learning strategies enhance students' social and interpersonal skills, leading to better academic performance and social development.

Self–esteem, social competencies, social development, ego–strength, coping with problems ability, and self-identity are enhanced by using cooperative strategies rather than individualistic and competitive efforts. Multiple achievements and results are attained because of the powerful effect of cooperation compared to individualistic and competitive instructional methods. Therefore, it can be said that the cooperative learning method is a powerful or the most valued tool for the learning of students and instruction for teachers (Azmin, N. H., 2016).

2.7 Methods of Cooperative learning

Since the 1970s, the explorers have been working and observing the applications of cooperative education principles and resultantly multiple methods of this strategy are being implemented until the present day. The most important methods are used as cooperative learning instructional strategies in the following strategies (Azmin, N. H., 2016).

2.7.1 Student Team Learning Methods (STLM)

This method was developed as an instructional strategy at John Hopkins University. The idea of STLM is aimed at emphasizing the application of group success and company goals that it can be possible when all associates are accountable and independent and get equal opportunities and simultaneous interaction while using this method of cooperative learnedness.

2.7.2 Student Team- Achievement Divisions (STAD) Method

This instructional learning method was developed by Salvin (2015), lasting competition between groups of cooperative education. This group was heterogeneous in terms of race, ability, and gender. Learners worked together as a team, but they were responsible for quizzes individually. This method is implemented for a variety of subjects. Students were heterogeneously indulged in competition through race, capability, ethnicity, and sex developed by educationist scholar Slavin. In this method, students learn and discuss through groups. The points depend upon the students' enhancement of the quiz, given to them group. This method is appropriate for multiple subjects (Slavin, 1988). Mathematics is a typical logic science subject, and it needs argumentation for getting insight. For getting learners interested, there are various student-centered learning strategies and there are many methods for implementing CL strategies in the school room but according to Slavin (1995), the pupils' team–success divisions is the best method in mathematical instruction as a cooperative learning strategy. It is not only most appropriate for teaching mathematics problem–solving skills, but also most suitable for social sciences, language usage, mechanics, geography, map skills, and social studies. It is a very simple and general way of instructional method. Due to the simplest cooperative teaching method.

2.7.3. Teams – games – tournaments

This strategy was composed by Slavin (2015) and it is similar to the students' Team – Achievement Divisions (STAD) and is used for the learning of learners in the heterogeneous group by race, age, ability, ethnicity, and gender. It is also known as the TGT group in the classroom. In this method, quizzes replace the game for the learning purposes of the pupils. Students are kept as a team for six weeks in order to play games during the tournaments. He suggested the instructors that there should be marks for games

and tournaments to motivate the learners for participating in physical activities.

The Jigsaw method is a cooperative learning technique developed by Elliot Aronson in 1971. In this method, students are divided into small groups, and each group member is assigned a different piece of information to become an expert on. The group members then come together and share their knowledge to complete a larger task or project. This method promotes individual accountability, as each student is responsible for mastering their assigned information, while also encouraging cooperation and collaboration.

2.7.4 Jigsaw: In 1975 this, method was introduced by the great educational scholar and philosopher Aronson (jigsaw). After some time, the other two versions of this method, Jigsaw II and Jigsaw III came into the market to improve instructional methods of learning. In this method, every group consists of four or five members of learners. Every student in the batch gets a multiple part of the knowledge (Azmin, 2016; Karacop, &Doymus, 2013). Every student must know all the information about the group to be successful. Every member of each group having the same information makes another "Expert Group". In an expert group, all pupils keep the same information. In the expert group, each member discusses and studies the information to teach in the best way in his original group. After discussion, every student returns to his original group and teaches the knowledge to his peers. Single or individual members' performances show the group performance.

Jigsaw III: for the first time, Kagan used this cooperative learning for the instructional classroom. This method was introduced for bilingual classrooms. This group was arranged like one native speaker, one non-native speaker and one member was a bilingual pupil (Karacop, & Doymus, 2013).

Group Investigation: Sharan and Sharan (1990) manifested the above group method for cooperative learning strategy. The target of this method was given to students that how to learn and how to enquire during the learning. In this method, all work was divided among the group members equally. After getting the task, every student started his part of the investigation. Synthesizing and summarizing the performance and then presenting conclusions to the grade was the vision of the group (Ahmad, F., 2010).

Think – Pair – Share: As from the name of this method, the instructor in the classroom makes pairs and questions are given to them for thinking about a reasonable solution. This method is very simple and easy, was developed by Maryland University Professor Frank Lyman. After posing the question, students are instructed to think of a reasonable result and then share the answer with their classmates in front of the classroom and in the presence of the teacher (Azmin, 2016).

Discussion Groups: The discussion group is aimed at discussing the problem in the group with each other and then sharing the responses with the classmates in the presence of the teacher. The important thing in this group is the selection of the group leader where a leader is chosen based on leadership skills, potential, and abilities of the organization rather than only academic performance. It is the leader's responsibility to ensure the involvement of every group member (Slavin, 1995).

Group Project: In the cooperative group project, the purpose is the participation of every group member with full involvement and motivation to be accountable. In this method, everyone is obliged to accomplish his given task and present in front of the class in the presence of his instructor who observes the presentation and takes the mistakes for correction. Every group member is responsible not only for his work but also for the group. There is another way in a group project that if it is needed, the whole group may also present the presentation.

Team Accelerated Instruction (TAI) Method: This instructional learning strategy designed by Slavin and his mates for math, lasting on four elements with heterogeneous capability, getting knowledge from groups to work jointly and accomplish their learning contents, targets, and homework. In this mode of learning, learners work peculiarly or individually in the group, but team members check and facilitate each other's work (Sutherland, T. E., & Wehby, J. H. 2001).

Jigsaw Method: Aronson, E. (2002), first time developed this model of learning. Multiple research studies by the researchers found that there was constructive effect of Jigsaw method (Azmin, 2016), and (Karacop, & Doymus, 2013). Currently, two additional versions; Jigsaw I and Jigsaw II have been introduced and implemented as learning strategies in education. In the Jigsaw model, each group comprises five elements. Every

member of the Jigsaw or Arson's group is given a different task or piece of information regarding learning from the lesson. Every student works in two discrete teams; the original team and the expert team in every period. Pupils leave their original team to make another group in which all students having the same piece of work called the "expert group". In an expert group, members study the same task and decide to present the best information to their original team respondents. At last, learners leave their "expert group" and rejoin their original group and present the task in front of their peers. Their performance is based on their achievements and presentations.

2.8 Drawbacks of Cooperative Learning Strategies in Classroom

Education scholar Parveen et al. (2011), developed research of cooperative learning's impact on the educational success in social sciences of 8th-gradepupils. She faced while studying some hazards of cooperative learning as following.

Free-Rider Effect: According to Slavin, if cooperative learning is without planning developed and some respondents of team exercise given total asks and others are free to do anything and don't take any part so they can be free-riders. It will be their loss and this rider effect is very dangerous for the learners. These learners are called free – rider and it happens when the task is single for solution and presentation.

Diffusion of Responsibility: A diffusion of responsibility situation occurs when some group members are ignored due to being conceived to be less skillful or lack confidence, so their responsibilities are transmitted to another group member. Especially it happens in mathematics when problem-solving activity is continued in the classroom, the students who are good in mathematics take an interest, but others are tired of mathematics and their roles are deliberately shifted to other active members of the group.

Learning a Part of Task Specialization: Multiple research studies by researchers found that the Jigsaw cooperative learning's effect on students was constructive (Azmin, 2016; Karacop, & Doymus, 2013). This happens when only one part of the task is given to one group member, and he becomes an expert in the given task and he doesn't take an interest in the rest of the content. In this way, he becomes unaware of the other parts of the content because every group member gets specialization in the given task as it happens in

the jigsaw and group investigation method. However, these pitfalls may be controlled by using proper cooperative learning strategies by the vigilance instructor.

2.9 Implementation of CL in the Schoolroom

Many methods for implementing CL strategies in the schoolroom was observed but STAD is the best method in mathematical instruction as a cooperative learning strategy. Additionally, a study by Gao and Luo (2019) revealed that the STAD method enhanced students' critical thinking skills in a Chinese college setting.

Similarly, a study by Hosseini et al. (2019) found that the STAD method led to significant improvements in Iranian high school students' learning outcomes in physics. It is not only most appropriate for teaching mathematics problem–solving skills but also most suitable for social sciences, language usage, mechanics, geography, map skills, and social studies. It is a very simple and general way of instructional method. Due to the simplest cooperative teaching method. Overall, the STAD method has shown promise as an effective cooperative learning strategy in various educational contexts. Its emphasis on teamwork, individual accountability, and healthy competition can foster a positive and engaging learning environment that promotes academic success.

Overview: Teams, quizzes, class presentations, team recognition, and improvement scores are the main five components of the student teams – achievement divisions. When this method is used, the five above elements must be, considered by the teacher.

Class Presentations: The teacher must present the material through lecture-discussion, audiovisual and blended methods to inculcate the topic in the mind of the learners. Learners must pay full concentration to the instructor’s learning strategies so that they could be able to answer the quizzes and discussions and give a presentation on their turn.

Teams: In STAD, the group consists of four or five members who are likely to be the presenter of their group like a teacher. Learning, discussion, presenting, and leading is the real function of the team to answer the quizzes. In this team, everyone tries to do his best for his group and make sure the learning environment for his group mates. Correction and enhancing the group knowledge is the main target of the group members.

Accountability for every individual and credit for all group members is the principle of the team. Respect, mutual concern, self–esteem, and tolerance are the social values of the team.

Quizzes: The learners of the group are individually responsible for the quizzes because after one or two presentations and problem-solving drilling of the related topic, they are to answer the quizzes of the other group members and teachers and no one group member can help his group mates during the quizzes session.

Individual improvement scores: Every student can get more scores on the bases of his hard work and motivation and improve his quality of education by solving problem sheets and his new scoring points compare to his previous average scores. The student's past academic and quiz performance are the base average performances.

Team Recognition: On the basis of the students' average exceeding a certain bench, marks, rewards, and certificates can be given to the learners for their motivation regarding mathematics should be boosted.

Preparation: Material for STAD is taken from the textbook and other adapted published sources or the teacher can make it from different sources. The group of students makes a cross-section of the class with heterogeneous sex, race, age, and intellectual.

2.9.1 Schedule of activities

A regular cycle of activities is the main benefit of STAD which are as followings:

Teaching: Presentation in front of the class daily or as per period

Team Study: Daily working on the sheets with the group members to master the content.

Test: daily learners take singular quizzes.

Team Recognition: On daily or cycle bases, students are given certificates and rewards, appreciation in front of the class, and displaying the taken scores on the notice board is the way of team recognition.

2.9.2 Aspects of Cooperative Learning Strategies

On the basis of the following elements, cooperative learning strategies can be differentiated from competitive or individual learning strategies (Johnson & Johnson, 20142014).

Goal: In cooperative learning, the class is broken down into small groups to work together to get a common goal and ensure that all group members do the same.

Level of cooperation: In cooperative learning, everyone in the class has to learn the assigned material and ensure the common level of achievement of all students by level of cooperation.

Interaction Pattern: Students collaborate and debate with each other to promote each other success. Debating, explaining for completing the assigned material, listening to each other's explanations, encouraging others' work, and assessing academic matters is the motive of cooperative learning.

Evaluation of outcome: A benchmark criteria system is used for students' evaluation and assessment. The only academic achievement and learning outcome of individual learners is the main function of cooperative learning.

2.9.3 Aspects of competitive Learning Strategies

Cooperative learning strategies involve students working together in small groups to achieve a common academic goal. However, there are also competitive learning strategies that involve healthy competition among students. In this approach, students compete with each other to achieve academic goals or rewards. Some aspects of competitive learning strategies include motivation, engagement, and accountability.

One example of a competitive learning strategy is the Team Accelerated Instruction (TAI) method, which involves dividing students into teams and competing to answer questions related to academic content. The TAI method has been found to improve students' academic performance in various studies. For instance, a study by Al-Ammari (2016) found that the TAI method improved students' academic achievement in mathematics in a Saudi Arabian middle school.

Another example of competitive learning strategies is the use of games or game-based learning. Games can be designed to involve competition between students or teams, which can motivate students to engage with academic content. A study by Plass et al. (2014) found that game-based learning improved students' learning outcomes in science and mathematics.

While competitive learning strategies can be effective in improving students' academic

performance, it is important to balance them with cooperative learning strategies to promote collaboration and teamwork skills. A study by Slavin et al. (2008) found that a combination of cooperative and competitive learning strategies was more effective in improving students' academic achievement than either approach alone.

In conclusion, competitive learning strategies can be an effective way to motivate and engage students in academic content. However, it is important to balance them with cooperative learning strategies to promote teamwork and collaboration skills. It is also crucial to consider the individual needs and preferences of students when choosing the appropriate learning strategy.

On the bases of the following elements, cooperative learning strategies can be differentiated from competitive and individual learning strategies (Plass, J. L., Homer, B. D., & Hayward, E. O. 2014).

Goal: In competitive and individualistic learning, learners must perform better than other classmates or group members at any cost.

Level of cooperation: In competitive and individualistic learning, learners have to perform better than other classmates or group members at any cost and they do not help and promote others' work. They think only about their work and performance.

Interaction Pattern: Students do not collaborate and debate with each other to promote each other's success. Debating, explaining for completing the assigned material, listening to each other's explanations, encouraging others' work, and assessing academic matters is not the motive of competitive and individualistic learning.

Evaluation of outcome: There isa benchmark criteria system used for students' evaluation and assessment. Academic achievement and learning outcome of individual learners is the main function of competitive and individualistic learning.

2.10 Uniqueness of Cooperative Learning

According to Gull, F., and Shehzad, S. (2015), working on the results of cooperative learning on academic achievement of learners states that CL is not just group learning as some scholars said but it is more than it. By comparison, of cooperative learning and a small group, the value or importance of the cooperative learning method can evaluate.

Details are as under:

2.10.1 Importance of Cooperative group

i. It outcomes in social interaction, and positive interdependence.
ii. It results in individual accountability in which every learner is responsible for assigned work.
iii. Social-skill ability and equal participation are the outcomes of structured cooperative learning.
iv. Face-to-face interaction and feedback are the essences of it.
v. The instructor or teacher always monitors students' behavior.
vi. The teacher observes the pupils' behavior and rectifies it during classroom learning in cooperation.

2.10.2 Importance of Small Group

i. There is no interdependence. Even sometimes, students check work.
ii. In small groups, some students sometimes do all work and the rest of the learners who let them do the work called hitchhikers.
iii. Students' social-skill learning and equal participation are limited in small groups.
iv. There is no need for face-to-face and feedback interaction.
v. The teacher in the classroom does not always monitor students' behavior.
vi. There is no need of observing and rectifying the learners' behavior for long-term purposes.

2.11 Mathematical skills

A Malaysian research study administered the cooperative learning's results by Malaysian mathematical pupils, and they found an improvement in students' achievements (Zakaria et al., 2010). Mathematics is the mother of all subjects because it is used in every subject as a number or digital. Without mathematics manufacturing, transportation, construction, the insurance industry, medical research, government transportation, and construction are not possible. Especially when computer revelation came into existence in the electronic industry, the importance of mathematics was increased. Government traffic plans, policies, census, elections, identity card systems, and air traffic systems can be

implemented without mathematics. The students' career opportunity depends on mathematics concepts and implementation. Mathematical knowledge is highly important for children's future because today's kid is the leader not only of society but also of the country in the coming time. Therefore, the mathematical concept is as crucial as his life because he will have to become the economist, scientist, and calculator of the census for the future generation. For broad career opportunities, children have to become mathematicians and have to get a highly conceptual understanding of mathematics (Gok, T., & Sylay, I., 2010).

According to Johnson, and Johnson (1991), if any subject has to be part of the curriculum, that is mathematics at the lower secondary level: therefore, it should be mandatory for every child to have strong and suitable background regarding mathematics. For productive citizens, mathematics should be well understood by learners. Therefore, math is compulsory at the secondary level for every learner so that nuclear energy, environmental safety, space exploration, and global warming issues should be at least understandable for the children. Not only knowledge but also learning mathematical skills for kids is essential for solving personal problems, society, career, and country. It is because mathematical content should be more applicable in daily life, societal issues, and country. As far as students' fear is concerned regarding mathematics, there is one best way of teaching math, on the bases of considerable evidence, for not only getting better outcomes but also motivating and fostering the learners that are cooperative learning strategies and procedures (Botha, W., 2021). According to Johnson and Johnson, the use of cooperative learning in mathematical skills like cognitive or critical thinking skills, problem-solving skills, and communication skills is very important.

