Characteristics of Chemically Reactive Casson Nanofluid Flow with Mixed Convection

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Candidate of **Master of Science in Math (MS MATH)** at the National University of Modern Languages do hereby declare that the thesis **Characteristics of Chemically Reactive Casson Nanofluid Flow with Mixed Convection** submitted by me in partial fulfillment of MS MATH degree, is my original work, and has not been submitted or published earlier. I also solemnly declare that it shall not, in future, be submitted by me for obtaining any other degree from this or any other university or institution. I also understand that if evidence of plagiarism is found in my thesis/dissertation at any stage, even after the award of a degree, the work may be cancelled and the degree revoked.

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ABSTRACT

Title: Characteristics of Chemically Reactive Casson Nanofluid Flow with Mixed Convection

This detailed research of chemically reactive Casson nanofluid with mixed convection is being covered in this thesis. It examines how radiation affects magnetohydrodynamic Casson fluid flow on an exponentially stretchable sheet. The effects of frictional heating and viscous dissipation on heat transfer are taken into consideration. The governing partial differential equations are transformed into ordinary differential equations using proper similarity transformation. We obtained confluent hypergeometric solutions to the heat and mass transport equations as well as zero-order analytical solutions to the momentum equation. The accuracy of analytical solutions is confirmed by numerical results obtained using a shooting strategy and the bvp4c integration scheme. Momentum, heat, and pressure are affected by the radiation parameter, the magnetic parameter, the Gebhart, Grashof, Prandtl, Eckert, and Schmidt numbers.

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LIST OF ABBREVIATIONS

MHD	-	Magnetohydrodynamic
ODE	-	Ordinary Differential Equation
PDE	-	Partial Differential Equation

LIST OF SYMBOLS

u, v	-	Velocity components in x and y directions
X, Y	-	Axial and normal coordinates
ν	-	Kinematics viscosity
Q_w	-	Gravity acceleration
B_T	-	Thermal expansion coefficient
T_∞	-	Ambient Temperature
C_∞	-	Ambient concentration
T	-	Fluid temperature
C	-	Fluid concentration
T_0	-	Constant
C_0	-	Constant
σ	-	Electrical conductivity
$B(x)$	-	Variable Magnetic field strength
ρ	-	Fluid density
k	-	Thermal conductivity
C_p	-	Specific heat at constant pressure
L	-	Characteristic length
σ^*	-	the Stefan -Boltzmann constant
k^*	-	Mean observation constant
Q_0	-	Heat generation coefficient
U_0	-	Reference velocity
U	-	Stretching velocity
D_m	-	Mass diffusivity
k_1	-	Chemical reaction rate
N	-	Exponential parameter
M	-	Magnetic parameter
G_r	-	Thermal Grashof number
P_r	-	Prandtl number

E_c	-	Eckert number
S_c	-	Schmidt number
S	-	Suction / injection parameter
F	-	Dimensionless velocity

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DEDICATION

This thesis work is dedicated to my parents, brothers and my teachers throughout my education career who have not only loved me unconditionally, but whose good examples have taught me to work hard for the things that I aspire to achieve.

CHAPTER 1

INTRODUCTION

Fluid is a substance that can flow, such as a liquid or gas. Fluids can be characterized by their density, viscosity, and pressure. They are affected by gravity and can be in a state of equilibrium or in motion. Fluids are important in many natural and industrial processes, including weather patterns, ocean currents, and hydraulic systems. Discipline of mathematics that concerns with the study of liquids and gases in motion is termed as fluid mechanics. It is an interdisciplinary field that draws on concepts from physics, engineering, and mathematics to understand the behavior of fluids under different conditions. One of the key concepts in fluid mechanics is the concept of fluid flow. This refers to the movement of a fluid from one point to another, and can take many different forms, such as laminar flow (smooth and orderly flow) or turbulent flow (chaotic and unpredictable flow). Another important concept is the concept of viscosity, which is the measure of fluid's resistance to flow. Fluid mechanics has many practical applications in real life. For example, it is used in the design of aircraft and ships, as well as in the design of industrial machinery such as pumps and turbines. In the field of medicine, fluid mechanics is used to understand blood flow through the human body, and in the field of meteorology, it is used to understand the movement of air and water in the atmosphere. Fluid mechanics is also used in the oil and gas industry to optimize the extraction of resources, and in the design of water treatment facilities.

In addition, the study of fluid mechanics is also essential for the understanding of natural phenomena such as ocean currents, atmospheric circulation, and weather patterns. It also plays a crucial role in the study of environmental issues such as pollution, climate change and erosion. In summary, fluid mechanics is a branch of mathematics that deals with the study of liquids and gases in motion, and it has a wide scale of applications in many fields such as aerospace, mechanical, civil, chemical and environmental engineering, physics and meteorology. The knowledge of fluid mechanics is essential in understanding and solving problems related to fluid flow in natural and man-made systems. Fluids can exist as liquids, gases, or plasma Climbala [19]. It is a substance with disappearing shear modulus, which means that it cannot withstand any applied force. Fluid is a basic requirement in everyday life and because of its relevance in many natural processes, scientists from all over the world are seeking to reveal numerous truths concerning fluid movement. Interest in the research of heat transmission with various physical features has increased during the past ten years. In the realm of chemical engineering, the issue of heat and mass transmission of a boundary layer over a stretching sheet in a porous medium has a significant applications Climbala et al [19],[63],[64]. Also, how forces affect fluid movement. It explains how stars, oceans, currents, tectonic plates, and blood circulation evolve. Wind turbines, oil pipelines, rocket engines, and air-conditioning systems are few examples of essential fluid flow uses, including hydrodynamic machinery, chemical processing devices, lubrication systems, polymer processing, crop freezing prevention and fog generation and dispersion. Archimedes was the first mathematician to establish the principle of fluid statics, which is now considered the fundamentals of fluid mechanics.

1.1 Nanofluid

Nanofluids are a relatively new class of fluids that have been the subject of intense research in recent years. They are composed of very small nanoparticles (typically on the order of a few nanometers in size) suspended in a liquid carrier. These nanoparticles can be made of a wide variety of materials, including metals, ceramics, and even polymers. A nanofluid is a common fluid with low thermal conductivity that is made up of nanoparticles with widths less than 100 nm. Choi [63] invented the phrase nanofluid to describe a new type of fluid. Compared to the basic fluids, these fluids are supposed to have superior thermal conductivity. By adding gold, copper, silver and other metal nanoparticles to the base fluid, it is feasible to enhance the nanofluid's thermal efficiency. Nanofluids could also be employed as refrigerants in the digital technologies and heavy vehicle industries. One of the key properties of nanofluids is that they have a much higher thermal conductivity than the base fluid in which they are suspended. This is because the small size of the nanoparticles allows them to be near one another, which increases the overall heat transfer between the particles.

This improved thermal conductivity of nanofluids has led to a wide range of potential applications, including use in heat exchangers, coolants for electronics and engines, and even in solar thermal energy systems. In addition to thermal conductivity, researchers have also observed increased viscosity and enhanced heat transfer due to natural convection in nanofluids. Another important property of nanofluids is their ability to improve the efficiency of various industrial processes. For example, they can be used as lubricants in machinery to reduce friction and wear, and they can also be used as catalysts to speed up chemical reactions. Despite the many potential benefits of nanofluids, there are still several challenges that must be overcome in order to fully realize their potential. One major challenge is the tendency of the nanoparticles to aggregate or settle out of the fluid over time, which can greatly reduce the fluid's effectiveness. Researchers are currently working on developing methods to stabilize nanoparticles in the fluid and prevent aggregation. Additionally, the toxicity of certain types of nanoparticles used in nanofluids is also a concern. Some nanoparticles, such as those made of certain metals, can be toxic if they are released into the environment or if they encounter human skin. Therefore, it is important to carefully consider the materials used in the nanoparticles and to take appropriate precautions when working with these fluids. Despite these challenges, nanofluids show great promise to enhance thermal conductivity and improve the efficiency of various industrial processes. Numerous study findings about the several ways that Nano fluids can be used to control flow are produced by Almatroud et al [5]. With ongoing research and development, it is likely that we will see increasing use of these fluids in many applications in the coming years.

1.2 Megnetohydrodynamics (MHD)

Megnetohydrodynamics (MHD) is the exploration in the behavior of electrically conductive fluids in the existence of magnetic fields. It is used to describe a wide range of phenomena, including the dynamics of the Earth's magnetic field, the behavior of plasmas in stars and galaxies, and the behavior of liquid metals in fusion reactors. One of the key principles of MHD is that a magnetic field can exert a force on a fluid. This is known as the Lorentz force, and it is given by the equation $F = J\,x\,B$, where J represents the current density and B represents the magnetic field. The Lorentz force can cause the fluid to move in a certain direction, and it can also generate heat through a process known as Joule heating.

Another important principle of MHD is that a fluid can also generate a magnetic field. This is known as the induced magnetic field, and it is given by the equation $B = \mu^0 * (J\ x\ V)$, where μ^0 is the permeability of free space and V is the velocity of the fluid. This means that when a fluid is in motion, it can generate a magnetic field that is perpendicular to both the fluid's velocity and the current density.

The combination of these two principles leads to a wide range of phenomena that are described by MHD. One of the most well-known is the MHD generator. It is an apparatus that transforms fluid kinetic energy into electrical energy. This is done by using a magnetic field to force a fluid to move in a certain direction, which in turn generates a current that can be used to generate electricity. Another important application of MHD is in the study of the dynamics of plasmas. Plasmas are extremely hot, ionized gases that are found in stars and galaxies. The behavior of plasmas is determined by the balance between the pressure gradient force and the Lorentz force. This balance leads to a wide range of phenomena, including the formation of stars and galaxies, the dynamics of solar flares, and the behavior of supernovae. In addition to these applications, MHD is also used to study a wide range of other phenomena, including the behavior of liquid metals in fusion reactors, the dynamics of the Earth's magnetic field, and the behavior of the solar wind.

MHD is an interdisciplinary field, and researchers from a wide range of fields, including physics, engineering, and astrophysics, contribute to its development. The stagnation point is a place in a fluid flow when the fluid's velocity is zero in fluid dynamics. This point is also known as a "dead water" or "dead air" point, as the fluid is not moving at this location. The stagnation point is an important concept in fluid dynamics as it plays a crucial role in the analysis of fluid flow patterns. Analyses that how chemical processes affect the mass transmission and the free movement of an incompressible viscous, electrically conducting fluid on a tensile surface under a constant transverse magnetic field by Affify [2]

1.3 Casson Fluid

Casson fluid is a non-Newtonian fluid with a yield stress, which means that the liquid will only start flowing when a specific level of stress is applied. This contrasts with Newtonian fluids, such as water and oil, which illustrate that stress and strain are linearly related, and begin to flow as soon as an external force is applied. Casson fluid is named after its discoverer, Anthony Casson. This kind of fluid is commonly found in industrial and biological systems. In industrial applications, Casson fluid is often used in the manufacturing of certain types of lubricants and in the oil and gas industry. In biological systems, Casson fluid can be found in the flow of blood in the human body and in the movement of certain types of fluids in plants. The uses of non-Newtonian fluids are vast, including oil recovery filtration, polymer engineering, ceramics production and petroleum production. Additionally, it is crucial for designing solid grids, producing heat for geothermal energy, recycling nuclear waste, and developing petroleum reservoirs, among other things. Due to the nonlinear interaction between stress and strain, non-Newtonian fluids are much complicated as compared to Newtonian fluids. To explore non-Newtonian fluids, several frameworks have been devised, however no one model can describe all their features. The behavior of Casson fluid can be modeled using the Casson equation, this explains the connection between the fluid's shear rate, yield stress, and shear stress. This equation can be used to predict the behavior of the fluid under different conditions and to design systems that utilize Casson fluid. One of the key properties of Casson fluid is its viscoelastic behavior, meaning that it exhibits both viscous and elastic behavior. This is because the fluid will resist flow until a certain amount of stress is applied, at which point it will begin to flow and behave like a viscous fluid. However, once the stress is removed, the fluid will return to its initial state, behaving like an elastic solid.