The cooperative learning method creates interest, ensuring motivation not only for the kids but also for teachers, and exploring communication with the group members.

Research studies and their findings of cooperative learning about mathematics performance indicate that the way the performance of learners improves while using the above strategies of instruction because working together, encouraging, supporting, and helping each other, and sharing problems and their solutions are the reasons for the success of the learners. The performance of the students was increased by using cooperative learning in the classroom as an instructional strategy (Tran, & Lewis, 2012) revealed that

the achievements in mathematics of the students are because of their helping behavior and receiving help from others while using the method. CL is a community where learners work together in order to get their assigned goal. Besides mathematical skills, multiple results from the survey of cooperative method in mathematical subject are also observed like improving social interpersonal skills, becoming active in learning rather than passive, fostering self-esteem, and motivation in the children at the elementary level.

2.11.1 Mathematical Problem-Solving Skills

A central place is merited for cooperative learning in math teaching. In mathematics problem solving, everyone becomes isolated, individualistic, and competitive to solve the assigned worksheets and tries to understand collectively. This is the reason for fear of math in students, and that creates math anxiety, tiredness of mathematics, and fear of math. These problems can be resolved or addressed by using small group cooperative learning in several ways (Parveen et al., 2017).

The zone of proximal development is defined by Vygotsky (1978) that the distance between the potential development level (as determined by problem solving in groups with peers) and the actual development level (determined by independent problem-solving skills) known as the Zone of Proximal Development (ZPD). Hartman discussed the bearing of Piaget's concept of "assimilation and accommodation" with cooperative learning. In assimilation, learners can make prediction and confronting misconceptions by activating prior knowledge. They promote cooperative learnedness for firm collaboration with more capable peers as compared to independent problem-solving skills.

Assimilation and accommodation are the concept of Piaget's mental development argued with cooperative learning by Hartman (1997). Incorporation of new knowledge into an existing schema called assimilation; resultantly, misconceptions are created in the mind of the learners. In addition, creating new knowledge or changing the existing schema is called accommodation; resultantly, creativity and invention are exhibited in the mind of the learners. Both processes mean assimilation and accommodation, and a cooperative learning strategy is the best way for the students in order to clear misconceptions and start imagination and problem-solving. In cooperative learning, students interact with teammates, dissents come into existence, and learners try to resolve matters.

Problem-solving skills refer to the ability to identify, analyze, and solve problems effectively. They are an essential set of skills for students to develop, as they are necessary for success in academic and professional settings. In educational settings, problem-solving skills are often developed through various teaching strategies, such as problem-based learning, project-based learning, and cooperative learning.

Research has shown that developing problem-solving skills can have a positive impact on students' academic performance and future success. For example, a study conducted by Hsu et al. (2019) found that problem-based learning improved students' problem-solving skills and academic achievement in a Taiwanese high school. Another study conducted by Mundy et al. (2019) found that project-based learning improved students' problem-solving skills and creativity in an Australian primary school.

Furthermore, the development of problem-solving skills has been identified as a key component of 21st-century learning. According to the Partnership for 21st Century Learning (P21), problem-solving skills are one of the four critical thinking and problem-solving skills necessary for success in the 21st century workforce (P21, n.d.).

i. When students interact in cooperative learning, disequilibrium and dissents come into existence, which can lead to better understanding.
ii. Misconceptions are dropped by the motivations of peculiar and create more powerful and concrete concepts.
iii. Critical thinking is encouraged by the teammates in the cooperative learning strategies.
iv. Due to constructive controversy, the higher-order cognitive process of the learners is promoted by cooperative learnedness.
v. Unavoidably encouraging students to vocalize ideas enhance the performance of the learners.

The developmental perspectives are related closely to practical cooperative learning methods as argued before in the multiple topics. Interaction, arguing, and disagreements with teammates describe the confidence and potential of learning in the learners in the group. The developmental perspective enhances the problem-solving skills of the learners in class through cooperative learning strategies. The Zone of Proximal

Development enhances an individual's problem-solving skills through cooperative learning (Vygotsky, 1978).

2.11.2 Mathematical Critical Thinking skills

This perspective, according to Calkins et al. (2020), studies that by applying reflection of peer-assisted in mathematics to cultivate brainstorming and exchanging ideas skills only belongs to cognitive rather than motivational and social interaction. This perspective describes learning and getting knowledge. In this perspective, the learner starts with thinking and goes to critical thinking, and then creativity. It increases students' mental process of interaction and information that develops the cognitive aspect.

Critical thinking is a self-regulatory judgment that results in understanding, applying, analyzing, synthesizing, and evaluating which depends upon the above reflection Camacho-Minuche et al. (2021). Critical thinking skills refer to the ability to analyze, evaluate, and synthesize information to form judgments and make decisions. It involves questioning assumptions, considering different perspectives, and using logic and reasoning to arrive at conclusions. Research has shown that critical thinking skills are crucial for academic success and are highly valued in the workplace. For instance, a study conducted by Ennis (2011) found that teaching critical thinking skills in the classroom improved students' overall academic performance. Another study conducted by PISA (2018) found that critical thinking skills were essential for success in the 21st century workforce.

There are various strategies that can be used to develop critical thinking skills, including cooperative learning, problem-based learning, and inquiry-based learning. These strategies encourage students actively engage with the material and to think critically about the information is presented.

Moreover, research has also shown that critical thinking skills can be improved through explicit instruction and practice. A study conducted by Abrami et al. (2008) found that explicit instruction in critical thinking skills improved students' critical thinking abilities.

2.11.3 Mathematical Communication Skills

In 2018, Vietnam's Ministry of Education and Training regarding mathematical communication skills stated that autonomy, self-study, and seven other professional

competencies are three general competencies used at each level of the education system, in order to improve the skills of students. These competencies used to groom ability to mathematical analytical skills, brainstorming skills, and especially exchanging ideas skills (ability to use text, terms, numbers, tables, graphs, diagrams, and symbols to share and interact with ideas). The foundation of mathematical communication skills depends upon mathematical knowledge, attitude, and skills, while interacting with students and their ideas more effectively, completely, and accurately with their instructor, small group work, and presentation (Karim, A. A. 2012). There are very important elements of communication skills for the improvement of sharing, positivity, and collaboration in mathematical language as follows:

i. It is highly important to remember and recognize the mathematical knowledge and letters and symbols for mathematical verbal (speaking and writing) communication.
ii. The second and most important element of mathematical communication is the exchange of ideas, contents, and solutions with each other and through writing or orals by solving mathematical problems.
iii. The third element of mathematical communication is to combine charts, terms, letters, symbols, graphs, and diagrams with daily-life language through interacting with each other or paper sheets.
iv. The fourth and most important element of mathematical communication is demonstrating self–assurance while interacting (discussing and debating), presenting, and evaluating mathematical concepts and ideas.

According to the Steering Committee of the National Engineering Education of Malaysia (2006), it is recommended that there are multiple competencies that are required in order to study as important general skills; critical thinking skills, creativity, problem-solving skills, analysis, and communication. These skills needed to investigate, and their relations are studied. Especially generic competence; problem-solving skills, critical thinking skills, and communication were considered mathematical skills as the goal of higher education and engineering education.

The ability to communicate mathematical ideas and problem-solving, and explain or present the situation with terms, charts, graphs, symbols, tables, and diagrams to others

is called mathematical communication (Depdiknas, 2007). According to Sabariah et al. (2007) and Rehman, R. A. (2009), it can empower the pupils in thinking, solving problems, enhancing the understanding of learners acquiring knowledge, applying the mathematical ideas, and sharing those ideas and interacting with the co-learners and their instructors. Therefore, there is a need of developing some innovative and creative instructing approaches. For the presentation of problem solutions in a written or spoken way, there is a need of improving students' mathematical communication skills. When students change their minds with ideas, and cognizance then they get achievements in their studies and these accomplishments depict their cognitive behavior. Mathematics is the foundation for all education in every part of the world, which shows the value of learning it. Therefore, it can be said that if one learner gets excellence in mathematics, he can excel in other domains of education as well. Cognizing mathematics is the competence of the other subject of the curriculum and can construct and develop society genuinely (Johnson & Johnson, 2008).

Education varies continuously for children: focusing on traditional strategies to modern strategies, and lecture methods to activity methods. Therefore, the internationally immerging teaching strategy among researchers and getting value in education, especially social science studies is a cooperative learning strategy. According to Roselainy et al. (2010), there are three important aspects of education including mathematical thinking phenomenon, general or generic skills, and knowledge development of mathematics. They emphasized the students about these aspects to improve their problem-solving skills, critical thinking skills, building up their knowledge, and general skills like self–directed education, communication, and teamwork.

Mathematical problems and ideas could have consolidated, organized, and explored through communication skills by the learners. Therefore, there is a need of interacting (debating and arguing) with the instructor and co-learners to improve the best communication and understanding regarding math content. Teachers should provide feedback on interacting answers to become fruitful for them and enhance their communication skills. The ability of students by applying mathematics as a tool while communicating and deliver the mathematical contents as a message (mathematical language) is known as mathematical communication skills (NCTM, 2000). Therefore, it is mandatory for students to improve their mathematical communication skills because

without good communication, the development of math may be limited, and the teaching-learning process of mathematics can be hampered in the absence of effective communication. The use of effective mathematical language (letters, symbols, terms, charts, graphs, diagrams, tables) as communication skills can improve the sharing and presenting of the ideas of math while the learning process takes place. Communication skills in mathematics are to be in the form of written, oral, or visual mediums. Effective communication can interpret, represent, and describe the relationship between ideas and terms, and problems. It is the skill to present math ideas through writing, describing, demonstrating, and speaking using mathematical tools. The communication skills of the pupils can be improved by cooperative learning. In this teaching strategy, students get more occasions for discussion, presentation, communication, and sharing ideas in order to improve their mathematical communication skills. Synthesis of lower secondary, secondary, and primary mathematics and reading programs that structured on a cooperative learning approach is the best evidence in education. The use of innovative curricula, textbooks, and technology in mathematics and reading is a sign of the importance of cooperative learning (Calkins et al., 2020).

2.12 Research in Cooperative Learning

Cooperative learning study has especially been starting since 2000 because of its effective implementation in instructional education. Researchers found excessive empirical favor in applying cooperative learning in multiple fields of knowledge like social relations, health, and especially in education. Most research in cooperative learning have been done in the UK, USA, Germany, Italy, Australia, Spain, Israel, Japan, and other developing countries in Asia and Africa.

2.12.1 Foreign Research of Cooperative Learning

Capar and Tarim (2015) examined the study on the impact of CL and concluded that it is a good learning strategy in which learners work jointly, help each other, and take interest in the given task. Dekor and Agbornu (2020) examined a research study on, “Effect of cooperative learning on learners’ achievements in Mathematic at middle level in the USA”. Findings of the above study were supported in the favor of the implementation of a cooperative teaching strategy. It boosted the performance of the students learning

mathematics and developed their interest in mathematical understanding for both male and female learners.

Sulisworo et al. (2016) examined a study on cooperative learning and found that learning in a cooperative style with other students is better to achieve educational objectives. They developed a study on cooperative learning impact and concluded that cooperative learnedness instructional strategy consisting of small groups is an effective teaching and learning strategy in which students become fruitful with various levels of ability. This instructional strategy can be implemented in different fields like mathematics, language, geography, and science.

Botha, W. (2021), researched cooperative learning effect on learners' skills and behavior, and he revealed the merits and demerits of applying a cooperative learning strategy in the classroom. According to Botha, W. (2021), in small group teaching, the Jigsaw method of cooperative learning technique is a good way of learning for learners. Jigsaw is the simplest type of teaching strategy in which groups consisting of four or five representative, and learners are dependent on each other to be successful. In addition, an instructor assigns a topic then the topic is broken down into subtopics to assign every group independently. Jigsaw cooperative learning technique is extremely useful and beneficial for the learners who establish the learning strategy. It will be an exciting and motivating strategy for children in the classroom as well as for the teacher.

Multiple research studies found that Jigsaw cooperative learning's outcomes on pupils' learning were productive. Azmin, (2016); Karacop and Doymus, (2013) revealed that using cooperative learning involvement and activeness increased the positive interdependence of students on each other and also the teacher's performance towards pupils improves. Fixed and allocated time differently the assigned for work and they are bound to do their assigned task; they can be free riders, and time is useful for them. This is also beneficial for the management of the classroom is enhanced using cooperative learning. According to them, some techniques of cooperative learning favor collaborative learning strategies and do not relax the students during the learning and learners become confused and feel stressed. Most of the learners are deprived of cognitive learning and lack expertise.

Conclusion: Buchs et al. (2017) favored cooperative learning by saying that it is a well–established technique for learning and facilitation. Problems of assigned work solved by the students; cognitive thinking skills and communication skills of the students are improved. Physical life experiences and critical thinking processes promoted in jigsaw strategy. Cooperative learning strategy using the Jigsaw technique is extremely helpful for the learners because this gives them opportunities for leadership duties. Students are allowed to take responsibility and are free to share their arguments in front of the other group members and in the presence of their teacher. This characteristic of sharing their assigned work creates opportunities for the learners to become the leaders of the future. It maximizes and maintains good working relationships among the members and focuses on the goal achievement. It happens when the same group members assist, encourage, and promotes each other.

Wang et al. (2017) conducted a research study and their findings were favorable to cooperative learning and revealed that children enjoyed clear and structured guidance in which their assigned task is clear, and this can happen through collaboration cum cooperative learning strategies. Heinonen et al. (2020), described that educational goals should be ensured in a structured path of learning and keeping guidance over the students at all times during learning in the classroom.

Heimbuch et al. (2018), revealed that children's engagement and learning in terms of studying is more beneficial and fruitful schooling strategy at the secondary level. Cooperative learning, according to Popov et al. (2019), is more beneficial in terms of bringing cultural gaps among learners where children are permitted in order to express and share different opinions, various beliefs, and ideas, and entertain their leadership through discussion among the group and in front of the class. Investigating how the application of cooperative learning techniques may enhance learning outcomes, many research studies have been conducted for this purpose for many years. Lin (2020), and Lee (2018), supported small group learning activities in terms of learning in the classroom. According to them, the critical thinking skills of the children may be increased by using small group techniques of collaborative group discussion methods. There is a lack of literature on effective techniques for guiding meaningful arguments and cultivating students' critical thinking skills at the secondary level rather than the university level (Saputra et al., 2019).

According to great renowned education scholars, Deutsch's (1949) Social Interdependence Theory, Bandura's (1965) Social Learnedness Theory, and Vygotsky's (1978) Cognitive Developmental Theory of sociocultural, critical thinking is essential for social learning environment so that kids start problem-solving, then reach imagination with involvement in the class. Critical insight is extremely important for the development of new concepts and essential to how motivational aspects cultivate learners' engagement with the group mates of their class. The foundation for insight into how to promote interdependence among the group mates provided by cooperative learning strategies.

2.12.2. Cooperative Learning Research in Pakistan

In Pakistan Munawar, S. &Chaudhary, A. H. (2019), determined in their study that that it was instituted in the subject English writing implementing cooperative learning that experimental group results were higher than control group results of seventh-grade school students. This research study was done on science subjects on class nine pupils who were 40 in number. An experimental model of pre-test and a post-test was used for scoring. Duration of the study consisted of two weeks and highly positive effect of the instructional strategy were found on science students' achievements. Motivation, positive attitude, the feeling of personal importance, and acceptance of heterogeneity of learners amplified in multiple fields of studies by the implementation of cooperative learning. During cooperative learning, students collaborate, share, and accept each other in the group. It examined the impacts of cooperative learnedness on ninth-class learners' mathematics understanding, distributing students into small groups. Findings were in the favor of cooperative learning in terms of amplifying acceptance of mixed ethnicity, positive attitude, and social interaction.

In 2004, Muhammad Iqbal conducted a study on outcome of cooperative learning in mathematics achievement at the secondary level in Pakistan. He used the Students Team Achievement Division method in his experimental study, which lasted for six weeks. The results of his study showed that the implementation of a cooperative teaching-learning strategy was more effective than the traditional instructional method. He concluded that cooperative teaching is a highly effective teaching technique.

In a 2017 study, Parveen et al. investigated the impact of cooperative learning on

academic achievement and student perceptions of the technique in Pakistan. The results indicated that the treatment had a positive effect on students' academic performance.

Similarly, Ahangari and Samadian (2014) conducted research in Iran on the effectiveness of cooperative learning in improving students' writing abilities, concluding that this instructional approach was superior to other methods. More than 1000 Studies and meta-analyses regarding the implementation and effectiveness of cooperative learning strategy conducted mostly in western countries and very few in Asian countries in the last 100 years. In our country, mathematics phobia exists excessively. This trend provokes us to study about implementation and effectiveness of the strategy. This is the study “Effect of cooperative learning on students’ mathematical skills”.

Cooperative learning is a teaching approach that has gained significant attention in Pakistan over the past few decades. Several research studies conducted to explore the effectiveness of cooperative learning in different educational settings. Here are a few examples of national cooperative learning research conducted in Pakistan:

It was worked on “Effectiveness of Cooperative Learning on Academic Achievement on Secondary School Students in Pakistan" (S. Iqbal & M. Akhtar 2013). This study investigated the impact of cooperative learning on the academic achievement of secondary school students in Pakistan. The results showed that cooperative learning had a positive impact on students' academic achievement.

M. Aslam and M. Ahsan (2012) researched it on “Impact of Cooperative Learning on Academic Achievement of Undergraduate Students”. This research examined the effectiveness of cooperative learning on the academic achievement of undergraduate students in Pakistan. The study found that cooperative learning had a significant positive effect on students' academic achievement.

It was studied on "Cooperative Learning in Higher Education: A Study of Pakistani Students' Perspectives" by A. Rehman and N. Malik (2014). This study explored Pakistani students' perceptions of cooperative learning in higher education. The results showed that students had a positive attitude towards cooperative learning and believed that it enhanced their learning experience.

It was worked on "Effectiveness of Cooperative Learning in Mathematics: A Study of Pakistani Secondary School Students" by S. Hussain, S. Khatoon, and S. Fatima (2017). This research investigated the impact of cooperative learning on mathematics achievement of secondary school students in Pakistan. The results showed that cooperative learning had a significant positive effect on students' mathematics achievement.

These are just a few examples of national cooperative learning research conducted in Pakistan. There are many other studies available in academic journals and publications that explore the effectiveness of cooperative learning in different educational contexts in Pakistan.

2.13 Summary of the review

The research studies, as mentioned throughout this literature, emphasize the need for critical thinking skills, problem-solving skills, and communication skills of pupils in terms of learning for the 21st century. The issues of trustworthiness and untrustworthiness, bias and unbiased, and tolerance and intolerance among children are prominent in their teenage years. For career opportunities and requirements for jobs in which analysis, reasoning, problem-solving, good communication, and tolerance is extremely required for the successful person. Employers always seek a candidate who possesses such skills, and these skills may be created through collaboration in cooperative learning strategies. It is also informed in the literature that the assigned task designed in a problematic way in which students are indirectly and willingly compelled to accomplish their work as an accepted challenge. A well-designed structure always promotes peer discourse, assures greater responsibility, shares problems, explores the content and encourages the expression of various ideas in the presence of the class teacher, and engages the learners in their given task in the classroom. In contrast, a lack of design destroys all potential and can create ambiguity and disengagement among the learners. Deutsch (1949) favored cooperative learning strategies for kids' learning in his famous social interdependence theory, but as well as he, made students' achievement conditional that pupils should be actively engaged and efforts of each group member should be involved consciously. The sense of responsibility in every group member creates a sense of individual accountability through

social interdepended theory.

Gok and Sylay (2010) revealed in their research studies that for the betterment, of problem-solving skills, there is the best way of learning technique is cooperative learning. It is a useful teaching strategy for complex skills and complex problems to solve by the cooperative learning strategy within groups in the classroom. As a way of structuring goals, which determines how the interaction of individuals turns into the groups' achievement. Johnson (2003) explained social interdependence theory. Appropriate use of social skills, positive interdependence, personal responsibility that turns into promoting interaction, and singular accountability are the elements of the above-said theory of Johnson. The purpose of this theory is to make great efforts in-group processing in order to get more achievement. It promotes a cooperative learning process rather than competitive and individualistic learning. Positive interdependence leads to individual accountability that encourages group interaction to get more outcomes. Multiple research studies on cooperative learning techniques not only in foreign countries but also in Asian states have done over the last 100 years. Meta–analysis of cooperative learning methods by various researchers and scholars revealed the importance of the strategy that encourages us to implement and experiment with collaborative or cooperative group learning in the classrooms. Besides the worth of cooperative learning, our students' fear of mathematics and teachers' lack of interest also compelled us to assess the consequences of research studies on it. This research study is, managed in order to determine cooperative learning effect on students' mathematical skills at middle level.

CHAPTER-3

RESEARCH METHODOLOGY

The research study based on a quantitative approach with the application of the experimental method. The cooperative learning method used as a treatment for measuring the students' mathematical skills, problem-solving skills, critical thinking skills, and communication skills. Cook, and Campbell (1979), first time introduced a quasi-experimental research study for the first time. There are three important quasi-experimental designs: non-equivalent group design, pre-test, and post-test design, and interrupted time series design (Fraenkel et al., 2012). This research design is a combination of pre-test, post-test design and Quasi-experimental time-series design This study was quantitative with a quasi-experimental time-series design having six treatment waves as described by Edmonds, & Kennedy (2016).

The purpose of the pilot test was to analyze the reliability of the developed test and to assess cooperative learning's results on pupils' mathematical skills at eighth class students (Kustati, M., &Yuhardi. 2014). The doctoral committee and mathematics subject specialists assessed a test of the content validity. Eight multiple choice and two written text test items developed for every test of selected four topics-- Sets, Algebra, Basic Statistics, and Geometry.

A mathematics teacher from Islamabad Model School for Boys Herno Thanda Pani Nilor Sector Islamabad was selected for the implementation of cooperative learning. Before starting the experiment, the mathematics teacher was provided 4-day training in order to implement cooperative learning in the classroom.

The trainer was a Ph.D. doctor in Mathematics, worked as resource person and master trainer to teach at elementary and secondary levels at the forms of FDE, NISTE, Edlink, CIDA, British Council, FBISE, and NDMA. Author of mathematics books for classes I, II, V, VI, VII, VIII, IX, and X. He is a principal at present time.

This chapter includes the nature of the study and encloses the population, sample, sampling technique, instrument development, validity, pilot study, and reliability. The nature of the study includes further two parts, research approach, and study design.

A quantitative approach using the experimental method was applied in the study. The quantitative approach gives measurement numerically which is accepted more accurate approach as compared to other approaches. This method is the best in experimental studies in the eyes of researchers. The four topics were taught during the experiment period and before starting any unit a pre-test was used to administer to measure the learners' mathematical skills. For every test development, the consensus of experts, doctors, and class teacher were valued. The whole experiment was done in 6 weeks at the elementary level in a public school in Nilore, an area in the ambit of the Federal Directorate of Education Islamabad, Pakistan.

3.1 Research Design

This study was quantitative with a quasi-experimental time-series design having six treatment waves as described by (Edmonds, & Kennedy, 2016). The graphical presentation of this design is as under:

Table 3.1

Single Exp.	O1-X1-O2	O3-X2-O4	O5-X3-O6	O7-X4-O8	O9-X5-O10

Where Single Exp. = Single Experimental group, X = Treatment, O= Observation

O_1, O_3, O_5, O_7, O_9 = Pre – test Observations respectively, X_1, X_2, X_3, X_4, X_5 = treatments respectively, and O_2, O_4, O_6, O_8, O_{10} = Post– test Observations respectively.

This research design is a combination of pre-test, post-test design, and Quasi-experimental time-series design presented by Fraenkel et al. (2012). It allowed the researcher to select the subjects for only a single experimental group without random sampling or disturbing the prevailing classroom setup. Because the researcher had no freedom to select students randomly and form groups for the experiment that are required for a true experimental design in the institution selected for study, the single-group quasi-experimental time-series design was considered the most suitable for the present study. Furthermore, this research design has the capability to control various internal-validity'

risks of the research. For example, testing, repeatedly, allows control over the maturation effect and it minimizes the historical effect as well. The use of the same subjects in all comparisons provides over the selection bias of participants and other participants-related factors (Tuckman, & Harper, 2012; Edmonds & Kennedy, 2016).

If X1, X2, X3, etc. are the same treatments, then there may be repeated presentations that may undermine the external validity of the design. However, the researcher used different contents to teach at every repeated treatment (Edmond, & Kennedy, 2016). The combined strengths of quasi-experimental design and time-series design made this study relatively more suitable.

3.1.1 Procedure

This research study administrated over a six-week time. This duration of six weeks used to give the pre-test, treatment session, and post-test process. The four areas of mathematics, set, algebra, basic statistics, and geometry, were converted into six parts for the research study; set-1, set-2, algebra-1, algebra-2, basic statistics, and geometry were the part 1, part 2, part 3, part 4, part-5, and part-6 respectively. In this experimental study, the first week served as a pilot test for part 1 of the students' mathematical skills. The second, third, fourth, fifth, and sixth weeks used as pre-tests, treatments, and post-tests for parts 2, 3, 4, 5, and 6, respectively. Overall, the students underwent six rounds of pre-tests, treatments, and post-tests to evaluate their mathematical skills.

3.1.2 Treatment

By following these steps, the cooperative learning method can effectively have applied in a classroom setting, promoting active learning, and enhancing student engagement. The cooperative learning treatment initiated from the second until the fifth week and included 24 periods where every period consisted of 40 minutes, which was the duration of CL application.

Teacher trained by a doctor who is one of mathematic authors of National Book Foundation at elementary level and certificate and module of training are attached. The teacher was using Gagne’s lesson plan during the instructional strategy. In the instructional phase of Gagne’s lesson plane problem solving skills, critical thinking skills, and communication skills questions existing: Q-1, Q-2, and Q-3 respectively.

The pre-test taken and the next four days assigned to the treatment and Saturday assigned for the post-test. Learners guided to complete the test in 40 minutes for the test examination. The student Teams Achievement Division (STAD) method used in the six groups of the 30-student classroom to measure the learner's mathematical skills during the treatment. There were six groups formed out of 30 students, each team consisted of five students. Each group had one high achiever, 3 moderators and one low achiever, selected through a random selecting technique. Learners familiarized with their roles and responsibilities in the group and this implemented the STAD method. Every member of the group assigned a role and responsibility as per session and that changed in every session in order to play all the roles and responsibilities during the CL session.

3.2 Population

Table 3.2

Summary of population of the study

Sr. #	School	8th-grade students
1	IMSB Nilore	45
2	IMSB Herno Thanda Pani	30
3	IMSG Nilore	101
4	IMSG Herno Thanda Pani	104
	Total	280

There were four public schools at the elementary level in the Nilore area. The Population of this research study constituted four public elementary schools the total strength was two hundred and eighty (280) students of grade-8 level, Nilore area, under FDE, Islamabad in 2022. The sources of the population were the principals, and administration related officials of the related schools.

3.3 Sample Size

Table 3.3

Sample size of the study

Ser. #	School	8th-grade students
1	IMSB Herno Thanda Pani	30

The sample size of the study was thirty students (30) taken fromgrade-8 from a public school at the elementary level in Nilore area Islamabad. The first test score of set – 1 of grade – 8 taken for a pilot study to measure the test reliability. Data assembled from one class that comprised 30 students (males). These learners were of the mixed ability of intermediate and low proficiency levels in mathematics.

3.4 Sampling Technique

The simple random sampling technique used for collecting data for the research study from public schools in the Nilore area, Islamabad. There were four public schools in the area of Nilore and research was experimental. Therefore, the researcher used random sampling technique for selecting one from the four schools at first stage. The selected section was grade – 8 students for collecting the sample data.

3.5 Instrument Development

Sets, Algebra, Geometry, and Basic Statics were the selected topics for the research study. Gagne's lesson plan used for instructional strategies for the study. Ten items were prepared from each topic for tests. There were three parts to every test: part I was made for measuring mathematical problem skills, part II was made for measuring mathematical critical thinking skills; part III was made for measuring mathematical communication skills. The research instruments were the pre-test and post-test skills applied in this study in order to measure the mathematical skills of grade – 8 students based on the syllabus of the federal textbook board Islamabad published by the National Book Foundation. The students were familiar with these tests, as they had gone through the previous class grade 8. The topics evaluated in the tests were sets, algebra, geometry, and basic statistics.

Self-developed Tests: five tests (sets, algebra-1, algebra-2, basic statics, geometry). **Every test**: Three parts (Part 1, part -2, Part 3), and with ten items each. Part 1: Mathematical problem-solving skills; Part -2: Mathematical critical thinking skills; Part -3: Mathematical communication skills. (Muzio, et. al., 2001, Bloom et al., 1956, p. 2,)

Table 3.4

Table of specification for instrument development

Content		**Objectives**						**Total**
Area	Weight	Knowledge 10%	Comprehension 20 %	Application 10%	Analysis 30%	Synthesis 10%	Evolution 20%	Total 100 %
Problem-solving skills	33.3 %		Q-1	Q-2	Q-3		Q-4	4
Critical thinking skills	33.3 %	Q-1	Q-2	-	Q-3		Q-4	4
Communication skills	33.3 %	-		-	Q-1	Q-2		2
Total	100%	1	2	1	3	1	2	10

(Nairn, A., & Dew, A. 2007)

3.6 Validity

Two experts having Ph.D. (one from education and one from mathematics) checked validity of the instrument.

3.7 Pilot Study

After designing the test instrument, a pilot study constructed to improve the validity and reliability. For the pilot study, the experts related to the same public school situated in Nilore, Islamabad, took the assessment of the research study (cooperative learning) at

grade-8 level students. The supervisor before the starting experiment of grade-8 students judged the demonstration of three lessons.

3.8 Reliability

The Cronbach alpha reliability test is an important tool for measuring the internal consistency of a test or questionnaire and used to assess the reliability of research findings. It was 0.82 and its procedure is as following: **Cronbach alpha reliability test**: To perform the Cronbach alpha reliability test in SPSS.

Table 3.5 *Reliability of the study*

Sr.#	Scale	Items	Cronbach's Alpha Reliability
1	Mathematical problem-solving skills	04	0.90
2	Mathematical problem-solving skills	04	0.71
3	Mathematical problem-solving skills	02	0.85
	total	10	0.82

3.9 Experimental Time

The experiment of the research study started on January 15, 2022 to February 22, 2022. The study administered with one experimental group during the months of January and February in the academic session 2021- 2022.

3.10 Variables

There were two types of variables used in the study: the first type was the Independent variable where cooperative learning used as an independent variable, and the second type was the dependent variable where mathematical skills used as dependent variables.

3.11 Data Collection

The data collected through tests by experimental method study consisting of six weeks from grade-8 level students consisting of thirty students from a public school Nilore area, Islamabad.

3.12 Data Analysis

Paired sample t-test was used for the mean score comparison. The pre-test and the post-test used for data analysis. Interpretations of the collected data carried out through SPSS. The pre-test and post-test techniques, by applying paired sample t-test, used to measure the significant difference in students' mathematical skills. The components measured were communication, brainstorming, and analytical skills before and after the application of treatment in the 8^{th}-grade classroom. Students' mathematical skill was the dependent variable and CL instructional strategy was the independent variable in the mathematical classroom. SPSS, the statistical software applied for the inferential statistics and descriptive statistics used to analyze pre–test, and posttest by calculating the standard deviation, mean, and percentage of each component. A bar graph also used for comparing the means of all three mathematical skill components. The data was collected at two different times, pre-test and post-test data, by using paired sampled–test.

3.13 Ethical Consideration

Before the research study experiment, a consent form taken from the parents of the grade – 8 students, the mathematics teacher, and the Head of the institution. In the ethical form, the name of the scholar, the experimental research study, and the purpose of the study as mathematical skills of grade – eight students and types of mathematical skills clearly mentioned. The duration of the experimental study consisted of six weeks. The consent form also attached as an appendix – D.

CHAPTER – 4

DATA ANALYSIS AND INTERPRETATION

The research administered in order to assess the cooperative learning method at the grade – 8 level. The quasi-test research study was demeanor to assess "The Effect of Cooperative Learning on Students' Mathematical Skills at Elementary School Level: A Quasi-Experimental Study". In the experimental study, pretest and posttest designs was introduced. The research study was managed in Islamabad Model School for Boys (I-X) Herno Thanda Pani FA Nilore, Islamabad under the ambit of FDE. The sample of this study was thirty students (30) of the eighth class, considered as one group, and this group was further divided into five subgroups. The data was collected before and after treatment.