Casson fluid has a high viscosity and approaches infinity at zero rate of shear. If the fluid's shear stress does not exceed its yield stress, the Casson fluid behaves like a solid Zhou et al [74],[64] investigated the thorough effects of heat and mass transport of magneto hydrodynamic Casson fluid over a moving wedge with slip, nonlinear thermal radiation, a constant source and sink of heat, and a chemical reaction. Relative to the Newtonian fluid, the Casson fluid is more excellent at the heat transfer phenomena.

1.4 Thesis Contributions

In this thesis, we present a thorough review of the research work by kamerwarsan [38] The study is extended by including other effects such as non-linear mixed convection, chemical reaction effect, and an inclined magnetic field. In this research work, we converted the system of partial difference equations (PDEs) by using similarity transformations into nonlinear ordinary differential equations (ODEs). We apply the shooting method is to get the numerical results. We used MATLAB software package to implement this and find the results. All the impacts of above mentioned are represented in this work in chapter 5 in the form of graphs and results are shown in the form of tables. Several appropriate physical parameters have been discussed in this thesis. And the conclusion we find from this extended work are mentioned in chapter 6.

1.5 Layout of Thesis

This dissertation is further composed of the following chapters as below:

Chapter 2

In this chapter literature review is discussed.

Chapter 3

In this chapter, we have written basic definitions, laws and concepts that will help to understand the upcoming work. The mathematical model and shooting method are also mentioned on the final page of this chapter.

Chapter 4

This chapter provides review of work done by Kamerwarsan [38] in detail. The mathematical results are achieved by solving the system of ODEs and applying shooting technique after an appropriate similarity transformation converts the PDEs in this study into ODEs.

Chapter 5

This chapter includes the extension of the work discussed in chapter 4. We have added non-linear mixed convection and the effects of chemical reaction to the model. The mathematical results are achieved by applying similar techniques used in the review work. All the obtained results are presented in the form of table and graphical behavior of different parameters added are shown by graphs.

Chapter 6

This chapter provides detailed conclusion achieved in this research work. Finally, the references used in this study are expressed in reference section.

CHAPTER 2

LITERATURE REVIEW

2.1 Non-Newtonian

A non-Newtonian fluid is one that deviates from Newton's law of viscosity, which specifies that viscosity should remain constant regardless of stress. Viscosity can transform from liquid to solid when non-Newtonian fluids are pressed. For instance, ketchup changes when shaken from being a Newtonian fluid to becoming runnier. Molten polymers and salt solutions are examples of non-Newtonian fluids, also numerous everyday items including toothpaste, custard, blood, starch suspensions, paint, corn starch, shampoo and melted butter. In fluid mechanics, fluid's shear characteristics can be described by the viscosity of the fluid but at times it is insufficient to describe non-Newtonian fluids. The easiest way to study them is to use a range of different rheological properties, which relate stress and strain rate tensors in a variety of flow scenarios, such as oscillatory shear or extensional flow, and are measured using various tools or rheometers. Stretching a sheet is the process of applying force on a flat, thin material, such as fabric or metal, to extend its length and decrease its width. In manufacturing and construction, this procedure is frequently used to shape and size materials for specific needs. Stretching a sheet causes the fibers or molecules within the material to be drawn apart and restructured, causing the sheet to become thinner and longer. Stretching a sheet can be accomplished using mechanical equipment, heat, or chemical treatments. Stretching a sheet can also be used to increase the qualities of the material, such as its strength, ductility, and flexibility. Research by Abo-El [1] has focused on the boundary layer concept in micropolar fluids as they flow past a linear stretching surface. This study provides valuable insights into the characteristics and behavior of micropolar fluids and can be applied to a wide range of industrial and biological applications. The results of this study can also be used to improve the understanding and modeling of non-Newtonian fluid flows, and to develop new techniques for controlling and manipulating these types of fluids.

It is preferable to examine the characteristics using tensor-valued constitutive equations, which are common in the field of continuum mechanics. Kempannagari [40] studied the convective heat transfer properties of a micropolar fluid flowing over an exponentially curved surface at its magnetohydrodynamic stagnation point. Pramanik [57] established that increasing the Casson parameter will increase the surface stress and described the heat transfer flow past in a stretching sheet with radiation. Hayat [30] presented a study on heat transfer analysis of Casson fluid flowing past a stretched sheet when a magnetic field is present. C.sulochanna [16] discussed the slip effects in convective MHD flow towards a spreading sheet. They discovered that raising the magnetic field parameter values reduced fluid velocity.

2.2 Megnetohydrodynamics

The study of MHD (magnetohydrodynamics) flow and heat transfer problems has gained a significant amount of attention in recent years, due to its numerous engineering applications. One of the most notable applications of MHD is in the cooling of continuous strips or filaments, such as in the annealing and tinning of copper wires. These industrial processes involve drawing the strips through a quiescent fluid, and sometimes stretching them as well. The properties of the final product, such as its cooling rate, greatly depend on the rate of cooling. Another important application of MHD in the field of metallurgy is the purification of molten metals from non-metallic inclusions through the use of a magnetic field investigated by Datti [22]. Pavlov [55] has studied the MHD boundary layer flow of an electrically conducting fluid in the presence of a magnetic field due to the stretching sheet. The results of these studies have significant implications for the understanding and control of MHD flow and heat transfer processes and can help to improve the efficiency and effectiveness of industrial processes that rely on MHD.

In recent years, many scientists and engineers have devoted significant time and effort to studying the effects of heat and mass transfer, both analytically and numerically, due to their practical applications. These effects are commonly studied in fields such as chemical processing, oceanic circulation, emergency cooling systems for advanced nuclear reactors, cooling processes for plastic sheets, fog formation and dissipation, food processing and drying, temperature distribution and moisture in agricultural fields, and polymer production. Non-Newtonian fluids, which have more complex rheological behavior than Newtonian fluids, are widely used in industrial and technical processes, making the study of heat and mass transport in these fluids an important theoretical and practical issue. Additionally, multiple researchers have examined the effect of MHD flow on various fluid models with convective boundary flows across the peripheral layer, which are used in technologies such as manufacturing and nuclear reactors.

2.3 Porous medium

A porous medium is a material that contains a network of pores, or small openings, that allow fluids to flow through it. These pores can be found in natural materials such as soil and rock, as well as in artificial materials such as ceramics and metal foams. The study of fluid flow and heat transfer in porous media is known as porous media fluid mechanics. Porosity can vary widely in nature, from less than 1% in some rocks to more than 50% in some soils. The permeability of a porous medium, which is a measure of the ease with which a fluid can flow through it, is closely related to porosity. Another important characteristic of porous media is the pore size distribution, which describes the range of pore sizes present in the material. Pore size distribution can be measured using a variety of techniques, such as mercury intrusion porosimetry. The pore size distribution can have a significant impact on the fluid flow and heat transfer characteristics of a porous medium. One of the most important applications of porous media fluid mechanics is in the study of subsurface flow and transport. In this context, porous media are used to model the flow and transport of fluids, such as water and oil, through soil and rock formations. This is important in understanding and predicting the behavior of subsurface systems, such as aquifers, oil reservoirs, and geothermal systems. Important application of porous media fluid mechanics is in the study of heat transfer in porous materials. Another important application of porous media fluid mechanics is in the study of heat transfer in porous materials. Porous materials are often used as thermal insulation and understanding the heat transfer characteristics of these materials is important for optimizing their thermal performance. Porous media can also be used as heat exchangers and understanding the heat transfer characteristics of these materials is important for improving their efficiency. Porous media fluid mechanics also has applications in various industrial processes, such as filtration, catalysis, and biotechnology. In filtration, porous media are used to remove particles and impurities from fluids. In catalysis, porous media are used to increase the surface area available for chemical reactions to take place. In biotechnology, porous media are used to culture cells and tissues.

Porous media can also be used to model the flow and transport of fluids in biological systems, such as in the study of blood flow through the cardiovascular system or the flow of fluids through the lungs. The study of heat transfer in elastic fluids flowing over a stretching sheet has been a topic of research for several authors. Subhas, [67] have conducted research on this topic, specifically focusing on the heat move of an elastic fluid over a stretching sheet. Similarly, Bujurke, [14] have studied heat exchange phenomena in second order fluid flow across a stretched surface with internal heating and viscous dissipation. This study provides valuable insights into the complexities of heat transfer in elastic fluids and how it is affected by internal heat generation and viscous dissipation. Prasad, [58] have also studied this topic, specifically analyzing the problem of elastic fluid flow and heat transfer in a porous medium over a non-isothermal stretching sheet with variable thermal conductivity. This study provides a deeper understanding of the heat transfer in elastic fluids in porous media and how it is affected by variable thermal conductivity. Additionally, K. Prasad, [37] have briefly shows the diffusion of a chemically reactive species of a non-Newtonian fluid immersed in a permeable medium over a extending sheet. This study provides insight into the chemical reactions and diffusion in non-Newtonian fluids in porous media and how it is affected by a stretching sheet. These studies provide valuable information for understanding and modeling heat transfer in elastic fluids and can be applied to many industrial processes.

2.4 Magnetic field

When it comes to specific issues involving the flow of conductive physiological fluids, such as blood and blood pumping devices, the impact of magnetic fields on peristaltic mechanisms are significant. The effects of a magnetic field on peristaltic flow have been the subject of discussions among numerous researchers. Malga et al [50]. Benazir[33] described the consequences of a non-steady magneto hydrodynamic Casson fluid flow across a flat plate and vertical shape with a non-uniform heat source. Anatha et al [6] also described the effects of Casson fluid by assuming the laminar fluid motion. Nadeem [52] investigated the effects of MHD on heat and mass transfer in a Casson fluid flowing past a stretching surface. Sandeep [17] examined the impact of aligned magnetic fields on the flow of nanofluids in thin films. Gireesha [26] proposed a numerical solution for boundary layer MHD heat and mass transfer across a stretching sheet, taking into account the effects of chemical reactions. M Khan [41] studied the effect of magnetic behavior on the MHD flow of a Carreau fluid under convective boundary conditions.

2.5 Radiation

When something emits energy, such as electromagnetic waves or particles, it is said to be radiating. This energy can take many forms, including x-rays, gamma rays, radio waves, heat, and light. Radiation is present all around us and is used in a variety of applications, such as in medicine, industry, and energy production. Ionizing radiation is one of the most well-known types of radiation. It is the kind of radiation with enough energy to release firmly bonded electrons from atoms, resulting in ions. Along with alpha and beta particles, this type of radiation also includes x-rays and gamma rays. In order to move heat away from the surface, thermal radiation is essential. It is used in industrial fields like helicopters, spacecraft and high-temperature systems. Reddy [61]examined how the flow of a non-linear stretched sheet of MHD Nano fluid was affected by thermal radiation and suction. There is heat radiation present, Kothandapani [44] a tapered asymmetric channel of a Williamson nanofluid that exhibits peristaltic transport. Using a radiative Carreau nanofluid over a stretched surface, Soret and suction/blowing effects on MHD stagnation point flow were examined.Sulochanna [68]. Kho [43] studied how radiation affected MHD heat transfer and mass transfer Casson on a porous stretched sheet, nanofluid flows. Dessi [23] evaluated how radiation and velocity slip affected a linear elongated sheet's Casson fluid magnetohydrodynamic stagnation point stream with convective boundary conditions.