Table 4.1

*Comparison between pre-test and post-test scores of Students on **Sets***

	Pre-test		Post-test		Paired sample t-test	
	M	*SD*	*M*	*SD*	*t*	*p*
Set-2 part-1	5.03	2.82	3.42	2.38	2.84	0.008
Set-2 part-2	4.52	2.36	4.90	2.47	-0.95	0.351
Set-2 part-3	5.03	2.53	4.80	2.85	0.47	0.644
Set-2 Total	11.71	5.96	14.35	6.81	-2.096	0.045

Table 4.1 presents the results of the pre-test and post-test applied to the different parts of the sets. There is significant difference between the pre-test and post-test results of Set-2 part 1 (t=2.84, p = .008). The mean scores of the pre-test (Mean=5.03, SD=2.82) are higher than the mean scores of the post-test ((Mean=3.42, SD=3.42). There is no significant difference between the pre-test and post-test results of Set-2 part 2.

Fig. 4.1

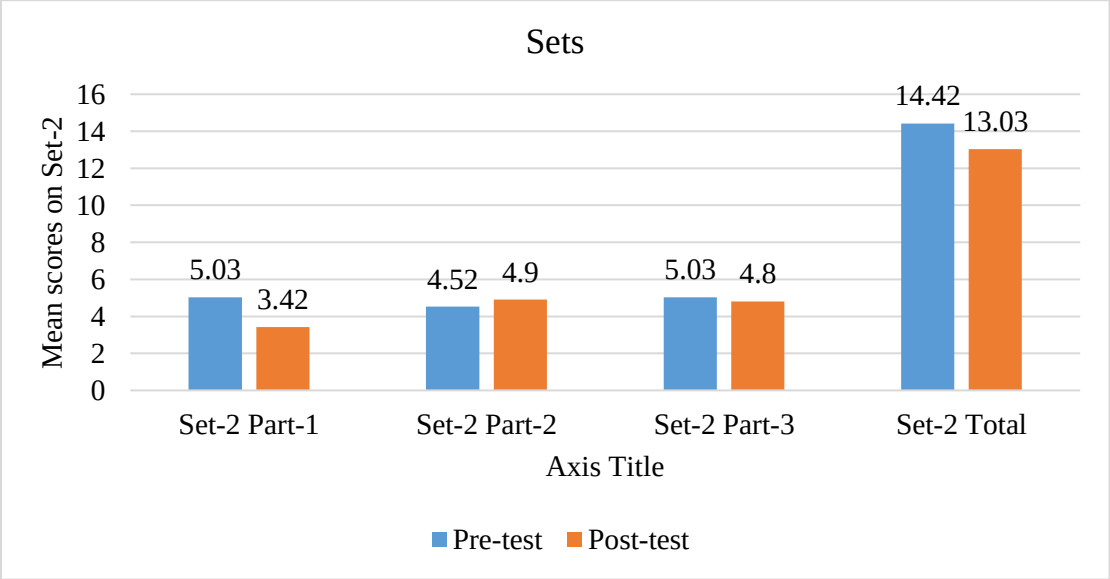


*Figure 4.1*Comparison between pre-test and post-test scores of Students' mathematical skills on **Sets**

Table 4.2

Comparison between pre-test and post-test scores of Students in Algebra-1

	Pre-test		Post-test		Paired sample t-test	
	M	*SD*	*M*	*SD*	*t*	*P*
Algebra-1 part-1	6.39	2.22	6.19	2.65	0.34	0.738
Algebra -1 part-2	6.52	2.36	6.00	2.53	1.28	0.211
Algebra -1 part-3	4.58	2.28	5.84	2.37	-2.67	0.012
Algebra -1 Total	17.48	5.51	18.03	6.60	-0.52	0.609

Table 4.2 presents the results of the pre-test and post-test applied to the different parts of Algebra 1. There is no significant difference between the pre-test and post-test results of Algebra-1 Part 1. There is no significant difference between the pre-test and post-test results of Algebra-1 Part 2. There is a significant difference between the pre-test (*M*= 4.58, *SD* = 2.28) and post-test (*M*= 5.84, *SD* = 2.37) results of Algebra-1 Part 3 (*t* =-2.67, *p* =. 012). The mean score of the post-test is greater than the pre-test.

Fig. 4.2

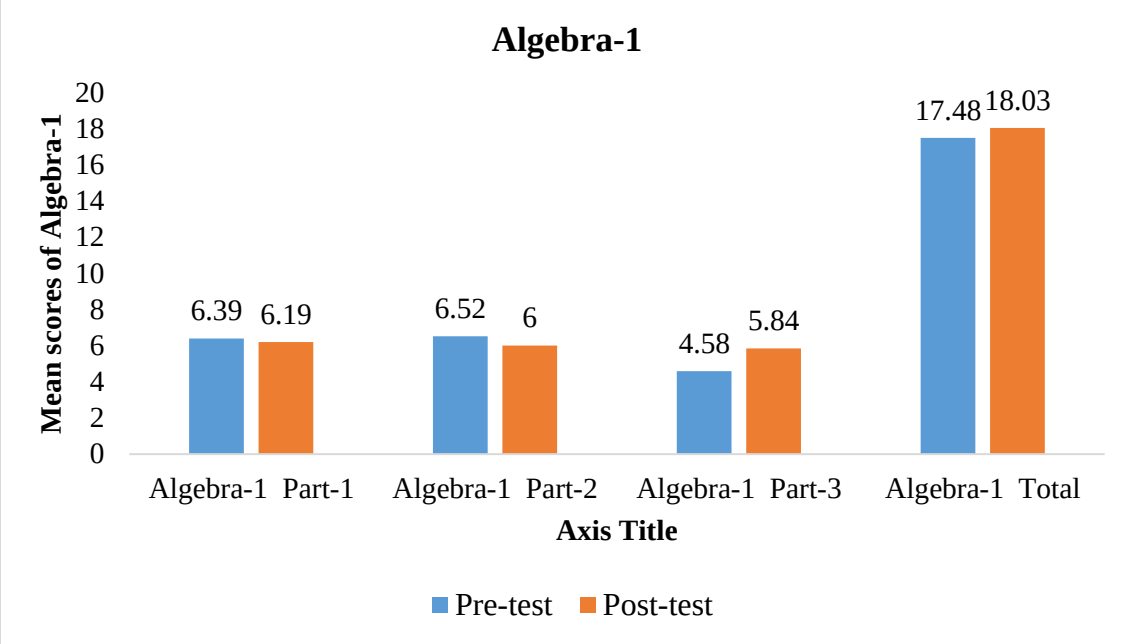


Figure 4.2 Comparison between pre-test and post-test scores of Students inAlgebra-1

Table 4.3

Comparison between pre-test and post-test scores of Students in Algebra-2

	Pre-test		Post-test		Paired sample t-test	
	M	*SD*	*M*	*SD*	*t*	*P*
Algebra -2 part-1	5.07	2.39	5.20	2.61	-0.235	0.816
Algebra-2 part-2	2.60	2.04	4.13	2.29	-2.986	0.006
Algebra -2 part-3	4.43	2.64	4.97	2.85	-1.038	0.308
Algebra -2 Total	12.10	5.64	14.30	6.92	-1.802	0.082

Table 4.3 presents the results of the pre-test and post-test applied to the different parts of Algebra 2. There is no significant difference between the pre-test and post-test results of Algebra-2 Part 1. There is a significant difference between the pre-test and post-test results of Algebra-2 Part 2 ($p = 0.006$, t=2.986, SD=2.29).

Fig. 4.3

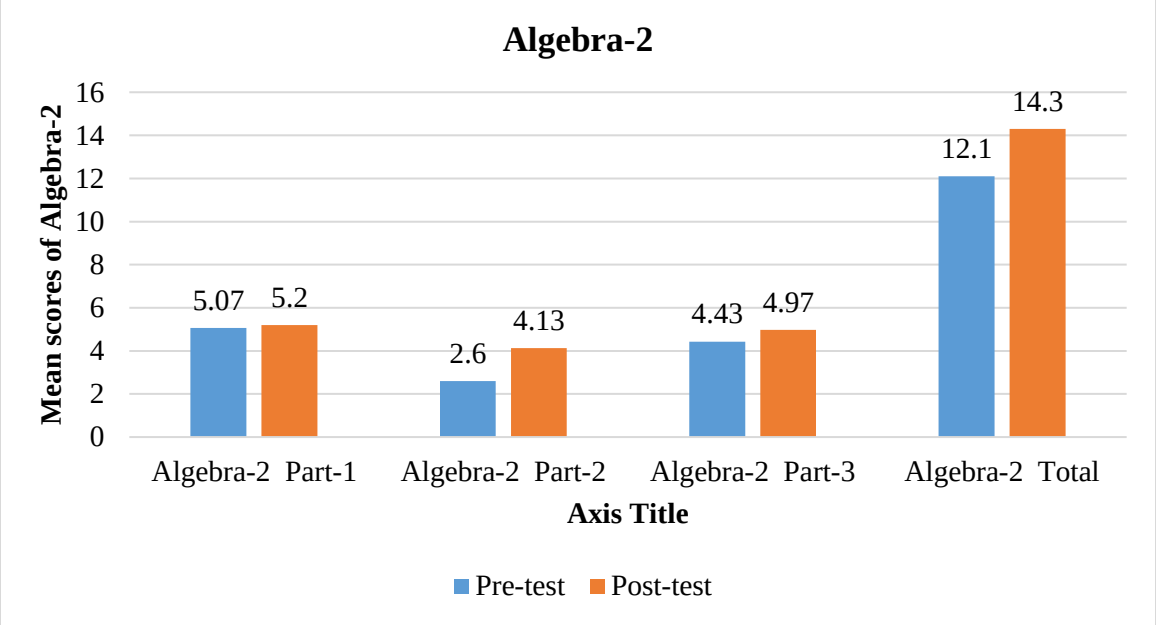


Figure 4.3 Comparison between pre-test and post-test scores of Students in Algebra-2

Table 4.4

Comparison between pre-test and post-test scores of Students on Basic Statistics

	Pre-test		Post-test		Paired sample t-test	
	M	*SD*	*M*	*SD*	*t*	*P*
Basic Stat-1 part-1	2.84	2.82	5.55	2.95	-4.030	.001
Basic Stat- 1 part-2	3.16	2.05	5.81	3.07	-3.582	0.001
Basic Stat- 1 part-3	1.97	2.09	5.03	2.52	-6.034	.001
Basic Stat- 1 Total	7.97	5.38	16.39	7.68	-5.428	.001

Table 4.4 presents the results of the pre-test and post-test applied to different parts of Basic Stat-1. There is statistically significant difference between the pre-test (*M*= 2.84, *SD* = 2.82) and the post-test (*M*= 5.55, *SD* = 2.95) results of Basic Stat-1 Part 1 (*t* =-4.03, *p*<. 05). The mean score of the post-test is greater than pre-test. There is a significant difference between the pre-test (*M*= 3.16, *SD* = 2.05) and post-test (*M*= 5.81, *SD* = 3.07) results of Basic Stat-1 Part 2 (*t* =-3.59, *p*<. 05).

Fig. 4.4

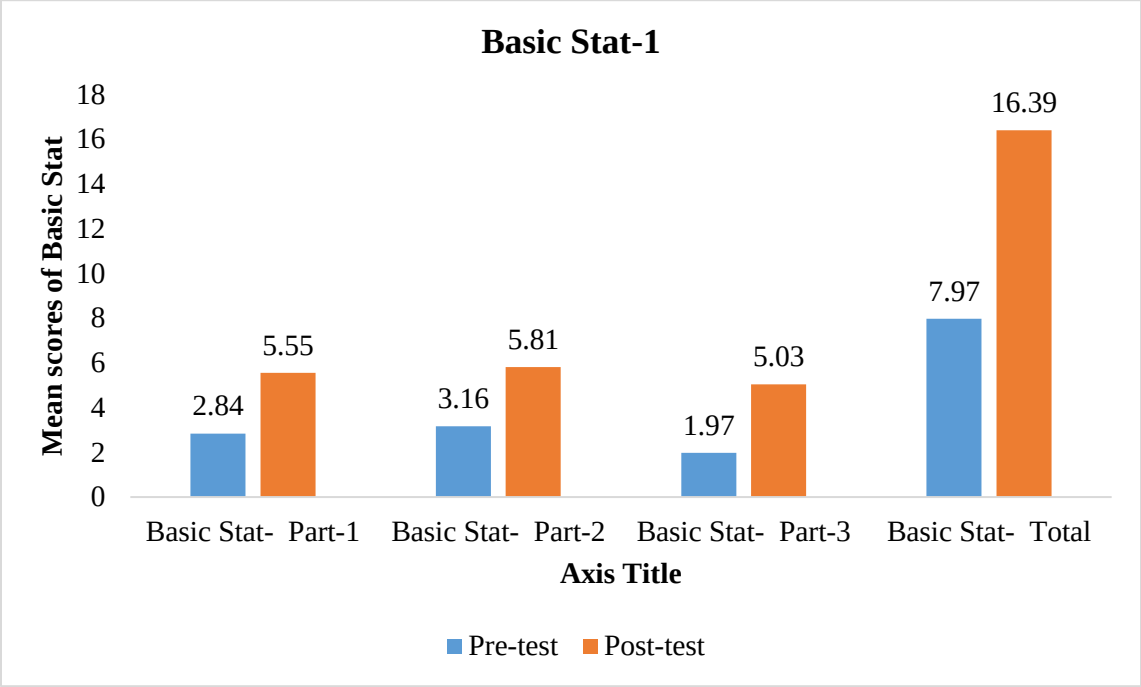


*Figure 4.4*Comparison between pre-test and post-test scores of Students on Basic Stat

Table 4.5

Comparison between pre-test and post-test scores of Students in Geometry

	Pre-test		Post-test		Paired sample t-test	
	M	*SD*	*M*	*SD*	*t*	*p*
Geometry part-1	6.61	1.73	7.42	1.18	-2.81	0.009
Geometry part-2	4.71	2.28	5.48	1.71	-2.04	0.050
Geometry part-3	3.68	1.66	6.45	1.73	-7.62	0.001
Geometry Total	15.00	3.61	19.35	2.87	-8.13	0.001

Table 4.5 presents the results of the pre-test and post-test applied to Geometry. There is a significant difference between the pre-test (M = 6.61, SD = 1.73) and post-test (M = 7.42, SD = 1.18) results of Geometry Part 1 (t =-2.81, p<. 05). The mean score of post-test is greater than pre-test. There is no significant difference between the pre-test and post-test results of Geometry part 2. There is a significant difference between the pre-test (M = 3.68, SD =1.66) and post-test (M = 6.45, SD =1.73) results of Geometry (t=7.62, p<. 05).

Fig. 4.5

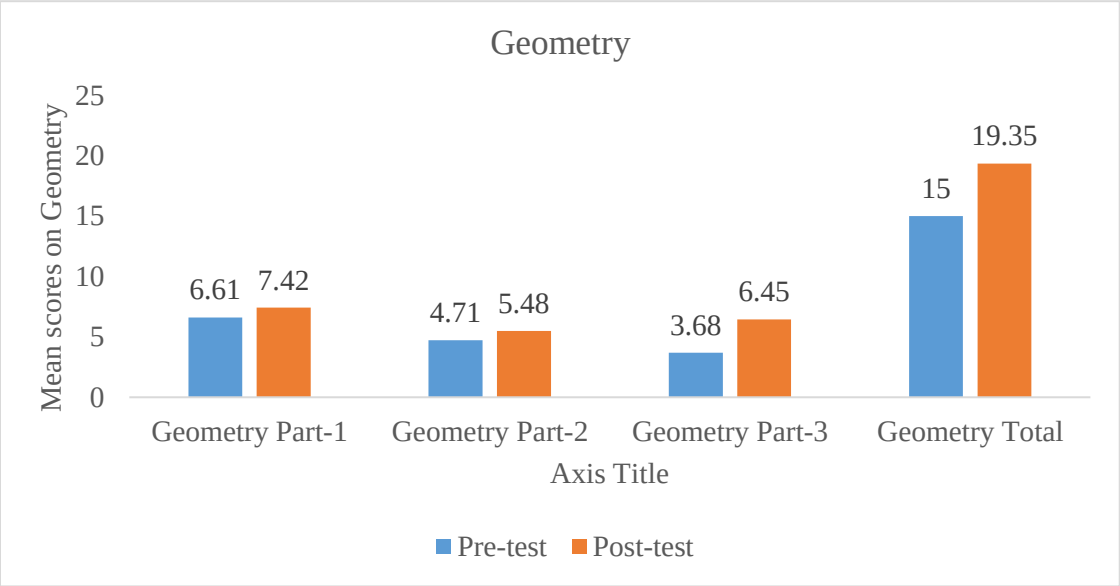


Figure 4.5 Comparison between pre-test and post-test scores of Students in Geometry

4.1 Test of Normality

For the normal distribution assumption, *Normality Tests: Kolmogorov-Smirnov and Shapiro-Wilk* used.