2.6 Fluid kinematics

The study of fluid motion without taking into account the forces causing the motion is known as fluid kinematics. It deals with the description of fluid flow in terms of velocity, acceleration, and other related parameters. In fluid kinematics, we use concepts such as flow, velocity, streamline, turbulence, viscosity, Reynolds Number, laminar flow, turbulent flow, stream function, pressure, density, continuity equation, Navier-Stokes equations, Bernoulli's principle, Poiseuille's law, shear stress, compressible flow, incompressible flow, Mach number, friction factor, boundary layer, and vorticity to understand the behavior of fluids in motion. The study of fluid kinematics is important in various fields such as aerospace, mechanical engineering, chemical engineering, and many more. The understanding of fluid kinematics is crucial in the design and analysis of fluid systems such as pipes, pumps, and turbines. The study of fluid kinematics is also important in understanding natural phenomena such as ocean currents, weather patterns, and fluid flow in the human body. Bhatti [13] . The effects of radiation on Williamson nanofluid moving through a porous surface were examined. Farooq [25] assessed how non-linear radiation affected the MHD flow of a viscoelastic nano liquid that was directed by a stretched surface. Gireesha [27] employed a shot technique to investigate the Maxwell fluid's heat transfer properties with non-linear radiation. Khan [41] addressed heat transfer across a heated non-linear surface in the presence of non-linear radiation. Similar solutions for nonlinear radiative rotating flow of ferrofluid with Joule heating were presented by Kumar [45] . Pal [53] investigated the impact of thermal radiation on the flow of a stretched sheet-prescribed nonnewtonian micropolar nano liquid. Mahanthish [48] explored the Maxwell nanofluid's nonlinear convection flow and radiative heat transfer. The RK technique was utilized to provide numerical solutions. Researchers looked at non-Newtonian fluid flowing across an extended sheet by nonlinear convection in the presence of nonlinear thermal radiation Mahanthisha [49].

2.7 Brownian motion

Brownian motion, also known as "random walk," is the arbitrary motion of particles floating in a fluid, triggered by the collision of the particles with the fluid molecules. This phenomenon was first observed by Robert Brown in 1827 and is a fundamental concept in fluid mechanics. The motion of the particles is characterized by a random walk in which the particles move in a random direction and distance at each step. The distance and direction of each step are determined by the collisions of the particles with the fluid molecules. The magnitude of the displacement of each step is determined by the temperature of the fluid, with higher temperatures leading to larger displacements. In a fluid, the random motion of the elements can be explained by the diffusion equation, which describes the change in the concentration of particles over time. The diffusion equation is governed by the coefficient of diffusion, which is determined by the properties of the fluid and the size of the elements.

Brownian motion is important in fluid mechanics because it plays a role in many natural phenomena, such as the transport of heat and mass in fluids. For example, in a fluid, heat is transported by the random motion of particles, and this process is known as thermal diffusion. Similarly, mass is transported by the random motion of particles, and this process is known as diffusion. In addition, Brownian motion plays an important role in a wide range of industrial and biological processes, such as particle separations, mixing, and chemical reactions. For example, in particle separations, Brownian motion causes the particles to randomly move around and eventually settle down based on their size, which allows for separation of different particle sizes. In mixing, the random motion of particles causes them to move around and come into contact with one another, which leads to more efficient mixing. In chemical reactions, Brownian motion allows for the particles to come into contact with one another, which increases the likelihood of a chemical reaction occurring. Brownian motion also plays a role in the behavior of colloids, which are suspensions of particles in a fluid.

Colloids are important in many industrial and biological applications, such as paint, ink, and food. The random motion of the particles in a colloid causes them to move around and come into contact with one another, which leads to the formation of aggregates.

Brownian motion is also important in the behavior of suspensions, which are fluids containing particles that are large enough to be seen with the naked eye. Suspension is important in many industrial and biological applications, such as paint, ink, and food. The random motion of the particles in a suspension causes them to move around and come into contact with one another, which leads to the formation of aggregates. Brownian motion is also important in the behavior of polymers, which are long chains of molecules. Polymers are important in many industrial and biological applications, such as plastics, rubber, and biological macromolecules. The random motion of the polymer molecules causes them to move around and come into contact with one another, which leads to the formation of aggregates. In summary, Brownian motion is a fundamental concept in fluid mechanics that describes the random motion of particles suspended in a fluid. It plays a role in many natural phenomena, such as the transport of heat and mass in fluids and has practical applications in a wide range of industrial and biological processes, such as particle separations, mixing, chemical reactions, colloids, suspensions, and polymers. Understanding the principles of Brownian motion can help engineers and scientists to design more efficient and effective processes in various fields.

2.8 Suction

Suction in fluid dynamics refers to the creation of a partial vacuum or low-pressure region within a fluid flow. This low-pressure region is created by the movement of the fluid, and it can result in a force that pulls other fluids or objects towards the vacuum. Suction is a fundamental concept in fluid dynamics and is used in a wide range of applications, from industrial processes to biological systems. One of the most common ways that suction is created in fluid dynamics is using a pump. A pump is a device that moves a fluid from one location to another by creating a pressure difference. The pressure difference is created by the pump's impeller, which rotates and creates a low-pressure region on the inlet side of the pump. This low-pressure region creates suction that pulls the fluid into the pump and pushes it out on the outlet side. Another way that suction is created in fluid dynamics is using a venturi. A venturi is a device that is used to increase the speed of a fluid by constricting its flow. As the fluid flows through the constriction, its velocity increases, and its pressure decreases. This decrease in pressure creates a low-pressure region that creates suction. Suction can also be created in fluid dynamics using a nozzle. Through the reduction of a fluid's cross-sectional area, nozzles accelerate fluids. The pressure of the fluid drops as it moves faster through the nozzle. Suction is produced as a result of the low-pressure area that results from this pressure drop. In addition to these examples, suction is also found in natural phenomena such as ocean waves, river flows and even in animals and plants. For example, suction is used by fish to suck in food, and by some animals to extract nectar from flowers. Also, some plants use suction to transport water and nutrients from the soil to the leaves. Suction is also an important concept in industrial processes such as vacuum cleaning, and in (Heating, ventilation, and air conditioning) HVAC systems. In vacuum cleaning, suction is used to pull dirt and debris into the vacuum cleaner. In HVAC systems, suction is used to pull air into the system, where it is then heated or cooled before being distributed throughout a building. Fang [24] a stretched surface's unstable boundary layer flow was examined. Yao [73] examined the impact of a convective surface boundary condition on the boundary layer flow of a viscous fluid. Hayat [29] investigated the flow of a Maxwell fluid with heat and mass transfer in the presence of a chemical reaction. Hayat [30] proposed a study on the impact of radiation on the MHD flow created by a stretching sheet. Ahmed et al [3] examined the boundary layer flow of a second-grade fluid over an arbitrary velocity stiff sheet. Kandasmny [39] researched the flow of a viscous fluid across a porous shrinking sheet when there is suction.

In summary, suction in fluid dynamics refers to the creation of a partial vacuum or low-pressure region within a fluid flow. This low-pressure region is created by the movement of the fluid and can result in a force that pulls other fluids or objects towards the vacuum. Suction is used in a wide range of applications, from industrial processes to biological systems and natural phenomena, and is an essential concept in fluid dynamics. It is created by pumps, venture, nozzle and other devices that create pressure difference, velocity change or cross-sectional area change. Suction is used in everyday life in various ways such as vacuum cleaning and HVAC systems.

2.9 Non-Linear Mixed Convection

In fluid dynamics, the simultaneous occurrence of forced and natural convection is referred to as non-linear mixed convection. In forced convection, an outside force, such as a fan or pump, drives the fluid flow, as opposed to natural convection, where the fluid movement is driven by changes in density. Because the velocity and temperature fields are not proportional to one another, the combination of these two convection mechanisms creates a non-linear flow, which results in a more complex flow pattern. The production of vortices and turbulent flow is one of the primary characteristics of non-linear mixed convection. The interaction between induced and natural convection causes these vortices and turbulent flow, which can significantly increase heat and mass transmission. This is so that heat and mass may be transferred more effectively since the fluid is better mixed as a result of the vortices and turbulent flow. Heat exchangers, boilers, and condensers, among other industrial processes, frequently use non-linear mixed convection. Both natural convection, which is brought on by the temperature difference between the fluid and its surroundings, and forced convection, which is brought on by a fan or pump, drive the fluid flow in these processes. When these two convection mechanisms are combined, heat is transferred more effectively, which enhances the efficiency of the industrial process. The purpose of the Hayat et al [28] research is to describe the mixed convective boundary layer stretching flow caused by the combined action of heat and mass transfer in the presence of thermophoresis. The porous space is occupied by an incompressible Maxwell fluid. Previously, Prahabat [56] offered a comprehensive assessment of convective transport in nanofluids to find a suitable explanation for the aberrant rise in thermal conductivity and viscosity. Khan [42] investigated nanofluid flow across a stretched sheet. Das [21] created a computational solution for heat transfer and nanofluid boundary layer flow across a stretching surface with convective boundary conditions. Das investigated the flow of nanofluids across a nonlinear stretching sheet under partial slip circumstances. examined how radiation affected a nonlinearly stretched sheet of viscous nanofluid. The characteristics of a mixed convection flow of nanofluid on a vertical flat plate encased in a porous substance.

2.10 Injection

In fluid mechanics, injection refers to the process of introducing a fluid into a flow system. This can be done in a variety of ways, such as through a nozzle, a pipe, or a valve. The fluid being injected may be a liquid, a gas, or a mixture of both. One important aspect of injection is the concept of mass flow rate, which is the amount of mass of the fluid being injected per unit of time. This can be controlled using a valve or a nozzle, which can be adjusted to vary the size of the opening through which the fluid is flowing. Another important aspect of injection is the concept of momentum, which is the product of the mass and velocity of the fluid being injected. This can be used to control the direction and speed of the fluid as it is injected into the flow system.

Injections can also be used to create specific types of flow patterns, such as turbulent or laminar flow. Injections can also be used to create vortices, which are swirling patterns of fluid flow. Injections can also be used in a variety of practical applications, such as in the manufacturing of certain products, in the operation of power plants, and in the aerospace industry. In industrial processes, injection is commonly used to introduce a fluid into a reactor, where it can mix with other fluids to create a desired chemical reaction. In power plants, injection is used to cool the turbine blades, which prevents them from overheating and malfunctioning. In the aerospace industry, injection is used to control the movement of air around the surface of an aircraft, which helps to reduce drag and increase lift.

Injection can also be used for mixing fluids, either through laminar or turbulent flows. In laminar flows, the fluid injected tends to follow a streamline trajectory and tends to mix slowly. On the other hand, turbulent flows tend to mix fluids more quickly and efficiently.

This can be used in industrial processes such as chemical reactions, where it is important to ensure that the reactants are well mixed. Injection can also be used to enhance heat transfer in fluid systems. For example, in a nuclear power plant, coolant is injected into the reactor core to transfer heat away from the fuel rods and into the steam turbine. Injection can also be used to enhance heat transfer in other types of systems, such as in automobiles and electronics.

Injection can also be used to control the flow patterns of fluids in a flow system. For example, in a pipe, injection can be used to create a turbulent flow, which can help to reduce the buildup of sediment and other materials on the pipe walls. Injection can also be used to create a laminar flow, which can help to reduce friction and increase the efficiency of the flow system. Injection can also be used to control the pressure of a fluid in a flow system. For example, in a pipe, injection can be used to increase the pressure of a fluid, which can help to push it through the pipe more quickly. Injection can also be used to decrease the pressure of a fluid, which can help to reduce the amount of energy required to move it through the pipe.

Non-linear mixed convection is also found in natural phenomena such as volcanic eruptions and geysers. In volcanic eruptions, both natural convection, which is induced by the temperature variation between the magma and the neighboring rock, and forced convection drive the fluid movement, which is caused by the gas pressure within the magma. The combination of these two convection mechanisms leads to the formation of complex flow patterns and the development of turbulent flow. In addition, non-linear mixed convection also occurs in biological systems such as blood flow in the human body. Blood flow is driven by both natural convection, which is caused by the temperature difference between the blood and the surrounding tissue, and forced convection, which is caused by the heart's pumping action. The combination of these two convection mechanisms leads to a more efficient transport of oxygen and nutrients throughout the body. In summary, non-linear mixed convection in fluid dynamics refers to the flow of a fluid where both natural and forced convection mechanisms occur simultaneously. The combination of these two convection mechanisms leads to a more complex flow pattern, and the formation of vortices and turbulent flow. Non-linear mixed convection is commonly found in industrial processes such as heat exchangers, boilers, and condensers, as well as in natural phenomena such as volcanic eruptions and geysers, and in biological systems such as blood flow in the human body. The interaction between the natural and forced convection mechanisms enhances the mixing of the fluid, resulting in a more efficient transfer of heat and mass.

2.11 Chemical Reactions

Chemical reactions in fluid dynamics refer to the process of chemical substances undergoing changes in composition and properties as a result of interactions with other substances. These reactions can occur in both liquids and gases, and they can have a significant impact on the behavior of the fluid. Chemical reactions in fluids can be classified into two main categories: homogeneous and heterogeneous reactions. Heterogeneous reactions occur when the reactants are in different phases. An example of a homogeneous reaction is the reaction between hydrogen and oxygen to form water, while an example of a heterogeneous reaction is the reaction between a solid catalyst and a gas to form a new product. Interphase transport influenced the exchange, convection, and dispersion coefficients, according to the study. They also looked at unstable solute diffusion in an annulus, the pattern of dispersion and uptake of a gas within a bronchial wall, the transport of a tracer material through a wall layer, and the effects of retention and irreversible processes on dispersion. They also investigated the dispersion of a tracer in an annular flow, which may be applied to catheterized arteries, as well as the effect of permeable wall features on dispersion in an oscillatory flow in an annulus Jonnadula [34]. The effect of a reaction on the MHD heat and mass transfer boundary layer flow through a extending sheet was explored. Magnetohydrodynamic nanofluid flow with chemical reaction and power-law velocity was examined by utilizing a stretching technique by Hayat [28]. N. Pandya [54] studied the effects of chemical reaction, radiation, Soret and Dufour on unsteady MHD dusty fluid flow via an inclined porous plate in a porous medium. Qayyum [59] Under chemical reaction, heat, and mass transfer circumstances, the influence of Newtonian stagnation point flow on a magnetohydrodynamic Walters-B nanofluid was studied.Sreedhar [66] examined turbulent MHD flow with heat absorption and chemical reactivity through an infinitely porous vertical plate.

One of the key features of chemical reactions in fluids is that they can lead to changes in the fluid's temperature, density, and viscosity. These changes can have a significant impact on the fluid flow patterns and can lead to the formation of vortices and turbulent flow. For example, exothermic reactions (reactions that release heat) can cause an increase in the fluid's temperature, leading to an increase in its density and viscosity. Chemical reactions in fluids also play a crucial role in various industrial processes such as oil refining, pharmaceutical manufacturing, and chemical synthesis. In oil refining, chemical reactions are used to convert crude oil into useful products such as gasoline and diesel fuel. In pharmaceutical manufacturing, chemical reactions are used to synthesize active ingredients for drugs. In chemical synthesis, chemical reactions are used to produce new compounds with specific properties.

Chemical reactions in fluids also occur in natural systems such as the atmosphere and oceans. For example, in the atmosphere, chemical reactions between gases such as carbon dioxide and water vapor lead to the formation of acids such as carbonic acid. In the oceans, chemical reactions between dissolved gases such as carbon dioxide and water lead to the formation of carbonic acid and bicarbonate ions. These reactions play a crucial role in regulating the pH of the oceans and in the carbon cycle. In summary, chemical reactions in fluid dynamics refer to the process of chemical substances undergoing changes in composition and properties as a result of interactions with other substances. These reactions can occur in both liquids and gases and can be classified into two main categories: homogeneous and heterogeneous reactions. Chemical reactions in fluids can impact the temperature, density, and viscosity of the fluid, as well as the flow patterns of the fluid. They play a crucial role in a variety of industrial processes, such as oil refining, pharmaceutical manufacturing, and chemical synthesis, and are also found in natural systems, such as the atmosphere and oceans. The study of the instability of two-dimensional stagnation point flow in an incompressible viscous fluid over a stretching sheet has been a topic of research for several authors. Nazar was one of the first researchers to investigate this problem, using an analytical method known as homotropy analysis to solve the nonlinear partial differential equations involved. Sajid has also conducted research on this topic, specifically focusing on the unsteady axisymmetric flow and heat transmission across a stretching sheet, using homotropy analysis.

2.12 Exponential Sheet

An exponential sheet in fluid mechanics refers to a sheet or surface that changes in size or shape at a constant exponential rate. These types of surfaces are often used to model various phenomena, such as the flow and heat transfer of fluids over stretching or shrinking surfaces. One of the most important applications of exponential sheets in fluid mechanics is in the study of the boundary layer flow of fluids over these surfaces. The boundary layer flow refers to the flow of fluid in the immediate vicinity of a surface, and it is affected by the rate of change of the surface. The study of the boundary layer flow over exponential sheets can provide insight into the behavior of fluids in many industrial processes, such as in heat exchangers and fluid-based manufacturing processes.

Another important application of exponential sheets in fluid mechanics is in the study of the effect of surface tension on fluid flow. The surface tension of a fluid is a measure of the force exerted by the fluid on the surface, and it can have a significant impact on the flow of the fluid. The study of the effect of surface tension on fluid flow over exponential sheets can provide insight into the behavior of fluids in many industrial processes, such as in the production of thin films and coatings. Exponential sheets are also used to model the flow and heat transfer of non-Newtonian fluids, such as viscoelastic fluids, over stretching or shrinking surfaces. The study of the flow and heat transfer of these types of fluids over exponential sheets can provide insight into the behavior of fluids in many industrial processes, such as in the production of polymers and other viscoelastic materials. Exponential sheets are also important in the study of fluid-structure interactions, which refers to the interactions between a fluid and a solid structure. The study of fluid-structure interactions over exponential sheets can provide insight into the behavior of fluids in many industrial processes, such as in the design of offshore platforms and other structures that are exposed to fluid flow.

Exponential sheets also play a role in the study of mass transfer and diffusion of species in a fluid. This study provides insight into the chemical reactions and diffusion in non-Newtonian fluids in porous media and how it is affected by a stretching sheet. In summary, exponential sheets in fluid mechanics are surfaces that change in size or shape at a constant exponential rate. These types of surfaces are used to model various phenomena, such as the flow and heat transfer of fluids over stretching or shrinking surfaces. They are important in the study of boundary layer flow, surface tension, non-Newtonian fluids, fluid-structure interactions, mass transfer and diffusion of species in a fluid. Understanding the principles of exponential sheet can help engineers and scientists to design more efficient and effective processes in various fields. A mathematical analysis has been conducted to study the momentum and heat transfer in a viscoelastic fluid flow over an exponentially stretching impermeable sheet.

The analysis involved the use of mathematical techniques to convert the highly non-linear differential equations that describe the flow and heat transfer into a set of ordinary differential equations. This was achieved by applying similarity transformations to the equations, which allowed for the use of more manageable mathematical methods. The analysis then proceeded by obtaining analytical solutions for the transformed momentum equation through the repeated solving of a non-linear Riccati type equation. This equation is a complex mathematical equation that describes the flow and heat transfer in the viscoelastic fluid, and it was solved using a combination of mathematical techniques and numerical methods. The analysis also included the calculation of a zero-order approximate solution for the stream function, which is a mathematical function that describes the flow of the fluid. This calculation was an important step in understanding the behavior of the fluid flow and heat transfer over the exponentially stretching impermeable sheet. Overall, the mathematical analysis provided a detailed understanding of the momentum and heat transfer in a viscoelastic fluid flow over an exponentially stretching impermeable sheet. The analytical solutions obtained from the analysis can be used to predict the behavior of the fluid flow and heat transfer in similar situations and can provide valuable information for the design and optimization of industrial processes that involve viscoelastic fluids elaborate by Sanjayanad [65].Bhattacharyya [12] studied the boundary layer flow and heat transfer over an exponentially shrinking sheet .The boundary layer flow refers to the flow of fluid in the immediate vicinity of a surface and it is affected by the rate of change of the surface. An exponentially shrinking sheet is a surface that changes in size or shape at a constant exponential rate. The analysis of this type of flow is important in understanding various industrial processes that involve fluid flow and heat transfer over shrinking surfaces.

The characteristics of a fluid that change with temperature and pressure are referred to as heat space dependent parameters in fluid dynamics. These characteristics include specific heat, thermal conductivity, viscosity, and density. For effective modelling and simulation of fluid flow in a range of engineering and scientific applications, it's critical to comprehend how these properties change with temperature and pressure. For instance, density is a measurement of a fluid's mass per unit volume. The density of a fluid normally decreases as temperature rises, while the density typically rises when pressure rises. This relationship is important to consider when modeling fluid flow in systems such as pipelines, where changes in temperature and pressure can have a significant impact on the density of the fluid and overall flow rate. In a Nano liquid flow through a flexible spinning disc, Mahanthes [48] studies the amplification of heat transmission caused by nanoparticles, magnetic fields, thermal, and exponential space-dependent heat source features. Mahanthes [49] investigates the nonlinear gravitational and radiation properties of a nanoliquid with an exponentially variable viscosity and space-dependent heat source.

2.13 Specific Heat

Specific heat is the amount of energy required to raise a unit mass of a fluid's temperature by one degree Celsius. The specific heat of a fluid normally declines with temperature, requiring less heat to raise the fluid's temperature. This relationship is crucial to take into account in applications like refrigeration systems, where the fluid's specific heat can significantly affect the system's overall efficiency. Other heat-space dependent characteristics exist in addition to these that, depending on the application, may be crucial to take into account. For example, the thermal expansion coefficient, which measures the change in volume of a fluid per unit change in temperature, can be important in applications such as pipelines, where changes in temperature can cause the pipe to expand or contract.[74] research A critical examination of hybrid nanofluids' specific heat capacity for thermal energy implementations

Overall, understanding how heat space dependent parameters in fluid dynamics can change with temperature and pressure is crucial for the accurate modeling and simulation of fluid flow in a variety of engineering and scientific applications. By considering these relationships, engineers and scientists can design more efficient and effective systems for a wide range of applications, from pipelines to heat exchangers and beyond. Yao [73] research experimental research was done on the impact of the particle mixture ratio on the specific thermal conductivity and fluid viscosity of Al_2O_3-ZnO hybrid nanofluids..