Table 4.6

Summary of Normality Tests: Kolmogorov-Smirnov, Shapiro-Wilk

	Tests of Normality					
	Kolmogorov-Smirnov			Shapiro-Wilk		
	Statistic	*df*	Sig.	Statistic	*df*	Sig.
Sets Pre-test	0.109	29	0.200*	0.942	29	0.346
Sets Post-test	0.144	29	0.200*	0.977	29	0.921
Algebra-1 Pre-test	0.220	29	0.029	0.916	29	0.126
Algebra-1 Post-test	0.222	29	0.025	0.775	29	0.001
Algebra-2 Pre-test	0.245	29	0.008	0.911	29	0.103
Algebra-2 Post-test	0.187	29	0.118	0.928	29	0.200
Statistics Pre-test	0.148	29	0.200*	0.971	29	0.832
Statistics Post-test	0.253	29	0.005	0.863	29	0.017
Geometry Post-test	0.233	29	0.015	0.903	29	0.077
Geometry Pre-test	0.165	29	0.200*	0.946	29	0.397

Table 4.6 shows the results of the Normality test. Most of the data scores follow the normal distribution assumption. Only one data, Algebra-1 pre-test and statistics post- tests slightly shows a violation of normality. The reliability of this research-study measurement test tool was determined by using Cronbach's Alpha formula through SPSS, which was 0.82. This table favors the null hypothesis as p- values are greater than 0.5.

4.2 Paired Sample t-test

The main research hypothesis is based on the cooperative learning effect on students' mathematical skills. Paired sample t-test used to test the main research hypotheses, and the following table describes the t-test results:

Objective – 1: To determine effect of cooperative learning consequences on students' mathematical problem-solving skills at the elementary level.

Table 4.7

Effect of cooperative learning on students' problem-solving skills

Subject	Pre-test		Post-test		Paired sample t-test	
	M	*SD*	*M*	*SD*	t	p
Sets	5.03	2.82	3.42	2.38	2.84	0.008
Algebra - 1	6.39	2.22	6.19	2.65	0.34	0.738
Algebra - 2	5.07	2.39	5.20	2.61	-0.235	0.016
Basic Statistics	2.84	2.82	5.55	2.95	-4.030	0.001
Geometry	6.61	1.73	7.42	1.18	-2.81	0.009

Table 4.7 presents five test results to assess cooperative learning consequences on learners' mathematical problem-solving skills mathematical problem-solving skills.

Sets: The table 4.7 indicates that there is a significant difference between pretest (M = 5.03, SD = 2.82) and posttest (M = 3.42, SD = 2.38) of problem solving skills, taught through Cooperative teaching method at t (29) = 2.84 at p = 0.008. **Algebra I:** the table 4.7 indicates that there is no significant difference between pretest (M = 6.39, SD = 2.22) and posttest (M = 6.19, SD = 2.65) taught through Cooperative Teaching method at t (29) = 0.34 at p = 0.738. **Algebra -2:** The table 4.7 indicates that there is a significant difference between pretest (M = 5.07, SD = 2.39) and posttest (M = 5.20, SD = 2.61), taught through Cooperative teaching method at t(29) = - 0.235 at $p = 0.016$. **Basic Statics:** The table 4.7 indicates that there is a significant difference between pretest (M = 2.84, SD = 2.82) and posttest (M = 5.5, S SD = 2.95), taught through Cooperative teaching method at t(29) = - 4.03 at $p = 0.001$. **Geometry:** The table 4.7 indicates that there is a significant difference between pretest (M = 6.61, SD = 1.73) and posttest (M = 7.42, SD = 1.18), taught through Cooperative teaching method at t(29) = - 2.81 at $p = 0.009$.

p values of four (sets = 0.008, algebra-2 = 0.016, basic statistics = 0.001, geometry = 0.009) from five test results were less than 0.005/5%. Therefore, it found from the above table that there is a constructive cooperative learning consequences on learners' mathematical

problem-solving skills mathematical problem-solving skills at the elementary level, which is first objective of the study and rejects null hypotheses.

Figure 4.6

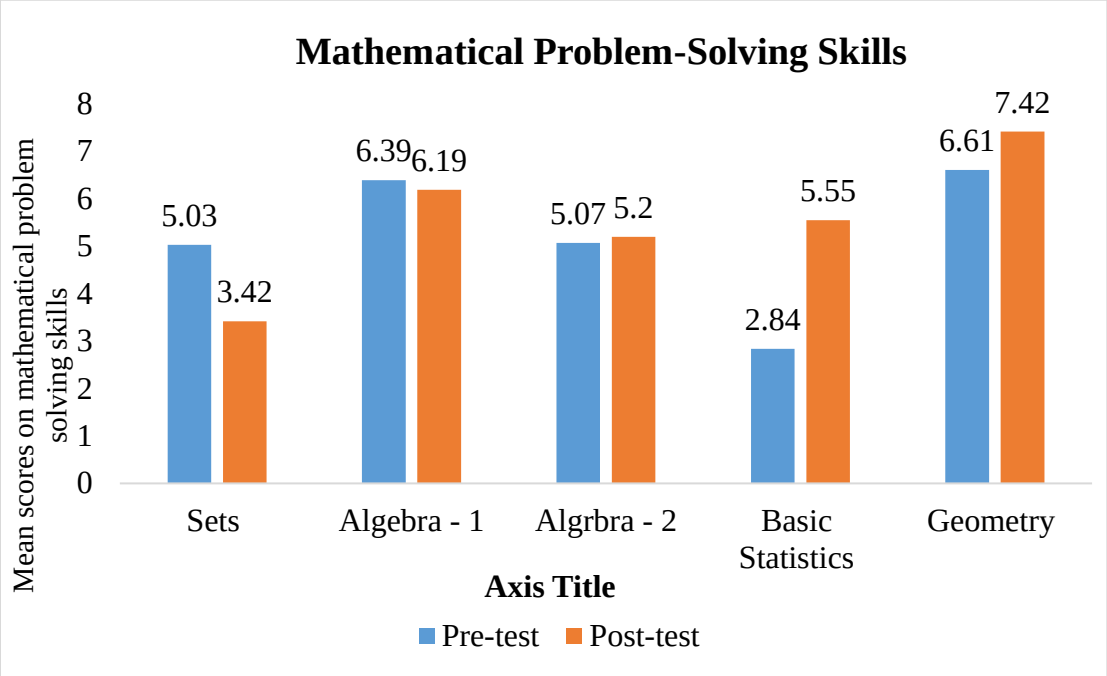


Fig 4.6 presents a comparison between the pre-test and post-test scores of Students in mathematical problem-solving skills.

Objective – 2: To examine the effect of cooperative learning on students' mathematical critical thinking skills at the elementary level.

Table 4.8

Effect of cooperative learning on students' mathematical critical thinking skills

Subject	Pre-test		Post-test		Paired sample t-test	
	M	*SD*	*M*	*SD*	*T*	*p*
Sets	4.52	2.36	4.90	2.47	-0.95	0.351
Algebra – 1	6.52	2.36	6.00	2.53	1.28	0.211
Algebra – 2	2.60	2.04	4.13	2.29	-2.986	0.006
Basic Statistics	3.16	2.05	5.81	3.07	-3.582	0.001
Geometry	4.71	2.28	5.48	1.71	-2.04	0.049

Table 4.8 presents five test results to determine cooperative learning consequences on

students' mathematical brainstorming skills. **Sets:** The table 4.8 indicates that there is no significant difference between pretest (*M* = 4.56, *SD* = 2.36) and posttest (*M* = 4.90, *SD* = 2.47) of problem solving skills, taught through Cooperative teaching method at *t* (29) =0.95 at *p* = 0.351. **Algebra I:** the table 4.8 indicates that there is no significant difference between pretest (*M* = 6.52, *SD* = 2.36) and posttest (*M* = 6.00, *SD* = 2.53) taught through Cooperative Teaching method at *t*(29) = 1.28 at *p* = 0.211. **Algebra -2:** The table 4.8 indicates that there is a significant difference between pretest (*M* = 2.60, *SD* = 2.04) and posttest (*M* = 5.20, SD=2.61), taught through Cooperative teaching method at *t*(29) = -2.986 at *p*=0.006. **Basic Statics:** The table 4.8 indicates that there is a significant difference between pretest *(M* = 3.16, *SD* = 2.05) and posttest *(M* = 5.81 *SD* = 3.07), taught through Cooperative teaching method at *t(*29*)* = - 3.582 at *p*=0.001. **Geometry:** The table 4.9 indicates that there is a significant difference between pretest *(M* = 4.71, *SD* = 2.28*)* and posttest *(M* = 5.48 *SD* =1.71*),* taught through Cooperative teaching method at *t(29)* = -2.04 *at p* = 0.049.

p values of three (algebra – 2 = 0.006, basic statistics = 0.001, geometry = 0.009) from five test results were less than 0.005/5%. Therefore, it found from the above table that there are cooperative learning's impacts on pupils' mathematical brainstorming skills at the elementary level, which is the second objective of the study and rejects null hypotheses.

Figure 4.7

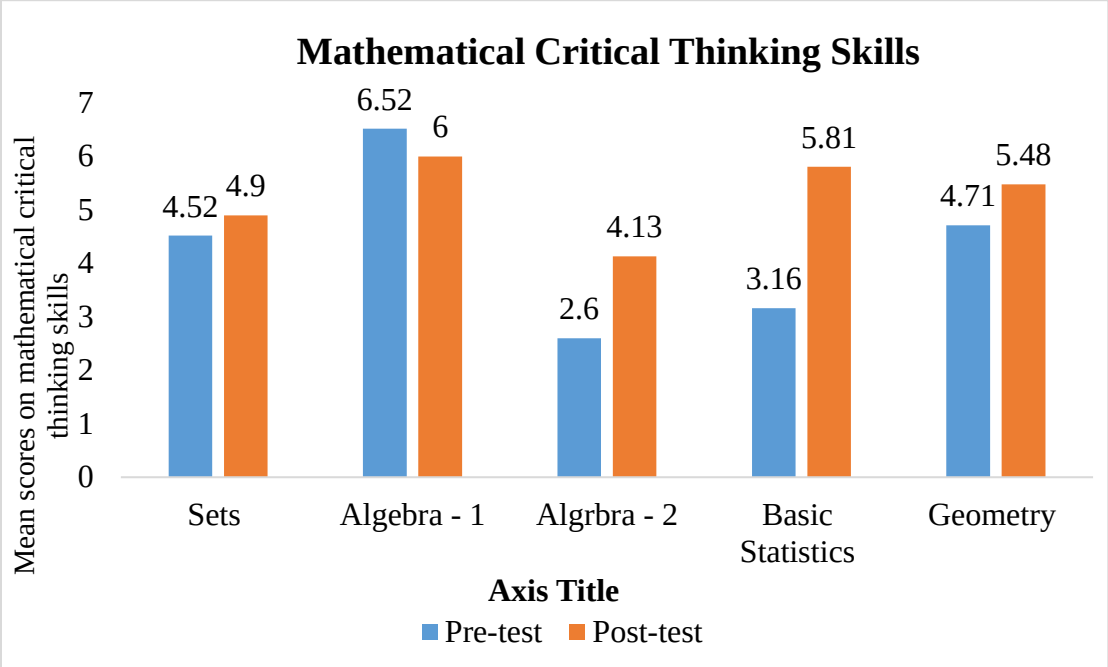


Fig 4.7 presents a comparison between the pre-test and post-test pupils' result on Brainstorming skills.

Objective – 3: To determine effect of cooperative learning students' mathematical communication skills at the elementary level.

Table 4.9

Effect of cooperative learning students' mathematical communication skills

Subject	Pre-test		Post-test		Paired sample t-test	
	M	*SD*	*M*	*SD*	*T*	*p*
Sets	5.03	2.53	4.80	2.85	0.47	0.644
Algebra – 1	4.58	2.28	5.84	2.37	-2.67	0.012
Algebra – 2	4.43	2.64	4.97	2.85	-1.038	0.308
Basic Statistics	1.97	2.09	5.03	2.52	-6.034	0.001
Geometry	3.68	1.66	6.45	1.73	-7.62	0.001

Table 4.9 presents five test results to assess the effect of cooperative learning on students' mathematical communication skills**. Sets:** The table 4.9 indicates that there is a significant difference between pretest (*M* = 5.03, *SD*= 2.53) and posttest (*M*=4.80, *SD*=2.85) of

problem solving skills, taught through Cooperative teaching method at *t*(29)=0.47 at *p*=0.644. **Algebra I:** the table 4.9 indicates that there is a significant difference between pretest (*M*=4.58, *SD*=2.28) and posttest (*M*=5.84, *SD*=2.37) taught through Cooperative Teaching method at *t*(29)=-2.67 at *p*=0.012. **Algebra -2:** The table 4.9 indicates that there is no significant difference between pretest (*M* = 4.43, *SD*= 2.64) and posttest (*M*=4.97, *SD*=2.85), taught through Cooperative teaching method at *t*(29)= - 1.038 at *p=0.308*. **Basic Statics:** The table 4.9 indicates that there is a significant difference between pretest (*M* =1.96, *SD*= 2.09) and posttest (*M*=5.03, *SD*=2.52), taught through Cooperative teaching method at t(29)= - 6.034 at p=0.001. **Geometry:** The table 4.9 indicates that there is a significant difference between pretest (*M* = 3.68, *SD*= 1.66) and posttest (*M*= 6.45, *SD*=1.73), taught through Cooperative teaching method at t(29)= - 7.62 at p=0.001.

P-values of three algebras – 1 = 0.012, basic statistics = 0.001, geometry = 0.001) from five test results were less than 0.005/5%. Therefore, it found from the above table that there is cooperative learning's outcome on pupils' mathematical presentation skills at middle level, which is third objective of the study and rejects null hypotheses.

Figure 4.8

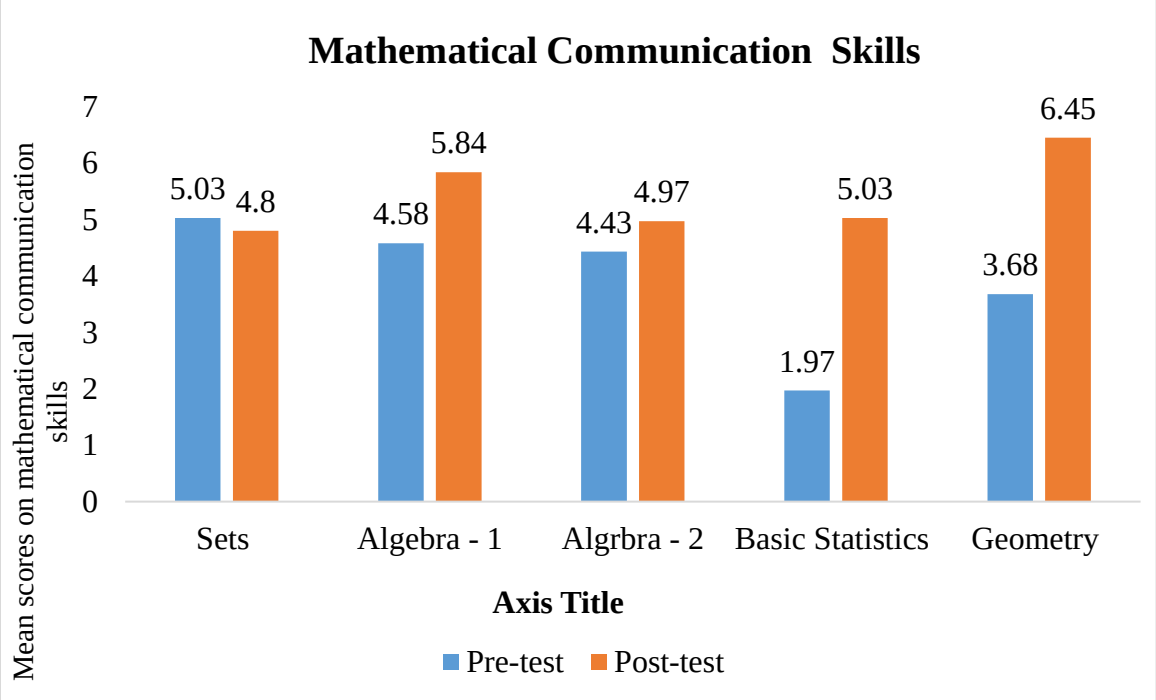


Fig 4.8 presents a comparison between the pre-test and post-test results on presentation skills.