CHAPTER 3

PRELIMINARIES

This chapter presents some fundamental definitions of regulating laws and non - dimensional elements that will be employed in later chapters. Dimensionless quantities, which are used in subsequent chapters, are also discussed. A brief description of the shooting approach used to find the numerical solution has also taken place.

3.1 Some Basic Definition

3.1.1 Fluid (White) [72]

A material can exist in three fundamental stages, solids, liquids and gases (at extremely high temperatures, it can also exist as plasma). A liquid or gaseous substance is referred to as a fluid. The ability of a substance to endure an applied shear stress that tends to change its form controls the distribution of a solid and fluid.

3.1.2 Fluid Kinematics

Fluid kinematics is concerned with depicting fluid motion without taking forces into account and moments that generate the motion.

3.1.3 Fluid Mechanics [72]

The field of science referred to as fluid mechanics investigates how liquids or gases behave both at rest and while moving. Therefore, this field of study concentrates on the static, kinematic, and dynamic characteristics of fluids.

3.1.4 Fluid dynamics [72]

It is the study of the movement of liquids, gases, and plasma from one area to another. Fluid dynamics has various applications, such as estimating aero-plane forces and moments, analyzing oil mass flow rates in pipelines, forecasting weather, and more.

3.1.5 Hydrodynamics [62]

The study of the motion of almost compact liquids and gases, especially water, that are practically incompressible at low speeds, also known as hydrodynamics.

3.1.6 Magnetohydrodynamic (MHD) [61]

The key emphasis of Megnetohydrodynamics is the circulation of fluids that transmit electricity (MHD). Fluids subjected to the influence of a magnetic field, which can be applied externally or developed inside the fluid by inductive action.

3.2 Physical properties of the Fluid [72]

The fluid's physical properties are listed below.

3.2.1 Pressure [46]

The definition of pressure as it pertains to fluids is the quantity of force per unit area pushed in a direction perpendicular to the area. Mathematically

$$p = \frac{F}{A}. \tag{3.1}$$

3.2.2 Stress [46]

The stress or stress vector is defined as the force of surface per unit area. Mathematically it is expressed as

$$S = \frac{F}{A}. \tag{3.2}$$

3.2.3 Temperature [44]

The degree of hotness or coldness of a body is called temperature. Heat always moves from a higher to a lower temperature location. Temperature affects the physical state of a substance. Water, for example, contrary to steam, which has even greater temperatures, ice possesses temperatures.

3.2.4 Density [23]

It can be stated as mass per unit volume. If m is the mass contained in volume v and the density is the same at all points in the liquid, the density is said to be uniform.

$$\rho = \frac{m}{v}. \qquad (3.3)$$

3.2.5 Compressibility [9]

The degree to which a fluid's volume alters in response to outside forces is known as the fluid's compressibility. If a fluid's concentration changes due to change in pressure or temperature, it is said to be compressible otherwise, it is incompressible.

3.2.6 Viscosity [10]

Is a physical attribute of fluids related to the shearing destruction of fluid particles caused by load applied.

3.2.7 Kinematic Viscosity [10]

The kinematic viscosity of the fluid is designated by V_S, which is the ratio of absolute viscosity to density.

$$V_s = \frac{\mu}{\rho}. \qquad (3.4)$$

3.2.8 Dynamic Viscosity [10]

Dynamic viscosity also known as absolute viscosity is the extent to which a fluid's resistance tends to cause it to flow. The attractive forces between the fluid's molecules cause this resistance. The viscosity of liquids and gases is usually greater than zero. It is showed by μ and mathematically it can be expressed as

$$\mu = \frac{shear\ stress}{shear\ strain}. \qquad (3.5)$$

3.3 Types of fluid flow [72]

3.3.1 Compressible and Incompressible Flows

Compressible flow (or gas dynamics) is the branch of fluid mechanics that deals with flows having significant changes in fluid density. While all flows are compressible Incompressible flow implies that the density remains constant within a parcel of fluid that moves with the flow velocity.

3.3.2 Steady/Unsteady Flow

A steady flow is one in which the conditions (velocity, pressure and cross- section) may differ from point to point but do not change with time. unsteady: If at any point in the fluid, the conditions change with time, the flow is described as unsteady.

3.3.3 Laminar and Turbulent Flow

Laminar flow is characterized as having a smooth course for fluid particles. A further rise in speed may result in instability, creating the turbulent form of flow, which is more erratic.

3.3.4 Internal and External Flow

Unbounded fluid movement over a surface, such as a plate, is known as external flow, a wire or a pipe. Internal flow occurs when a fluid is completely confined in a pipe or duct by hard surfaces.

3.3.5 Natural and Force Flow

Force from the outside, such as a pump or fan, causes fluid to flow over a surface or through a tube. The heated fluid rises and the cooled fluid falls because there is no need for an external force to cause any fluid motion in nature. The movement of the wind is natural for the planet, yet it is pushed on bodies exposed to the wind. irrespective of the source of the air flow (fan or wind).

3.4 Types of Fluid

3.4.1 Ideal Fluid

A fluid without viscosity, turbulence, or heat transfer is an ideal fluid. It is a theoretical idea that is applied to mathematical models in fluid dynamics to make them simpler. Ideal fluids always flow in a straight line and have a constant density and incompressibility. They lack internal energy and entropy as well.

3.4.2 Real Fluid

Real fluid is a term used to describe a fluid with viscosity. The majority of the fluids are actual fluids.

3.4.3 Newtonian Fluid

A fluid is considered Newtonian if the rate of shear stress and strain are directly related.

3.4.4 Non-Newtonian Fluid

If a substance in which the rate of shear strain and shear stress are not equal (or velocity gradient). It can be expressed mathematically as

$$\boldsymbol{\tau} = k\left(\frac{\partial u}{\partial y}\right)^{n}. \qquad (3.6)$$

3.4.5 Shear Thickening Fluids [73]

A small class of actual liquids known as shear thickening fluids sees their velocity increasing as the shear rate rises. Corn starch and slime are two examples.

3.4.6 Nano fluids [9]

A fluid is called a Nano fluid if it contains nanoparticles which are particles smaller than a nanometer. Nano fluids are conventional heat transfer fluids containing solid nanoparticles.

3.5 Mechanism and Properties of Heat transfer [26]

3.5.1 Heat

Heat is term used to describe the transfer of energy from one medium to another due to temperature (or the surroundings). Energy analysis, like calories or joules are used to measure heat.

3.5.2 Heat Transfer

The area of engineering science devoted to discussing the movement of energy between materials is called heat transfer.

3.5.3 Natural Convection

When a fluid moves as a result of density changes brought on by temperature changes, this is known as free or natural convection.

3.5.4 Forced Convection

Forced convection occurs when fluid flow is induced by an external force, such as pumping or blowing.

3.5.5 Mixed Convection

Mixed convection combines aspects of both strapped and true convection.

3.6 Modes of Heat Transfer

Conduction, convection, and radiation are the three forms of heat transport.

3.6.1 Conduction

Conduction Heat transfer occurs as a result of an exchange of energy from one molecule to another without the molecules moving or as a result of the motion of free electrons if they are available. This sort of heat transmission occurs in solids, liquids, and gases.

3.6.2 Convection

Objects in liquids and gases have free motion, and they carry energy with them as they move from hot to cold regions. Heat transfer via convection is the transmission of heat from one location to another produced by such macroscopic motion in a liquid or gas, in addition to energy transfer by conduction inside the fluid.

3.6.3 Radiation

All matter emits thermal radiation regardless of its temperature. This is the only method of heat transmission that does not require a medium. Thermal radiation is defined as the transmission of energy from the body's surface via electromagnetic waves.

3.7 Important Definitions

3.7.1 Streamlines

A streamline is defined as the path in all directions that is perpendicular to the velocity behavior. The gradient of the streamline in two-dimensional flows must equal the tangent of the velocity angled with the x-axis.

3.7.2 Stream Function

Fluid dynamics research is facilitated by the stream function. To comprehend the flow pattern closest to a body, the stream function is frequently used to create streamlines. The stream functions are

$$u = \frac{\partial \psi}{\partial y}, \qquad (3.7)$$

$$v = -\frac{\partial \psi}{\partial x}. \qquad (3.8)$$

3.7.3 Viscous Dissipation

Viscous dissipation is the irreversible (in the thermodynamic sense) conversion in thermodynamic terms.

3.7.4 Thermal Conductivity [23]

Thermal conductivity refers to the ability of a given material to conduct/transfer heat. It is generally denoted by the symbol 'k' but can also be denoted by 'λ' and 'κ'

$$k = \frac{Qd}{A\delta T}. \qquad (3.9)$$

Where k is thermal conductivity, Q is the amount of heat transferred, d is the distance between two isothermal planes, A is the area of surface and δT is the difference in temperature.

3.7.5 Joule Heating [23]

Joule heating refers to the energy loss caused by an electric current flowing through a resistor.

3.7.6 Thermal Diffusivity [23]

It compares a material's ability to begin thermal energy to energy retention capacity, indicating how rapidly or easily heat can permeate an object or matter.

Mathematically

$$\alpha = \frac{k}{\rho C_p}. \qquad (3.10)$$

k is thermal conductivity is ρ density and C_p is specific heat.

3.7.7 Newton's Law of Viscosity

It defines the relation between shear stress and deformation rate of the fluid is proportional. Mathematically

$$\tau = \mu \frac{\partial u}{\partial y}. \qquad (3.11)$$

3.7.8 Boundary Layer [53]

Viscous effects are particularly significant near solid surfaces, where the intense interaction of fluid molecules with solid molecules causes the relative velocity of the fluid and the solid to approach zero for a stationary layer. Therefore, the velocity near the wall must decrease in order to skip. This is referred to as non-slip conditions. In the state being described, the fluid and solid surface are not moving relative to each other at their point of contact. This means that the velocity of the flow near the wall is zero and increases as distance from the wall increases, resulting in large velocity gradients close to the wall. This area is typically very thin and is known as the boundary layer. It is important to understand the characteristics of this boundary layer in order to accurately predict and analyze the flow of fluids in various systems. Additionally, the boundary layer plays a crucial role in heat and mass transfer between the fluid and the solid surface, as well as in determining the overall energy efficiency of a system. Understanding the behavior of the boundary layer is important for many engineering and industrial applications, such as in the design of aerodynamic surfaces or heat exchangers.

3.8 Laws of Conservation [53]

Modeling the issues of fluid dynamics use integral or differential representations of three conservation rules. These laws' integral formulations account for changes in mass, momentum, or energy inside the control volume T C[70].

3.8.1 Continuity Equation

This equation illustrates mass conservation. The equation below expresses mass conservation for any fluid. Expressed by the equation

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla}.(\rho \boldsymbol{V}) = 0.$$

For the steady flow above equation can be written as

$$\boldsymbol{\nabla}.(\rho \boldsymbol{V}) = 0.$$

The equation is rewritten in terms of velocity potential for incompressible and irrotational flows, which is given by

$$\boldsymbol{\nabla}^{\boldsymbol{2}}.\psi = 0, \qquad (3.12)$$

is known as the Laplace equation.

3.8.2 Momentum Equation [53]

A body's simple momentum is outlined as the product of its mass and velocity. According to Newton's second law a body's acceleration is proportional to the net force acting on it and inversely proportional to its mass, and the rank of momentum change, and net force is identical. As a result, when the net force is constant, a system's momentum remains constant. And thus, such systems' momentum is retained. The momentum of any fluid is defined as the formula is

$$\frac{\partial(\rho V)}{\partial T} + \nabla((\rho V)V) - \nabla . T - \rho g = 0.$$

Since

$T = -pI + \tau$, the above equation becomes

$$\rho\left(\frac{\partial V}{\partial t} + V . \nabla V\right) = \nabla . (-pI + \tau) + \rho g. \quad (3.13)$$

According to the momentum equation, the acceleration of a particle is the outcome of a net force, which is sent by the gradient of gravitational, viscous, and pressure forces.

3.8.3 Energy Equation [53]

A system's energy conservation can be conveyed in estimated form as follows

$$\rho C \frac{\partial T}{\partial t} = \nabla(K \nabla T) \pm \hat{H}_r. \quad (3.14)$$

3.9 Dimensionless Quantities

3.9.1 Prandtl Number $(\boldsymbol{P_r})$

This value represents the proportion of momentum diffusivity (viscosity) to thermal diffusivity. mathematically

$$P_r = \frac{v}{\alpha}, \tag{3.15}$$

where ν represents kinematic viscosity and α symbols for thermal diffusive heat transport.

3.9.2 Eckert Number $(\boldsymbol{E_c})$

It illustrates the transformation of kinetic energy to thermal energy ratio.
Mathematically

$$E_c = \frac{\omega_\infty^2}{Cp\Delta T}. \tag{3.16}$$

where C_p is the specific heat, w_∞^2 fluid velocity distant from the body, ΔT the temperature difference.

3.9.3 Biot Number $(\boldsymbol{B_i})$

As the impedance to heat transfer varies between the inside and exterior of a material. The ratio is identified as the Biot number. Mathematically it can be expressed as

$$B_i = \frac{hL}{K}, \tag{3.17}$$

here, h is convective heat transfer, L is the characteristic length and k is the fluid's thermal conductivity.

3.9.4 Nusselt Number (N_u)

It is the ratio between total heat transfer and heat transmission through conduction. It depicts convective heat transfer between a fluid and its surroundings, or the relationship between heat transfer intensity and temperature field in a flow boundary layer. Mathematically

$$N_u = \frac{\alpha H}{\lambda}, \tag{3.18}$$

where H is the characteristic length, α is the heat transfer coefficient, and λ is thermal conductivity.

3.9.5 Reynolds Number (Re)

The ratio between fluid inertia force and molecular friction force (viscosity). It determines the character of the flow (laminar, turbulent, or transient). Mathematically it can be written as

$$Re = \frac{\rho \nu L}{\mu}. \tag{3.19}$$

Where ρ is density of fluid, ν is the flow speed, L is the characteristic linear dimension and μ is dynamic viscosity of fluid.

3.9.6 Schmidt Number (S_c)

This represents the relationship between momentum diffusivity (viscosity) and mass diffusivity. It can be written as

$$S_c = \frac{V}{Dm}. \tag{3.20}$$

where ν is the kinematic viscosity and D_m is mass diffusivity

3.9.7 Sherwood Number S_h

Sherwood number is the ratio of total mass transfer to diffusive mass transport. Mathematically

$$S_h = \frac{\beta L}{D}, \tag{3.21}$$

where β is the mass transfer coefficient, L denotes the characteristic length and D stands for molecular diffusivity.

3.4 Shooting Method

In mathematical physics, the shooting method is a strategy for resolving boundary value issues (BVPs). A BVP is a form of differential equation in which the solution must satisfy constraints or boundary values, at two distinct points inside the domain. The shooting method is utilized to approximate solutions for BVPs that cannot be solved analytically. The core idea behind the shooting method is to change a boundary value problem (BVP) into an initial value problem (IVP) by introducing a new unknown variable, known as the "shooting parameter." This parameter is used to modify the initial conditions of the IVP in order for the solution to satisfy the boundary requirements of the BVP.

The shooting method consists of two main steps: Guess an initial value for the shooting parameter and use it to find the solution of the IVP. Compare the boundary value of the solution obtained in step 1 with the desired boundary value and adjust the shooting parameter accordingly.

Repeat the procedure until the solution's boundary value is sufficiently near to the required boundary value. Approximate solution to the BVP is the final value of the shooting parameter. The shooting approach is applicable to a broad range of BVPs, including those using ordinary differential equations (ODEs) and partial differential equations (PDEs). However, it is essential to emphasize that the shooting method is a numerical methodology and so can only yield approximate results. It is also sensitive to the initial guess of the shooting parameter and may not converge for certain BVPs. Overall, the shooting method is a useful technique for solving boundary value problems that cannot be solved analytically. It is widely used in mathematical physics and engineering and has many potential applications in various fields.

CHAPTER 4

RADIATION EFFECTS ON HYDRO MAGNETIC NEWTONIAN LIQUID FLOW DUE TO AN EXPONENTIAL STRETCHING SHEET

4.1 Introduction

This chapter is a review of research work presented by Kameswaran [38]. Hydromagnetic Newtonian liquid flow with the radiation effect on an exponentially stretching sheet has been studied. Using some similarities transformations, the conversion of PDEs to ODEs. A shooting method has been employed. for compiling results, and effects of distinct parameters on velocity, temperature and concentration profile has been discussed in detail.

4.2 Mathematical Formulation

Here we have considered the magnetohydrodynamic movement of a Newtonian fluid in two dimensions across a stretched sheet. The system commences with the sheet-pulling slit. The x-axis, which points in the direction of motion, follows the surface that is always stretching. The plate is parallel to the y-axis. It is claimed that the velocity of a sheet is an exponential function of the slit distance x. Temperature and concentration are believed to be far from the fluid $T_\infty$and C_∞, respectively. A variable-intensity magnetic field $B(x)$ is applied to the sheet and the temperature and concentration gradients between the sheet and the surrounding air were believed to be exponential functions of the distance x from the slit. Under the conventional boundary layer approximation, the momentum, heat, and mass transfer equations are affected by radiation and viscous dissipation.

In the context being described, the x and y components of velocity are represented by u and v, respectively. The electrical conductivity of the fluid is represented by σ, its kinematic viscosity is represented by v, its density is represented by ρ, the temperature of the fluid is denoted by T, and the concentration of the fluid is represented by C. Additionally, the radiative heat flux is given by q_r, and the thermal diffusivity, which is represented by α, is calculated as the ratio of thermal conductivity (K) to the product of fluid density (ρ) and specific heat at constant pressure (Cp).

It is also assumed that the fluid has a weak electrical conductivity, resulting in a small induced magnetic field. The radiative heat flux, q_r, is modeled using Rosseland's approximation, based on the surface and atmospheric conditions, and T_0, C_0 are positive constants. U_0 represents the characteristic velocity and L represents the characteristic length. Additionally, the magnetic field, $B(x)$, is assumed to take a form that supports a similarity solution.

$$q_r = -\frac{4\sigma^*}{3K^*}\frac{\partial T^4}{\partial y}, \tag{4.1}$$

here σ^* is Stefan-Boltzman constant and K^* is mean absorption coefficient. Assuming that there aren't any significant temperature variations within the flow, T^4 be described as a straight line with temperature.

$$T_4 = 4T_\infty{}^3 T - 3T_\infty{}^4. \tag{4.2}$$

4.3 Governing Equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{4.3}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \left(v\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2 u}{\rho}\right), \tag{4.4}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{v}{C_p}\left(\frac{\partial u}{\partial y}\right)^2 - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} + \frac{\sigma B^2}{\rho Cp}u^2, \tag{4.5}$$

$$u\frac{\partial c}{\partial x} + v\frac{\partial c}{\partial y} = D_B\frac{\partial^2 c}{\partial y^2}, \tag{4.6}$$

With boundary conditions

$$\left.\begin{pmatrix} u = U_w = U_0 e^{\frac{x}{l}},\ v = 0, \\ C = C_w = C_\infty + C_0 e^{\left(\frac{2x}{L}\right)} \\ T = T_w = T_\infty + T_0 e^{\left(\frac{2x}{L}\right)} \end{pmatrix}\right., \text{ at } y = 0, \tag{4.7}$$

$$u \to 0,\ T \to T_\infty,\ C = C_\infty,\ \ at\ y \to \infty. \tag{4.8}$$

Similarity transformations used in this study are

$$\left.\begin{array}{lll} u = U_0 e^{\frac{x}{l}} f'(\xi), & v = -\sqrt{\frac{U_0 \nu}{2l}}\, e^{\frac{x}{2l}}[f(\eta) + \eta f'(\eta)], & \eta = \sqrt{\frac{U_0}{2\nu l}}\, y e^{\frac{x}{2l}}, \\ \psi = \sqrt{2U_0 \nu l}\ e^{\frac{x}{2l}} f(\eta), & T = T_\infty + T_0 e^{\frac{2x}{l}} \theta(\eta), & C = C_\infty + C_0 e^{\frac{2x}{l}} \phi(\eta). \end{array}\right\}$$

Equation (4.1) satisfies identically after using the above-mentioned transformations. Equation (4.2) transforms into ODE mentioned below

$$f''' = Mf' - 2f'^2 - ff''. \tag{4.9}$$

The energy and concentration equations become

$$\theta'' = \left(\frac{1}{1 + \frac{4}{3}R}\right)\left(P_r\left(4f' - \theta' f\right) + P_r G_b f''^2 - MG_b f'^2\right), \tag{4.10}$$

$$\phi'' = S_c(4f'\phi - \phi' f). \tag{4.11}$$

Equational constants that are not dimensionless in (4.7) - (4.9) are

$$M = \frac{2\sigma B_0^2 L}{\rho U_0},\ K = \frac{4\sigma^* T_\infty^3}{K^* K},\ P_r = \frac{\rho C p}{K},\ G_b = \frac{U_0^2}{C_p T_0},\ S_c = \frac{v}{D}.$$

4.4 Solution Methodology

System of nonlinear PDEs $(4.3), (4.4), (4.5), (4.6)$ along boundary conditions are converted into first order ODEs. Then system of first order ODEs with respective boundary conditions are solved by using shooting method by adopting following process

$f = y_1,$

$f' = y_2,$

$f'' = y_3,$

$y_3 = \left(My_2 + 2{y_2}^2 - y_1 y_3\right),$ $\qquad y_3(0) = u_1,$

$\theta = y_4,$

$\theta' = y_5,$

$y_5' = (\frac{1}{1+\frac{4}{3}R})(P_r(4y_2 - y_2y_1) + P_rG_by_3^2 - MG_by_2^2),$

$\phi = y_6,$

$\phi' = y_7,$

$\phi'' = y_7' \; = \; -S_c(y_7y_1 - \; 4y_6y_2),$ $\qquad\qquad\qquad\qquad y_7(0) = 1.$

4.5 Physical Quantities of Interest

In heat and mass transfer problems, important engineering variables include the local Nusselt numberN_{ux}, the skin friction coefficient C_f, and the local Sherwood number Sh_x. These parameters represent the rate of heat transfer at the wall, the resistance to fluid flow at the surface, and the rate of mass transfer, respectively. They are commonly used to evaluate the performance of various systems and devices such as heat exchangers, boilers and condensers.