Table 4.10

Effect of cooperative learning on students' mathematical skills

Subject	Pre-test		Post-test		Paired sample t-test	
	M	*SD*	*M*	*SD*	*t*	*p*
Sets	11.71	5.96	14.35	6.81	-2.096	0.045
Algebra - 1	17.48	5.51	18.03	6.60	-0.52	0.609
Algebra - 2	12.10	5.64	14.30	6.92	-1.802	0.082
Basic Statistics	7.97	5.38	16.39	7.68	-5.428	0.001
Geometry	15.00	3.61	19.35	2.87	-8.13	0.001

Table 4.10 presents five test results to assess cooperative learning's outcome on students' mathematical skills. Sets, algebra 1, algebra -2, basic statistics, and geometry were the five areas of the study at grade – 8 level mathematics. **Sets:** The table 4.10 indicates that there is a significant difference between pretest (M = 11.71, SD = 5.96) and posttest (M = 14.35, SD = 6.81) of problem solving skills, taught through Cooperative teaching method at t (29) = -2.096 at p = 0.045. **Algebra I:** the table 4.10 indicates that there is no significant difference between pretest (M = 17.48, SD = 5.51) and posttest (M = 18.03, SD = 6.60) taught through Cooperative Teaching method at t (29) = -0.52 at p = 0.609. **Algebra -2:** The table 4.10 indicates that there is no significant difference between pretest (M = 12.10, SD = 5.64) and posttest (M = 14.30, SD = 6.92), taught through Cooperative teaching method at t(29) = -1.802at p = 0.082. **Basic Statics:** The table 4.10 indicates that there is a significant difference between pretest (M = 7.97, SD = 5.38) and posttest (M = 16.39, S SD = 7.68), taught through Cooperative teaching method at t(29) = -5.428 at p = 0.001. **Geometry:** The table 4.10 indicates that there is a significant difference between pretest (M = 15.00, SD = 3.61) and posttest (M = 19.35, SD = 2.87), taught through Cooperative teaching method at t(29) = -8.13 at p = 0.001.

. p values of three (sets = 0.045, basic statistics = 0.001, geometry = 0.001) from five test results were less than 0.005/5%. Therefore, it was found from the above table that there is cooperative learning's results on learners' mathematical skills at lower secondary level, which is the title of the study.

Figure 4.9

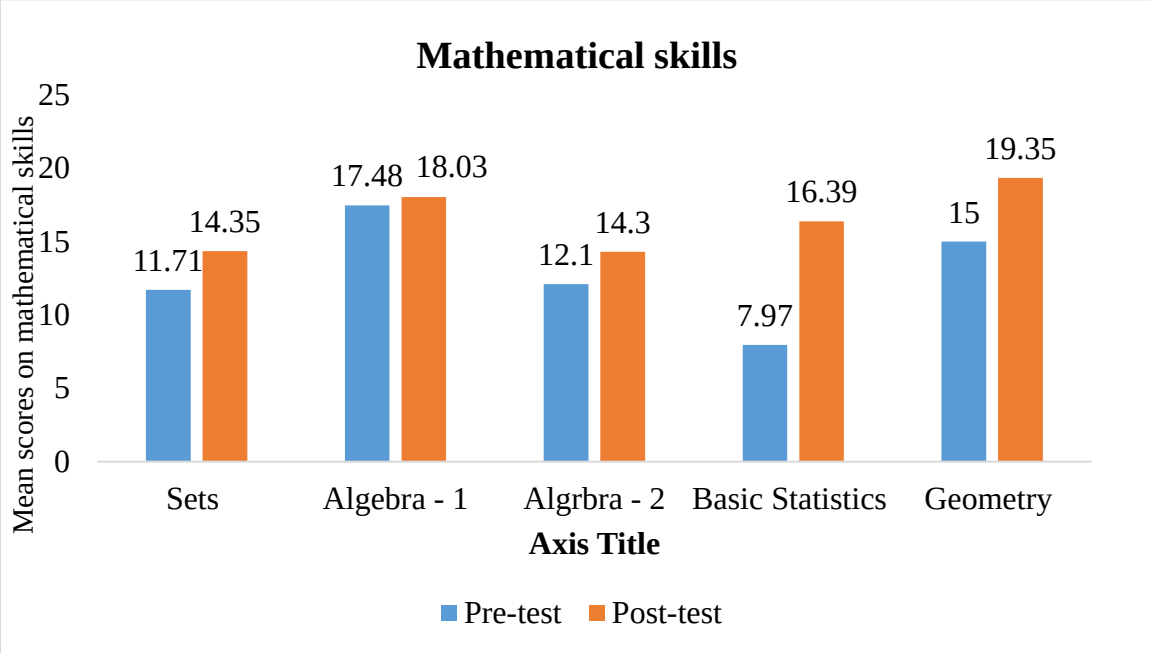


Fig 4.9 presents a comparison between the pre-test and post-test scores of Students in mathematical skills.

4.3. p value percentage

It may be easy to test the hypotheses of the study with the help of the percentage of p values of test scores of all areas of the study.

Table 4.11

Comparison p-value percentage of all parts and total tests scores of the study

Test	Sets	Algebra -1	Algebra-2	Basic Stats	Geometry	P< 5%	Percentage < 5%
Problem-solving skills	0.008	0.738	0.016	0.001	0.009	4	80%
Critical thinking skills	0.351	0.211	0.006	0.001	0.049	3	60%
Communication skills	0.644	0.012	0.308	0.001	0.001	3	60%
Mathematical skills	0.045	0.609	0.082	0.001	0.001	3	60%

Table 4.11 represents the p values in percentage to assess cooperative learning impacts on

pupils' mathematical skills, critical thinking skills, and communication skills. Four p values of problem-solving skills (80%), three p values of critical thinking skills (60%), three p values of communication skills (60%), and three p values of mathematical skills (60%) were less than 0.05/5%. It manifested that there is statistically significant difference exists and therefore the study rejects null hypotheses and favors alternate hypotheses.

i. Cooperative learning effect on pupils' problem-solving skills at lower secondary level is positive.

ii. Cooperative learning effect on pupils' critical thinking skills at lower secondary level is constructive.

iii. Cooperative learning effect on pupils' mathematical presentation skills at lower secondary level is positive.

4.4 Effect Size

To assess, the size of the effect of intervention or treatment Cohen's Criteria concluded Partial Effect Size by using Eta squared statistical formula.

Table 4.12

Comparison of the effect size for paired–samples t-test of all parts test scores

Test	Sets	Algebra-1	Algebra-2	Basic Stats	Geometry	Average	Eta squared
Problem-solving skills	0.2176	0.0039	0.0019	0.3589	0.2140	0.15	0.15
Critical thinking skills	0.0301	0.0534	0.2351	0.3067	0.1254	0.15	0.15
Communication skills	0.0075	0.1973	0.0358	0.5566	0.6669	0.29	0.29
Mathematical skills	0.1315	0.0092	0.1006	0.5039	0.6950	0.288	0.288

Table 4.12 represents the Effect Size Statistics of the cooperative learning strategy and the Partial Effect Size was concluded by Cohen's (Cohen, J., 1988) Criteria by using Eta squared statistical formula as below: Eta squared = $t^2 / t^2 + N - 1$; Small effect = 0.01 to

0.05, Moderate effect = 0.06 to0.13, Large effect = 0.14 to onward. It was concluded that the overall effect size of cooperative learning is large (Eta squared = 0.288). Problem-solving skills (Eta squared=0.15), Critical thinking skills (Eta squared = 0.15), communication skills (Eta squared =0.29), and mathematical skills (Eta squared = 0.288) and it may be used in instructional learning strategies. There were four objectives and the tables from 4.2 to 4.8 showed the assessment of four objectives through p-values and p–value percentages.

CHAPTER-5

SUMMARY, FINDINGS, DISCUSSIONS, CONCLUSIONS, AND RECOMMENDATIONS

5.1 Summary

The mastery of mathematical skills is vital, not only among schoolchildren but also for teachers. The implementation of cooperative learning has become increasingly popular in current years as pedagogy trends worldwide. It has shifted from teacher-centered to learner-centered methods. The students' mathematical skills scored on the three mathematical components: communication, analytical, and brainstorming skills. The achievements manifested that pupils had improved their scores in mathematical skills, and they discussed in the paper. The interpretation of the collected experimental research data was penned in detail in this chapter. The experimental research study was to examine "The Effect of Cooperative Learning on Students' Mathematical Skills at Elementary School Level: A Quasi-Experimental Study". In the experimental time series design was implemented by the researcher and core objectives of the experimental study were as following:

i. Assess cooperative learning's effect on learners' problem-solving skills in mathematics at the elementary level.
ii. Determine cooperative learning's effect of on students' mathematical critical thinking skills at the elementary level.
iii. Study the effect of cooperative learning on pupils' mathematical communication skills at the elementary level.

Mathematics is said to be the mother of all social sciences and natural sciences as, without it, the smooth running of daily life can be critical. It is not only exploring the universe but also leading logical science. On the schooling aspect, mathematics embraces typical argumentation and logic which need to be imparted through specific teaching-learning strategies. The research study was conducted in Islamabad Model School for Boys (I-X) HernoThanda Pani FA Nilore, Islamabad under the ambit of FDE. The sample of this study

was thirty (30) pupils of the eighth class, considered as one group, and this group was further divided into five subgroups.

Four topics Sets, Algebra, Geometry, and Basic Statics selected from the mathematics subject of 8th grade. Before the implementation of cooperative learning, pre–test (appendix – 1), following treatment, posttest (appendix-2) was taken by the students of the class. The test developed consisting of three parts: the first part was exploring students' problem–solving skills, the second part was critical thinking skills and the third part was communication skills. The duration of the experiment consisted of six (6) weeks (15 January 2022 to 25 February 2022). After the covering of four chapters through cooperative learning STAD as treatment, six post-tests (appendix-2) used as examining or assessment tools.

Pre-test and post-test applied as measuring tools during study for the assessment of cooperative learning outcome on students' mathematical skills. The pilot research study administered on 7th-class students as a reliability of test before the commencement of the main research study on 8th-class students. The reliability of the test turned up by applying Cronbach's alpha formula in SPSS. The reliability was determined to be 0.8. Applying paired sample–Test, it was calculated significant difference between the *M* scores of the pre-test and post-test. By using a Bar graph, the interaction effect was displayed. Overall Cronbach's Alpha reliability of cooperative learning results on the learners' mathematical skills (ECLSMS) at the elementary level was 0.7. The instrument was further divided into three more parts: Part A, Part B, and Part C.

There was total of 30 respondents from class 8 studying in Islamabad Model School for Boys (I-X) Herno Thanda Pani Nilore, Islamabad (IMSB). These students' test results were used as a pilot test and displayed the reliability of the tool. All of these respondents were Boys. There were 10 teachers in the school and only one (10%) teacher conducted the research method (cooperative learning) for the collection of data.

The teacher was M.Sc., M.Ed. and had more than 15-year experience in teaching Mathematics in the public schools of Islamabad in the ambit of FDE. The assumptions of Paired sample t-Test was analyzed by using SPSS software. The first assumption was independent subjects, the second assumption was that the pre-and post-measurements must

be taken from the same subject, and the third assumption was that the normal distribution must be significant. The research study consisted of six weeks: one week was used for pilot testing from 8th-grade students and the rest of five weeks were used for the implementation of cooperative learning through lesson plans and data collection testing from eighth pupils studying in IMSB (I-X) Herno Thanda Pani, Nilore Islamabad.

5.2 Findings

An experimental research study administered at the public school at the middle level to determine cooperative learning consequence on pupils' mathematical skills by applying a quasi-experimental time series design group under the ambit of FDE Islamabad. The following findings obtained from the study:

1. Table 4.7 presents five test results to assess cooperative learning's outcome on learners' problem-solving skills in mathematics. There was a significance difference between pretest and post-test of set regarding problem solving skills taught through cooperative learning method. As p values of four (sets = 0.008, algebra – 2 = 0.016, basic statistics = 0.001, geometry = 0.009) from five test results were less than 0.05/5%.

   Therefore, it found from the above table that there is a constructive cooperative learning consequences on learners' mathematical problem-solving skills at the elementary level, which is first objective of the study and rejects null hypotheses.

2. Table 4.8 presents five test results to assess the cooperative learning outcomes on learners' mathematical critical thinking skills. There was a significance difference between pretest and post-test of set regarding mathematical critical thinking skills taught through cooperative learning method. As p -values of three (algebra – 2 = 0.006, basic statistics = 0.001, geometry = 0.009) from five test results were less than 0.05/5%.

   Therefore, it found from the above table that there is a constructive cooperative learning consequences on learners' mathematical critical thinking skills the elementary level, which is second objective of the study and rejects null hypotheses.

3. Table 4.9 presents five test results to examine cooperative learning outcomes on learners' mathematical communication skills. There was a significance difference between pretest and post-test of set regarding mathematical communication skills

taught through cooperative learning method. As p -values of three (algebra – 2 = 0.006, basic statistics = 0.001, geometry = 0.009) from five test results were less than 0.05/5%.

Therefore, it found from the above table that there is a constructive cooperative learning consequences on learners' mathematical communication skills at the elementary level, which is third objective of the study and rejects null hypotheses.

4. Table 4.10 presents five test results to determine the CL effect on learners' mathematical skills. Sets, algebra 1, algebra -2, basic statistics, and geometry were the five areas of the study at grade – 8 level mathematics. Standard deviation (sets = 6.81, algebra – 1 = 6.6, algebra – 2 = 6.92, basic statistics = 7.68, geometry = 2.87). p values of three (sets = 0.045, basic statistics = 0.001, geometry = 0.001) from five test results were less than 0.05/5%.
5. Table 4.11 represents the p – values in percentage to determine cooperative learning's outcome on learners' mathematical skills, problem-solving skills in mathematics, critical thinking skills in mathematics, and communication skills. Four p values of problem-solving skills (80%), three p values of critical thinking skills (60%), three p values of communication skills (60%), and three p values of mathematical skills (60%) were less than 0.05/5%.
6. Table 4.12 represents the Effect Size Statistics of the cooperative learning strategy and the Partial Effect Size was concluded by Cohen's Criteria by using Eta squared statistical formula as below: Eta squared = $t^2 / t^2 + N - 1$; Small effect = 0.01 to 0.05, Moderate effect = 0.06 to0.13, Large effect = 0.14 to onward.

   i. Mathematical problem-solving skills (Eta squared=0.15) are greater than 0.14.

   ii. Mathematical critical thinking skills (Eta squared = 0.15) are greater than 0.14.

   iii. Mathematical communication skills (Eta squared =0.29) are greater than 0.14.

   iv. Mathematical skills (Eta squared = 0.29) are greater than 0.14.

5.3 Discussion

Multiple research studies have proved that Jigsaw's cooperative learning method impact on students' achievements were productive (Azmin, 2016; Karacop and Doymus,

2013). According to them, they have taken the significant benefits from research studies of cooperative learning as cooperative learning's outcome foster learners' problem-solving skills in math and elaborated problem-solving skills at the secondary or lower secondary level. Positive interdependence and social interaction of learners are encouraged by cooperative learning strategies.

The performance of the students increased by using cooperative learning in the classroom as an instructional strategy. Tran, & Lewis (2012), stated that the effect of cooperative learning are better than individual learning and competitive learning but both if compared, enhance problem-solving skills at all grade levels. Johnson et al. studied more than a hundred correlation studies and over six hundred experimental studies conducted on individualistic, competitive, and cooperative learning. The results from research studies regarding cooperative, competitive, and individualistic learning were better. Tran, and Lewis (2012), investigated in their research study on 80 students of mathematics in Vietnam that student–centered cooperative learning highly improved pupils' achievement and retention.