- **Skin friction Coefficient**

$$C_f = \frac{2\tau_w}{\rho u\, w^2}.$$

- **Shearing stress at the wall surface**

$$\tau \; = \; -u \left[\frac{\partial u}{\partial y}\right]_{y=0}.$$

- **Heat surface rate at the surface flux**

$$q_\omega = -k\left[\frac{\partial T}{q}\right]_{y=0},$$

$$\frac{Nu_x}{\sqrt{\mathrm{Re}\, x/2}\sqrt{\frac{x}{L}}} = -\theta(0).$$

- **Mass flux at the wall surface**

$$J\omega = -D\left(\frac{\partial C}{\partial y}\right)_{y=0},$$

$$J\omega = -D\,\frac{(C_w - C_\infty)}{\phi(\eta)}\,e^{\frac{-2x}{L}}e^{\frac{5x}{2L}}.$$

- **Sherwood Number is defined by**

$$Sh_x = \frac{x}{D}\frac{j_w}{C_w - C_\infty},$$

$$-\phi'(0)= \frac{Sh_x}{\sqrt{\frac{x}{L}}\sqrt{\mathrm{Re}\,x/2}}.$$

4.6 Results and Discussion

Solutions have been found for the effects of viscous dissipation and radiation on magnetohydrodynamic (MHD) flow across an exponentially stretched sheet through both computational and analytical means. The set of partial differential equations for flow, heat, and mass transport were transformed into a set of nonlinear ordinary differential equations by using similarity conversions. A zero-order approximation solution for the dimensionless stream function, f, was determined using an analytic method. The equations for the species and energy were solved using converging hypergeometric functions. The validity of the methodology was determined by comparing the analytical result with a numerical solution generated using a shooting approach that combines the Runge-Kutta and Newton-Raphson schemes. Tables (4.1 - 4.3) and graphs are provided to show the results of the study. Table 4.1 shows the skin friction coefficient at different magnetic parameter (M) values and the graphs illustrate the effects of viscous dissipation, magnetic parameter, and radiation parameter on velocity $f(\eta)$, concentration $\phi(\eta)$ and temperature $\theta(\eta)$profiles.

Table (4.2) demonstrates the impact of radiation, magnetic field and dissipation on the dimensionless wall temperature gradient. All three configurations demonstrably reduce the wall temperature gradient values. Whereas, in the table (4.3), in contrast to the influence of the magnetic parameter, an increase in the Schmidt number causes the dimensionless wall concentration gradient to rise. The results demonstrate a significant correlation between the analytical and numerical outcomes. In the absence of physical factors, the table displays the skin friction coefficients for various magnetic parameter values (i.e.,$P_r = S_c = K = G_b = 0$). We found that the friction coefficient increases as the magnetic parameter increases. It is crucial to remember that the wall skin-friction coefficient is the same for magnetic and nonmagnetic materials. ($M = 0$) and non-magnetic at ($M = 1$).

Figure (4.2) displays the velocity profile's fluctuation in respect to the magnetic parameter. In the boundary layer, the magnetic parameter decreases fluid velocity. This is because of the surge in Lorentz force, which corresponds to Darcy's drag for flow through a porous substrate. This opposing force decreases fluid mobility in the boundary layer. Figure (4.3) exhibits the influence of the magnetic parameter on the temperature distribution. As the value of the magnetic parameter increases, so does the thermal boundary layer thickness. Due to the opposing force of Lorentz drag, which amplifies frictional heating between fluid layers, heat energy is created. Consequently, causing the thermal boundary layer's thickness to increase. Figure (4.4) demonstrates the impact of the magnetic parameter on the concentration profile. It has been discovered that a rise in M induces a thickening of the species boundary layer. Figure (4.5) demonstrates the impact of the thermal radiation parameter on temperature K. Thermal radiation boosts the temperature in the area of the boundary layer without a doubt. In order to optimize cooling conditions, the level of radiation should be kept to a minimum. The radiation parameter determines the contribution of conduction heat transfer to thermal radiation transfer. K.

Figure (4.6) demonstrates the impact of the Gebhart number G_b on heat transportation. It is common knowledge that increasing the viscous dissipation parameter raises the boundary layer temperature. We also note that the factors impacting the energy equation are distinct from those affecting the equations for momentum and species conservation since the energy equation is somewhat decoupled from them. Gebhart number, the radiation parameter and Prandtl number are variables in the energy equation. have no effect on the velocity and concentration profiles. Tables 4.1 - 4.3 show a strong correlation between the analytic and statistical solutions.

4.7 Tables and Graphs

Table 4.1: An estimate of $-f''(0)$ obtained using the shooting approach and the analytical method for various M values.

M	$-f''(0)$
0.0	1.28180
1	1.62917
2	1.91262
3	2.15873
5	2.58113

Table 4.2: An examination of $\theta'(0)$for various values of M, G_b and K for $P_r = 7$.

K	G_b	M	$-\theta'(0)$
0.5	0.2	0.0	3.82250
0.5	0.2	1	3.48315
0.5	0.2	2	3.19113
0.5	0.2	3	2.92857
0.0	0.2	1	4.55621
0.5	0.2	1	3.48315

Table 4.3: A comparison of $-\phi(0)$ collected by the analytical method with the shooting technique for various values of M and S_c.

S_c	K	G_b	M	$\phi'(0)$
1.0	0.5	0.2	0.0	1.80568380
10	0.5	0.2	1	6.31856739
1.0	0.5	0.2	2	1.61139119
1.0	0.5	0.2	3	1.35849679
5.0	0.0	0.2	1	4.44825057
2.0	0.5	0.2	1	2.58904404

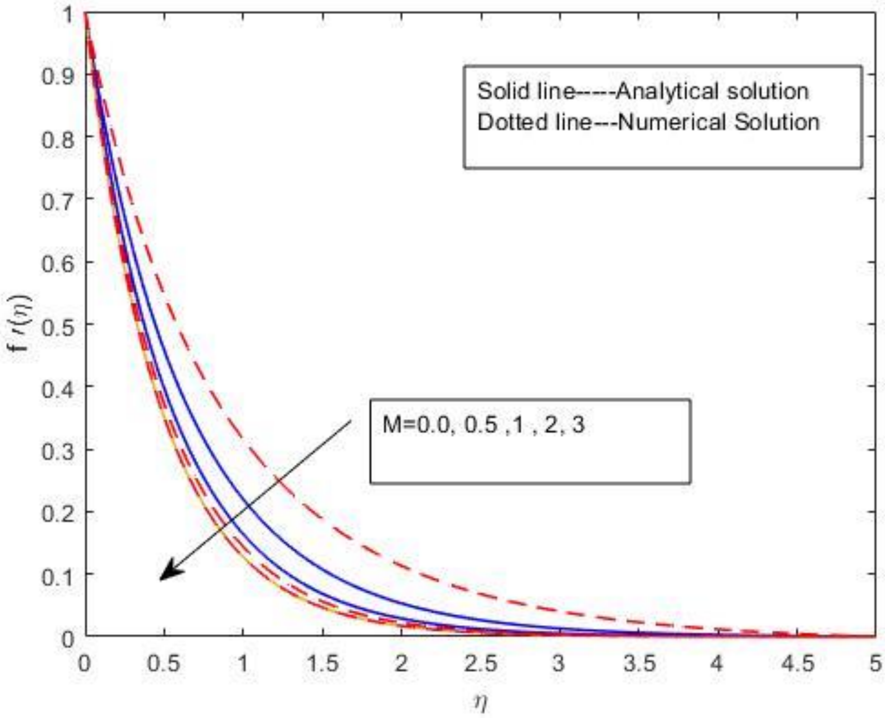

Figure 4.2: Impact of the magnetic parameter M.

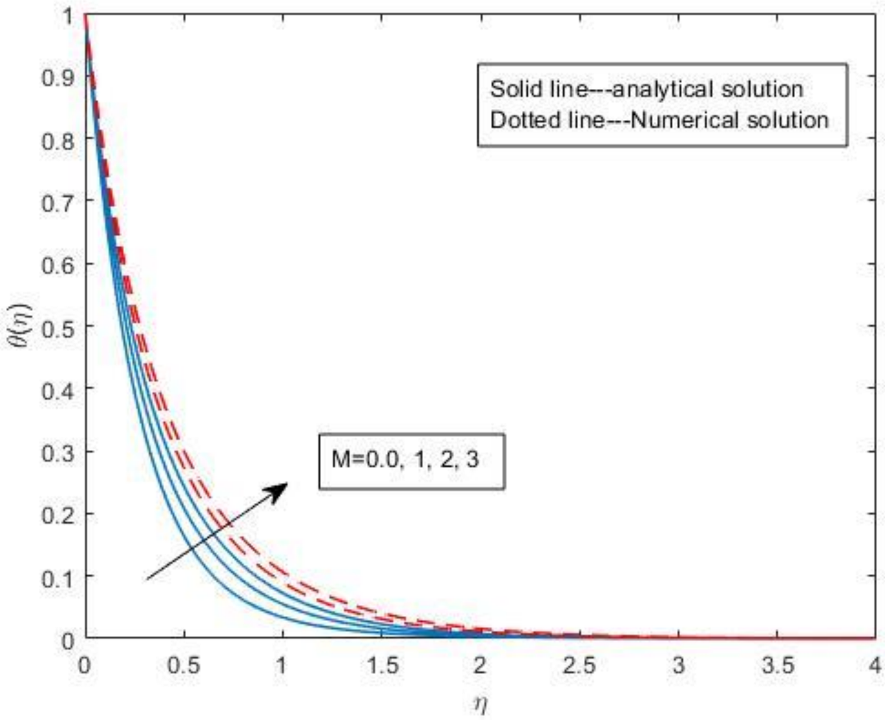

Figure 4.3: Effect of M on temperature.

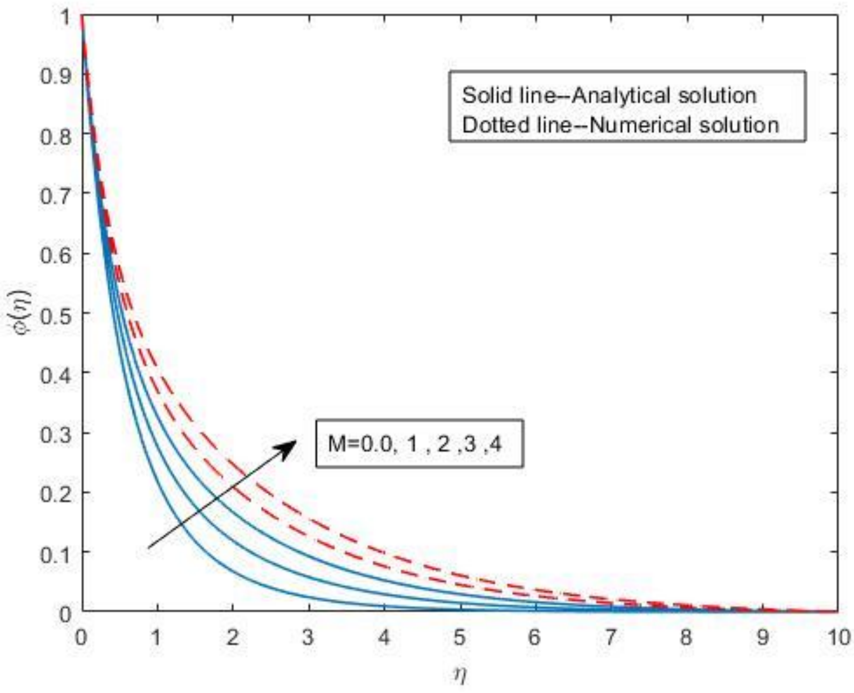

Figure 4.4: Impact of M on concentration.