In Pakistan, Munawar, S., & Chaudhary, A.H. (2019), determined in their study and that it was instituted in the subject of English writing by implementing cooperative learning that scores of experimental batch was better as compare to non-experimental batch seventh-grade school students. Pre-test, treatment, and then post-test quasai models used for scoring. Duration of the study was consisted of two weeks and found highly positive effect of the instructional strategy on science students' achievements.

Khan (2008), in Pakistan Ahangari, S., and Samadian (2014), in Iran, conducted research on CL consequences to grow the writing achievements of pupils and found that the cooperative learning technique is a better instructional strategy.

In 2004, Iqbal M. conducted a research study on the effect of cooperative learning mathematics achievement at the secondary level in Pakistan. He used the Students Team Achievement Division method in his experimental study and it lasted for six weeks. His findings were in the favor of the implementation of a cooperative teaching-learning strategy. According to him, cooperative teaching is a highly effective teaching technique rather than the traditional instructional method.

Edekor, and Agbornu (2020), in the USA, examined a research study on the impacts of cooperative learning strategy in Mathematics on pupils' achievements at lower secondary level. Findings of the study supported implementation of a cooperative teaching strategy. It boosted the performance of the students learning mathematics and developed their interest in mathematics understanding for both male and female learners. Sulisworo et al. (2016), examined studies on cooperative learning, and found that learning in a cooperative style with other students is better to achieve educational objectives and conducted. A research study on the impacts of cooperative learning, and concluded that cooperative learning instructional strategy consists of small groups is an effective teaching and learning strategy in which students get equipped with various level of ability. This instructional strategy can have implemented in different fields like mathematics, language, geography, and science.

Botha, W. (2021), administrated a study on CL impact on students' skills, and attitudes and he revealed the merits and demerits of applying a cooperative learning strategy in the classroom. According to Botha, W., in small group teaching, the Jigsaw method of cooperative learning technique is a good way of learning for learners.

Colosi and Zales (1998) revealed that by using cooperative learning, involvement, and activeness increased and the positive interdependence of students on each other and the teacher's effectiveness improves in his or her pupils.

Wang et al. (2017) conducted a research study and their findings were favorable to cooperative learning and revealed that children enjoyed clear and structured guidance in which their assigned task is clear, and this can happen through collaboration cum cooperative learning strategies.

Effendi Zakaria et al. (2010) researched Cl result on learners' mathematical attitude and performance in Malaysia. Heinemann et al. (2020) described that educational goals should ensure in a structured path of learning and students should guide during learning in the classroom.

Heimbuch et al. (2018) revealed that children's engagement and learning in terms of studying are more beneficial and fruitful in the term of implementing of cooperative learning at the secondary level.

Cooperative learning, according to Popov et al. (2019), is more beneficial in terms of bringing cultural gaps among learners where children permitted to express and share different opinions, various beliefs, and ideas, and entertain their leadership through discussion among the group and in front of the class.

A Malaysian research study administered cooperative learning's outcome on Malaysian learners in math, and found improvement in students' achievements (Zakaria et al., 2010). Different writers and scholars viewed multiple aspects of applying cooperative learning strategies. According to them, some techniques of cooperative learning favor the collaborative learning strategies and do not relax the students during the learning and learners get confused and feel stressed. Most of the learners deprived of cognitive learning and lack expertise.

5.4 Conclusions

CL effects, after findings and analysis of the research, on students' mathematical skills by applying time series experimental design; all four of the study objectives achieved and conclusions are drawn:

1. It observed that the post-test score was better than pre-test scores. Therefore, by comparing the responses of students, before and after implementing cooperative learning as a research tool for solving mathematical problems. It concluded that cooperative learning (Cl) is helpful and constructive in improving for students' mathematical problem-solving skills at the elementary level.
2. It observed that the post-test score was better than pre-test scores. It concluded that there was a productive change in the test scores and critical thinking skills of the experimental group after the treatment of cooperative learning. It concluded that cooperative learning is a better instructional method than the drilling teaching method, and that cooperative learning impact is constructive on students' critical thinking skills in math.
3. It concluded that the post-test score was better than pre-test scores. Based on responses of students in post-test results of writing communication skills and on the bases of the researcher's observation, that cooperative learning impact is constructive on pupils' mathematical communication skills at lower secondary

level.

4. There was a change in the group members' behavior while implementing a cooperative learning strategy. They were discussing, arguing, and leading rather than using their memory for gathering information about mathematical solutions. It concluded from the findings that there is an impact of CL on learners' mathematical skills at middle level and concluded that Cooperative learning is better than the drilling teaching method.
5. On the bases of effect size results from findings, it concluded that the overall effect size of cooperative learning is positive for students' mathematical skills, Communication, Critical thinking, and Problem-solving skills in mathematics.

5.5 Recommendations

Keeping in mind the experimental study, statistical analysis, findings, and conclusions of this study, the following recommendations may arise:

1- Teachers should receive professional development and training on the principles and strategies of cooperative learning and how they specifically promote problem – solving skills in mathematics. This will equip them with the knowledge and skills needed to effectively design and facilitate cooperative learning activities that target mathematical problem solving skills.

2- Teachers should design tasks and activities that require students to engage in higher-order thinking processes, such as analyzing, evaluating, and synthesizing mathematical concepts. These tasks should be challenging and open-ended, encouraging students to think critically, explore multiple solutions, and justify their reasoning.

3- Design activities that require pupils to engage in purposeful and structured mathematical communication. These activities can include problem-solving tasks, discussions, debates, presentations, or peer teaching. Provide prompts, sentence starters, or graphic organizers to scaffold pupils' communication efforts.

5.6 Limitations

Limitations of study were as below:

i. The classroom which was used during the study was not enough spacious for applying of cooperative learning activities. Therefore, it may be a large hall so that the group activities may be carried out more effectively.

ii. The duration of the study may be comprised on two or more months in order to collect more data for more effective analysis.

iii. Absentee of students was creating hurdles during the experimental study.

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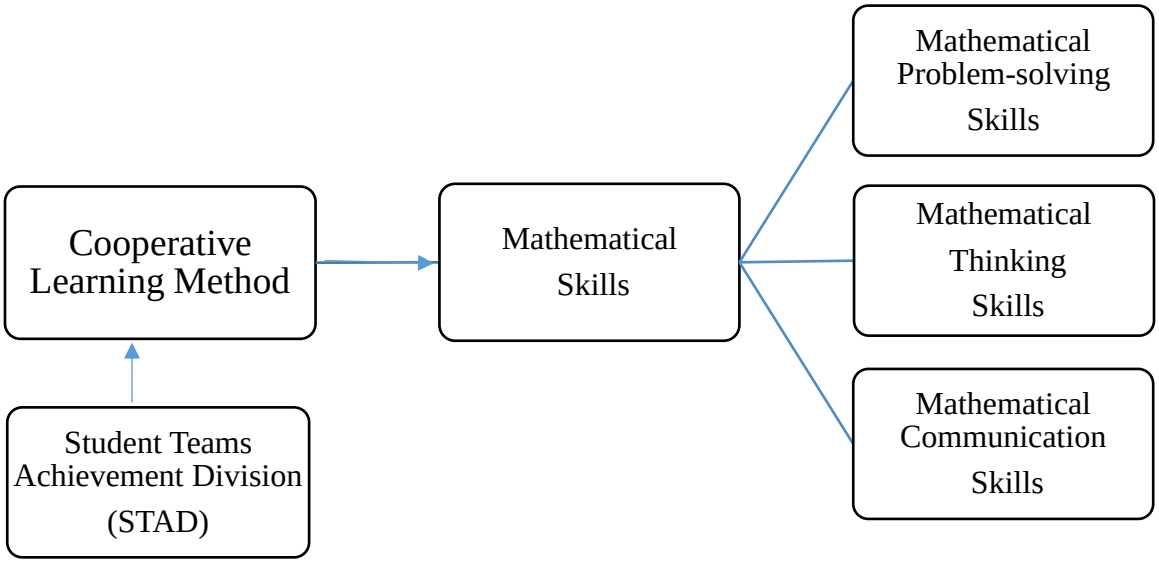
APPENDICES

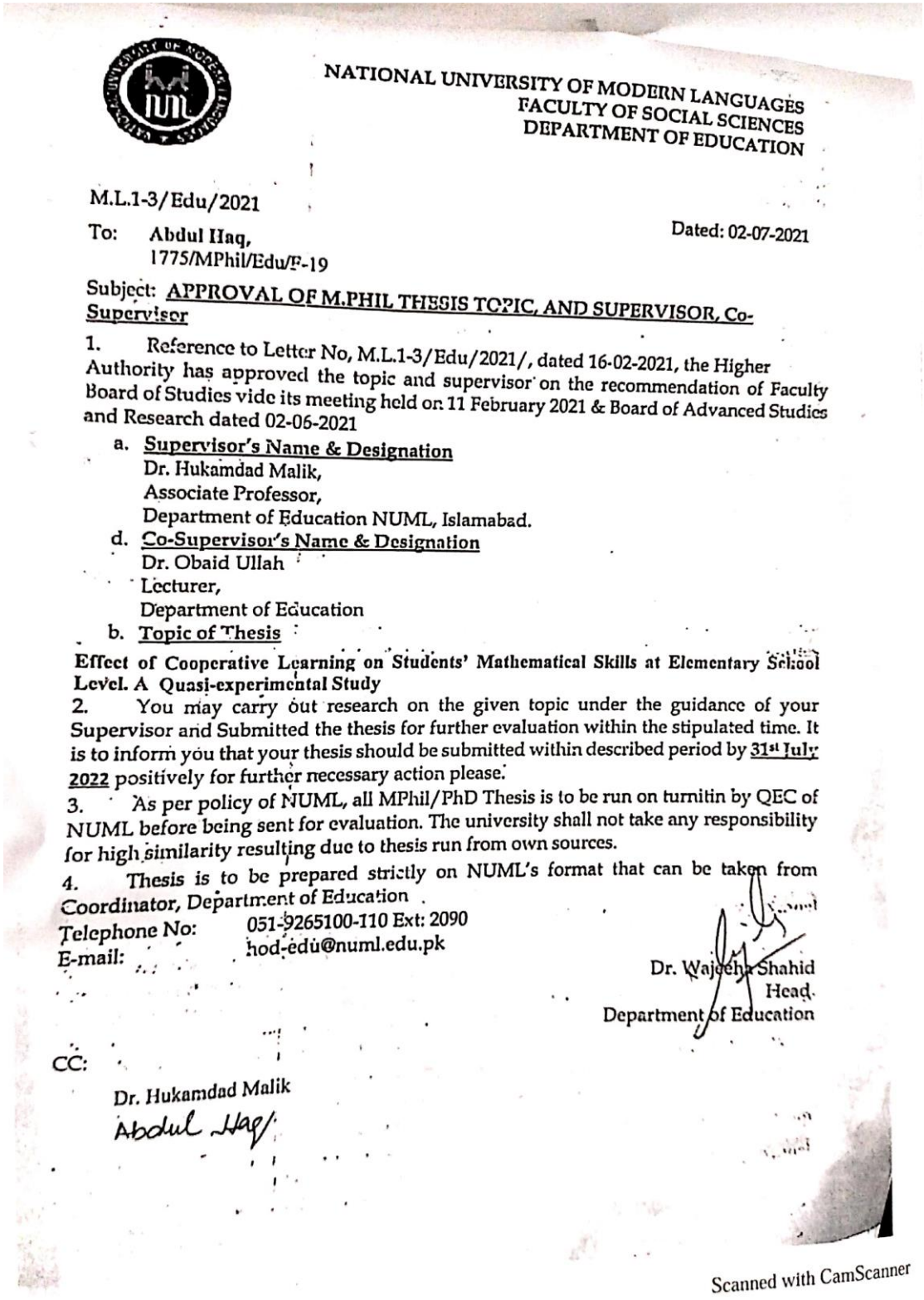
Appendix-A

CONCEPTUAL FRAMEWORK



Appendix-B

TOPIC APPROVAL LETTER



NATIONAL UNIVERSITY OF MODERN LANGUAGES
FACULTY OF SOCIAL SCIENCES
DEPARTMENT OF EDUCATION

M.L.1-3/Edu/2021

Dated: 02-07-2021

To: Abdul Haq,
1775/MPhil/Edu/F-19

Subject: APPROVAL OF M.PHIL THESIS TOPIC, AND SUPERVISOR, Co-Supervisor

1. Reference to Letter No, M.L.1-3/Edu/2021/, dated 16-02-2021, the Higher Authority has approved the topic and supervisor on the recommendation of Faculty Board of Studies vide its meeting held on 11 February 2021 & Board of Advanced Studies and Research dated 02-06-2021

a. Supervisor's Name & Designation
Dr. Hukamdad Malik,
Associate Professor,
Department of Education NUML, Islamabad.

d. Co-Supervisor's Name & Designation
Dr. Obaid Ullah
Lecturer,
Department of Education

b. Topic of Thesis

Effect of Cooperative Learning on Students' Mathematical Skills at Elementary School Level. A Quasi-experimental Study

2. You may carry out research on the given topic under the guidance of your Supervisor and Submitted the thesis for further evaluation within the stipulated time. It is to inform you that your thesis should be submitted within described period by 31st July 2022 positively for further necessary action please.

3. As per policy of NUML, all MPhil/PhD Thesis is to be run on turnitin by QEC of NUML before being sent for evaluation. The university shall not take any responsibility for high similarity resulting due to thesis run from own sources.

4. Thesis is to be prepared strictly on NUML's format that can be taken from Coordinator, Department of Education .

Telephone No: 051-9265100-110 Ext: 2090
E-mail: hod-edu@numl.edu.pk

Dr. Wajeeha Shahid
Head
Department of Education

CC:

Dr. Hukamdad Malik

Abdul Haq

Scanned with CamScanner

Appendix-C

DATA COLLECTION REFERENCE LETTER

Hazara University

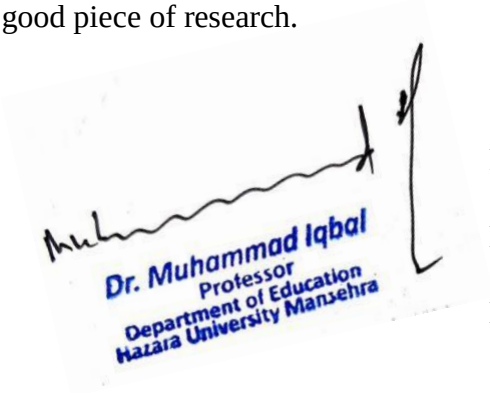
MANSEHRA,
PAKISTANDEPARTMENT OF
DUCATIONGARDENCAMPUS

VALIDATION OF RESEARCH TOOL

January17, 2022

I have validated research instruments (Pre-test & Post-test) constructed by Mr. Abdul Haq, M.Phil. Education Scholar, Registration # 1775/ M.Phil./Edu/F-19 from Department of Education, Faculty of Social Sciences, National University of Modern Languages. (NUML), Islamabad. These tests are regarding his research topic ‘**‘EFFECT OF COOPERATIVE LEARNING ON STUDENTS’ MATHEMATICAL SKILLS AT ELEMENTARY LEVEL: A QUASI-EXPERIMENTAL STUDY’.** The scholar has prepared these tests in the light of objectives of his experiment. I have thoroughly reviewed these tests for measuring mathematical skills, & indicated deficiencies. For the purpose of refinement, the scholar is suggested to construct scoring key reflecting mathematical skills against each step of solution/item under the guidance of supervisor.

I believe that the pre-test & post-test will serve the research purpose after incorporating the suggested improvement. The research topic is very pertinent, & I hope the student will produce good piece of research.



Prof. Dr. Muhammad IqbalMajoka

Department of Education,

Hazara University, Mansehra

Appendix-D

VALIDATION OF RESEARCH TOOL

January 01, 2022

I have validated research instruments (Pre-test & Post-test) constructed by Mr. Abdul Haq, M.Phil. Education Scholar, Registration # 1775/ M.Phil./Edu/F-19 from Department of Education, Faculty of Social Sciences, National University of Modern Languages.(NUML),Islamabad.Thesetestsareregardinghisresearchtopic**'EFFECT OF COOPERATIVE LEARNING ON STUDENTS' MATHEMATICAL SKILLS AT ELEMENTARY LEVEL:A QUASI-EXPERIMENTAL STUDY'.** The scholar has prepared these tests in the light of objectives of his experiment. I have thoroughly reviewed these tests for measuring mathematical skills, & indicated deficiencies. For the purpose of refinement, the scholar is suggested to construct scoring key reflecting mathematical skills against each step of solution/item under the guidance of supervisor.

I believe that the pre-test & post-test will serve the research purpose after incorporating the suggested improvement. The research topic is very pertinent, & I hope the student will produce good piece of research.

DR. KHALID MAHMOOD
Principal
Islamabad Model School for
Boys (VI-X), Jhang Syedan
(F.A), Islamabad

Appendix-E

CERTIFICATE OF COMPLETION

This is to certify that Mr. Irfan Abbas has successfully completed the training program on Cooperative Learning Method, conducted by on January 17 -22 in 2022. During the training, Mr. Irfan Abbas has demonstrated a comprehensive understanding of the principles and techniques of Cooperative Learning Method, and has actively participated in all the learning activities and discussions. By completing this training, Mr. Irfan Abbas has gained valuable knowledge and skills that can be applied in various educational settings to promote collaborative learning and enhance student achievement.

We congratulate Mr. Irfan Abbas on his achievement and wish him success in his future endeavors.

DR. KHALID MAHMOOD
Principal
Islamabad Model School for
Boys (VI-X), Jhang Syedan
(F.A), Islamabad

Appendix-F

COVERING LETTER FOR VALIDITY CERTIFICATE

Effect of Cooperative learning on students' mathematical skills at elementary level: Quasi experimental design.

Subject: **Request for validity certificate**

Respected Sir

I have attached my test self – developed for the purpose of research titled an "effect of cooperative learning on students' mathematical skills at elementary level: Quasi experimental design." The interrupted time – series design is based on pre –tests, treatment and the post –tests (Cook & Campbell, 1979). It is categorized into three parts problem solving skills, critical thinking skills and communication skills. The area of study is categorized into four further areas i.e. sets, algebra, geometry and basic statics. Kindly, check content validity of my self – developed test and provide your valuable suggestions for its improvement and certify its validity.

Abdul HaqBrohi
M. Phil. Scholar, Department of Education,
National University of Modern Languages,
Islamabad Pakistan

Appendix-G

CERTIFICATE FOR TOOL VALIDATION

Experiment test self -developed

For the Research Entitled As

EFFECT OF COOPERATIVE LEARNING ON STUDENTS' MATHEMATICAL SKILLS AT ELEMENTARY LEVEL: QUASI-EXPERIMENTAL DESIGN

By

Mr. Abdul Haq Brohi

M. Phil Scholar, Department of Education, Faculty of Social Sciences, National University of Modern Languages (NUML), H-9, Islamabad, Pakistan

This is certifying that the test developed by the scholar towards his thesis has been assessed by me and in find it that has been designed adequately to assess the students' mathematical skills based on three parts i.e. problem solving skills, critical thinking skills and communication skills. It is considered that the research instrument, developed for research for examining of effect of cooperative learning on students' mathematical skills at elementary level is according to the objectives of the research, assure adequate construct and content validity according to the purpose of research, and can be used for data collection by the researcher with fair amount of confidence.

Name _____

Designation _____

Institute _____

Signature _____

Date _____

Appendix-H

RESEARCH INSTURMENT

"EFFECT OF COOPERATIVE LEARNING ON STUDENTS' MATHEMATICAL SKILLS AT ELEMENTARY LEVEL: A QUASI-EXPERIMENTAL STUDY

PRE-TEST

Subject: Mathematics | Class: 8th

Student Id: _____ | Teacher Name: _____

Section: A | Total Marks 24

Class duration: 40 minutes | Student Name: _____

Topic/lesson: Sets

PART - I

NOTE: There are four alternative options for all the given problems. Circle the right answer after solving each question.

1. If set A = $\{0, 1, 5, 7, 8, 9, 10, 11\}$ & B = $\{-3, 0, 1, 2, 3, 4, 5, 6, 7\}$are given, then we get the answer of AUB:

   A. $\{0, 1, 5, 7, 8, 9, 10, 11\}$ B. $\{3, 0, 1, 5, 7, 8, 9, 10, 11\}$

   C. $\{3, 0,2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$ D. $\{-3, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

2. If set P = $\{0, 1, 5, 7, 8, 9, 10, 11\}$& Q = $\{0, 1, 2, 3, 4, 5, 6, 7\}$are given, then answer of Q∩ P was:

   A. $\{0, 1, 5, 7\}$ B. $\{0, 1, 5, 7, 8, 9, 10, 11\}$

   C. $\{0, 1, 2, 3, 4, 5, 6, 7\}$ D. $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

3. If set X = $\{0, 2, 4, 6, 8, 9, 10, 11\}$& Y = $\{0, 1, 2, 3, 4, 5, 6, 7\}$are given, then answer of $X-Y$ is:

   A. $\{1, 3, 5, 7\}$ B. $\{8, 9, 10, 11\}$

   C. $\{8, 9, 10, 11, 0, 1, 5, 7\}$ D. $\{0, 1, 5, 7, 8, 9, 10, 11\}$

4. If set P = $\{-1, 2, 4, 6, 8, 9, 10, 12\}$& Q = $\{0, 1, 2, 3, 4, 5, 6, 7, 12\}$are given, then answer of Q−Pwas:

   A. $\{-1, 8, 9, 10\}$ B. $\{0, 1, 3, 5, 7\}$

   C. $\{2, 4, 6, 12\}$ D. $\{-1, 0, 1, 3, 5, 7, 8, 9, 10\}$

PART - II

5. For set A = $\{-2, -1, 0, 2, 4, 6, 8, 10, 14, 15\}$& B = $\{-2, 2, 3, 4, 6, 7, 14, 16\}$the answer is $\{-2, 2, 4, 6, 10, 14\}$, then the following alternative is true:

   A. A−B B. AUB

   C. B∩ A D. B−A

6. The answer of P − Q is $\{-5, -4, -3, 3, 4, 6, \}$, if set Q = $\{-2, 0, 2, 5, 14\}$, then the elements of set P are:

   A. $\{-2, 0, 2, 5, 14\}$ B. $\{-5, -4, -3, 3, 4, 6, \}$

   C. $\{-5, -4, -3, 0, 2, 3, 4, 6, 14\}$ D. $\{-5, -4, -3, -2, 0, 2, 3, 4, 5, 6, 14\}$

7. If there are three sets X, Y, & Z. If the all elements of Y & Z are present in X, then the following answer is true:

   A. X is superset of Y & Z B. X is proper subset of Y

   C. X is proper subset of Z D. X is proper subset of Y & Z

8. If there are three sets P, Q, & R. If the all elements of R present in set P & set Q, then the following statement is true:

   A. P− Q = R B. PU Q = R

   C. P ∩ Q = R D. Q− P = R

PART - III

9. Solve (X U Y) U Z on the given sheet, when X = $\{-5, -4, -3, 3, 4, 6, 7, 8, 9, 20, 21\}$, Y = $\{-5, -4, -3, 3, 4, 6, \}$& Z = $\{8, 9, 10, 11, 20, 21, 25, 27\}$.

10. Solve (P − Q) ∩ R on the given sheet, when P = $\{-6, -4, -3, 3, 4, 6, 7\}$, Q = $\{-5, -4, -3, 3, 4, 6, 8\}$& R = $\{1, 2, 8, 9, 10, 11, 12, 13, 15, \}$.

11. Prove the commutative law for union on given sheet, when set A = $\{0, 1, 5, 7, 8, 9, 10, 11\}$ & B =$\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

Appendix-I

RESEARCH INSTURMENT

"EFFECT OF COOPERATIVE LEARNING ON STUDENTS' MATHEMATICAL SKILLS AT ELEMENTARY LEVEL: A QUASI-EXPERIMENTAL STUDY

POST - TEST

Subject: Mathematics | Class: 8^{th}

Student Id: _____ | Teacher Name: _____

Section: A | Total Marks 24

Class duration: 40 minutes | Student Name: _____

Topic/lesson: Sets

PART - I

NOTE: There are four alternative options for all the given problems. Circle the right answer after solving each question.

1. If set P = $\{-1, 2, 4, 6, 8, 9, 10, 12\}$ & Q = $\{0, 1, 2, 3, 4, 5, 6, 7, 12\}$are given, then answer of Q−Pwas:

   A. $\{-1, 8, 9, 10\}$ B. $\{0, 1, 3, 5, 7\}$

   C. $\{2, 4, 6, 12\}$ D. $\{-1, 0, 1, 3, 5, 7, 8, 9, 10\}$

2. If set A = $\{0, 1, 5, 7, 8, 9, 10, 11\}$& B = $\{-3, 0, 1, 2, 3, 4, 5, 6, 7\}$are given, then we get the answer of AUB:

   A. $\{0, 1, 5, 7, 8, 9, 10, 11\}$ B. $\{3, 0, 1, 5, 7, 8, 9, 10, 11\}$

   C. $\{3, 0,2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$ D. $\{-3, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

3. If set P = $\{0, 1, 5, 7, 8, 9, 10, 11\}$& Q = $\{0, 1, 2, 3, 4, 5, 6, 7\}$are given, then answer of Q∩ Pwas:

   A. $\{0, 1, 5, 7\}$ B. $\{0, 1, 5, 7, 8, 9, 10, 11\}$

   C. $\{0, 1, 2, 3, 4, 5, 6, 7\}$ D. $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

4. If set X = {0, 2, 4, 6, 8, 9, 10, 11}& Y = { 0, 1, 2, 3, 4, 5, 6, 7}are given, then answer of X−*Y* is:

   A. {1, 3, 5, 7} B. {8, 9, 10, 11}

   C. {8, 9, 10, 11, 0, 1, 5, 7} D. {0, 1, 5, 7, 8, 9, 10, 11}

PART – II

5. If there are three sets P, Q, & R. If the all elements of R present in set P & set Q, then the following statement is true:

   A. P− Q = R B. PU Q = R

   C. P ∩ Q = R D. Q− P = R

6. For set A = {−2, −1, 0, 2, 4, 6, 8, 10, 14,15}& B = {−2, 2, 3, 4, 6, 7, 14,16}the answer is {−2, 2, 4, 6, 10, 14}, then the following alternative is true:

   A. A−B B. AUB

   C. B∩ A D. B−A

7. The answer of P − Q is {−5, −4, −3, 3, 4, 6, }, if set Q = {−2,0, 2, 5, 14}, then the elements of set P are:

   A. {−2,0, 2, 5, 14} B. {−5, −4, −3, 3, 4, 6, }

   C. {−5, −4, −3, 0, 2, 3, 4, 6, 14} D. {−5, −4, −3, −2, 0, 2, 3, 4, 5, 6, 14 }

8. If there are three sets X, Y, & Z. If all elements of Y & Z are present in X, then the following answer is true:

   A. X is superset of Y & Z B. X is proper subset of Y

   C. X is proper subset of Z D. X is proper subset of Y & Z

PART – III

9. Prove the commutative law for union on given sheet, when set A = {0, 1, 5, 7, 8, 9, 10, 11} & B ={ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}.

10. Solve (X U Y) U Z on the given sheet, when X = {−5, −4, −3, 3, 4, 6, 7, 8, 9,20, 21}, Y = {−5, −4, −3, 3, 4, 6, }& Z = {8, 9, 10, 11, 20, 21, 25, 27}.

11. Solve (P − Q) ∩ R on the given sheet, when P = {−6, −4, −3, 3, 4, 6, 7}, Q = {−5, −4, −3, 3, 4, 6, 8}& R = {1, 2, 8, 9, 10, 11, 12, 13, 15, }.

Appendix-J

CONSENT FORM

Dear parents: I am, Abdul Haq, scholar student of M. Phil. Education, working on an experimental study regarding students' mathematical skills. This experiment can develop your children's critical thinking, problem solving & communication skills. The experiment was consisted on six weeks from 15-Aug-2021to 30- Sep-2021. According to research ethics, informed consent from your good self is required to complete the experiment.

Thank You

I ---------- understood about the experiment study that my son would attend the classroom on regular & punctual bases.

---------- ----------

(Signature of Parents) (Date)

---------- ----------

(Signature of Experimenter) (Date)

---------- ----------

(Signature of Principal) (Date)

Stamp

Appendix-K

LESSON PLAN FOR GAGNE'S THEORY OF LEARNING

Name of teacher ----------

Date ----------

Name of the school ----------

Period duration: 40 minutes

Subject: Mathematics

Topic of the lesson: Sets

Sub-topic: union & intersection of sets

Grade 8 (Eight)

Phases	Events	Instruction
Pre-instructional phase	Gain attention	The teacher will greet the student before starting the lesson & will show them video or demonstration about the sets for Introducing the topic
	Objectives of the lesson	At the end of the lesson the leaners were able to: i. Know about union & intersection of sets ii. Demonstrate the union & intersection of sets iii. Differentiate between union & intersection of sets
	Recall previous knowledge	Asking different question from previous lesson about sets: - i. What is a set? ii. What are the types of sets? iii. Why we make sets? iv. How to take union & intersection of sets?
Instructional phase	Presenting stimulus materials	While demonstrating the utensils, books, copies sets on a flip chart, asking what these sets are called
	providing learning guidance	Elaborating & demonstrating how sets are being union & intersection by performing activities. Student will observe while demonstrating & presenting activities
	Eliciting the performance (practice)	i. Students were invited to perform and solve the questions in the group activity.

		$\{0,1,5,7,8,9,10,11,12,13,\}u\{-3,5,6,10,11,12,13,\}$ ii. Students were explaining the activities with each other by the brainstorming. Take intersection between A=$\{-3,5,6,10,11,12\}$ and set B=$\{0,1,5,7,8,9,10,11\}$ iii. Students were presenting their activities in the form of writing text under the supervision of teacher. Presenting union and intersection of set $\{0,1,5,7,8,9,10,11,12,13,\}$ and $\{1,5,7,8,9,10,11,12,13\}$
Post instructional phase	Providing feed back	After the pupils' presentations, the teacher would give them the positive feedback.
Post instructional phase	Providing feed back	After the pupils' presentations, the teacher would give them the positive feedback.
	Assessing the performance	Students was assessing on the base of given demonstration activities. i. What are sets? ii. What are types of sets? iii. What makes a set & give examples from daily life? iv. How to union & intersection of sets?
	Enhancing retention & transfer	Homework was given to the students: i. Union of $\{0,1,5,7,8,9,10,11,12,13,\}$& $\{-3,5,6,10,11,12,13,\}$ ii. Write uses of different sets in daily life iii. Intersection of $\{0,1,5,7,10,11,12,13,\}$&$\{-3,5,6,10,11,12,13,\}$

Procedures

Demonstrating flip chart of different sets, i. Types of sets ii. Subtraction of sets, iii. Union of sets, iv. Intersection of sets.	i. union of $\{0,1,5,7,8,9,10,11,12,13,\}$& $\{-3,5,6,10,11,12,13,\}$ ii. uses of different sets in daily life iii. Intersection of $\{0,1,5,7,10,11,12,13,\}$&$\{-3,5,6,10,11,12,13,\}$
	
Sets of sets example from daily life	Union & intersection of sets
Subtraction of sets example from daily	Types of sets example from daily life

Appendix-L

Module Title: **COOPERATIVE LEARNING METHODS FOR ENHANCING STUDENTS' MATHEMATICAL SKILLS**

Module Overview: This module is designed to introduce teachers to the concept of cooperative learning and its application in enhancing students' mathematical skills. The module will provide an overview of the key elements of cooperative learning, including its benefits, principles, and strategies, as well as tips for implementing cooperative learning in a mathematics classroom.

Module Objectives:

1. Understand the concept of cooperative learning and its benefits.
2. Identify the principles and strategies of cooperative learning.
3. Apply cooperative learning techniques to enhance students' mathematical skills.
4. Evaluate the effectiveness of cooperative learning in a mathematics classroom.

Module Outline:

I. Introduction to Cooperative Learning

- Definition of cooperative learning
- Benefits of cooperative learning
- Principles of cooperative learning

II. Strategies for Implementing Cooperative Learning

- Group formation
- Roles and responsibilities
- Group processes
- Assessment and evaluation