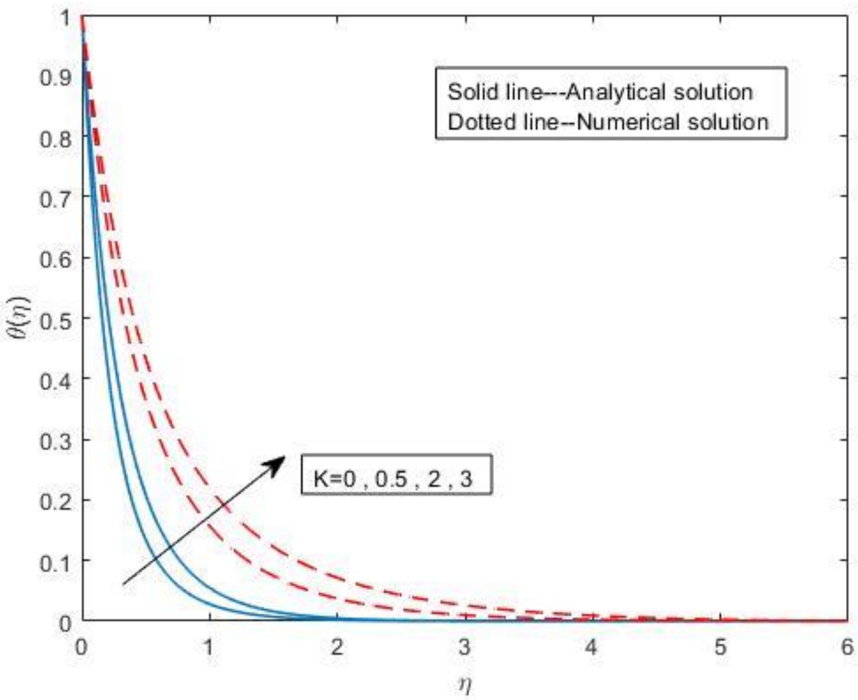

Figure 4.5: Impact of K on temperature profile.

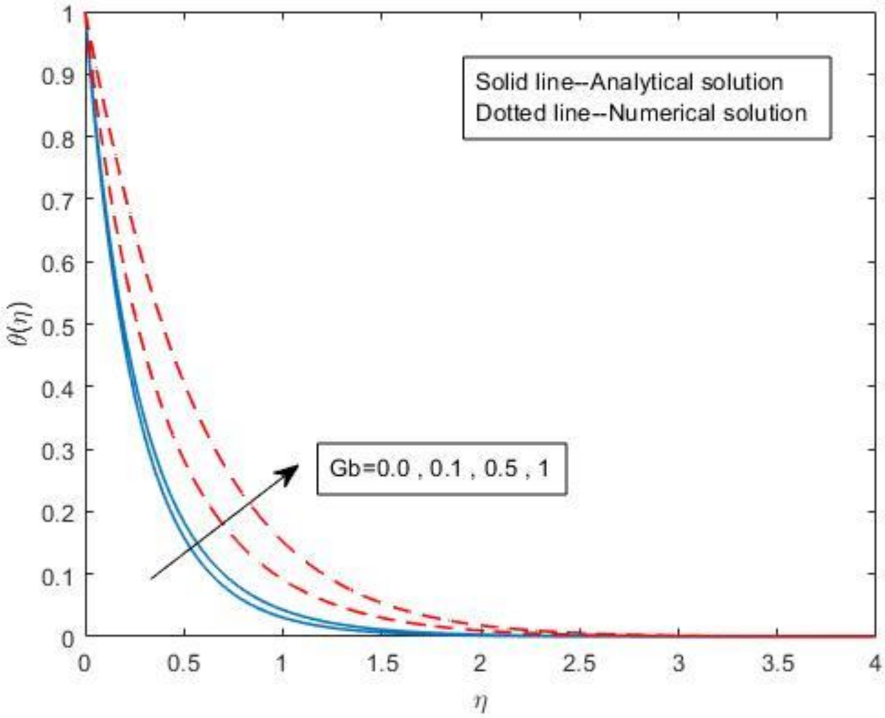

Figure 4.6: Impact of G_b on the temperature.

4.8 Conclusion

Hydromagnetic issues arise when radiation and viscous dissipation effects are present. Newtonian liquid flow caused by an exponentially stretching sheet has been studied. An analysis of the results of analytical and numerical methods revealed exacts solution in terms of hypergeometric functions.

- We found that in the boundary layer region, the magnetic parameter decreased the fluid’s velocity. Additionally, when the values of M rise, the boundary layer thickens. The mixed and separate impacts of the magnetic behavior M the radiation parameter K, along with the viscous dissipation parameter G_b increase heat transfer rates.
- In certain limiting conditions, when the parameters P_r, S_c, K, and G_b are all zero, the current findings are in good accord with those of earlier research.

CHAPTER 5

CHARACTERISTICS OF CHEMICALLY REACTIVE CASSON NANO FLUID FLOW WITH MIXED CONVECTION

5.1 Introduction

This chapter concerns the characteristics of chemically reacted Casson fluid flow with non-linear mix convection. Appropriate similarities transformations are used to convert PDEs to ODEs. The results have been investigated using ODEs and the shooting approach. Diverse parameter impacts according to velocity, temperature, and concentration depicted graphically and are analyzed in depth.

5.2 Mathematical formulation

Consider the two-dimensional Megnetohydrodynamics flow of a non-Newtonian fluid over an expanding sheet. The starting point of the system is located at the slit from which the sheet is being drawn. The x-axis, which is parallel to the surface of the continuously expanding sheet, is traced forward, and the y-axis is perpendicular to it. The assumption is that the velocity of the sheet increases exponentially as x, the distance from the slit, increases. It is also assumed that the temperature and concentration of the fluid are T_∞ and C_∞ respectively, far from the fluid and that the concentration and temperature differences between the sheet and the surrounding environment are represented by exponential functions of the x-distance from the slit. A variable magnetic field, $B(x)$, is applied normally to the sheet. The equations governing the transport of momentum, heat, and mass, taking into account radiation and viscous dissipation effects, are expressed under the standard boundary layer approximation.

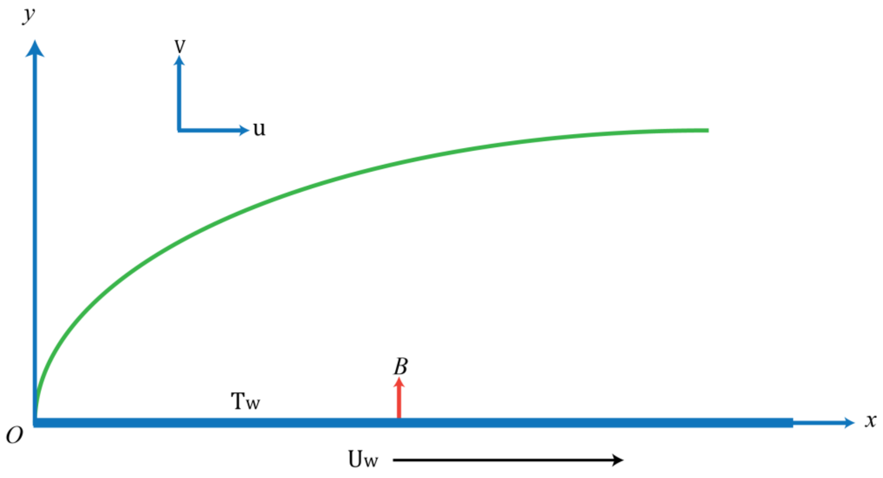

Figure 5.1: Schematics of the problem.

By considering the same assumptions as stated in Chapter 4,the model's governing equations are listed below. Magnetic field *B(x)* is believed to have the form to make it easier to find a similarity solution (4.1) and linear function of temperature (4.2) already used in chapter 4.

5.3 Governing Equations

Under the above stated assumptions, the governing equations of the problem takes the Following form

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{5.1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\begin{pmatrix} \nu\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2}+g(\beta_1(T-T_\infty)+\beta_2(T-T_\infty)^2) \\ +g(\beta_3(C-C_\infty)+\beta_4(C-C_\infty)^2)-\frac{\sigma B^2 u}{\rho} \end{pmatrix}, \tag{5.2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\begin{pmatrix} \alpha\frac{\partial^2 T}{\partial y^2}+\frac{\nu}{C_p}\left(1+\frac{1}{\beta}\right)\left(\frac{\partial u}{\partial y}\right)^2-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} \\ -\frac{1}{\rho C_p}Q_0(T-T_\infty)\exp[(-\frac{U_0}{2\,\nu L})^{\frac{1}{L}}e^{\frac{x}{2L}}] \\ +\tau\left(D_B\frac{\partial c}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right) \end{pmatrix}, \tag{5.3}$$

$$u\frac{\partial c}{\partial x}+v\frac{\partial c}{\partial y}=D_B\frac{\partial^2 c}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}-k_1(C-C_\infty)^n, \tag{5.4}$$

with boundary conditions are

$$\begin{pmatrix} u = U_w = U_0 e^{\frac{x}{l}},\ v = v_w = -V(x),\ T = T_w = T_\infty + T_0 e^{\left(\frac{x}{L}\right)}, \\ C = C_w = C_\infty + C_0 e^{\left(\frac{x}{L}\right)}, \end{pmatrix}, at\ y = 0, \quad (5.5)$$

$$u \to U_\infty = U_0 e^{\frac{x}{l}},\ T \to T_\infty,\ C = C_\infty,\ \ at\ y \to \infty. \quad (5.6)$$

Similarity transformation used in this research are given below.

$$\left.\begin{array}{lll} u = U_0 e^{\frac{x}{l}} f'(\xi), & v = -\sqrt{\frac{U_0 v}{2l}}\, e^{\frac{x}{2l}}[f(\xi) + \xi f'(\xi)], & \xi = \sqrt{\frac{U_0}{2vl}}\, y e^{\frac{x}{2l}}, \\ \psi = \sqrt{2U_0 vl}\, e^{\frac{x}{2l}} f(\xi), & T = T_\infty + T_0 e^{\frac{2x}{l}} \theta(\xi), & C = C_\infty + C_0 e^{\frac{2x}{l}} \phi(\xi). \end{array}\right\}$$

The continuity equation satisfies identically while the momentum, energy and concentration equations are as follows

$$f''' = \left(\frac{1}{1+\frac{1}{\beta}}\right)\begin{pmatrix} Mf' - \lambda\theta(1 + \beta_1\theta) - N\lambda\phi(1 + \beta_2\phi) \\ + 2f'^2 - ff'' \end{pmatrix}, \quad (5.7)$$

$$\theta'' = \left(\frac{1}{1+\frac{4}{3}R}\right)\begin{pmatrix} \mathrm{P_r}(4f' - \theta' f) + \left(\frac{1}{1+\frac{1}{\beta}}\right) P_r E_c f''^2 \\ -\mathrm{P_r}(\mathrm{N_b}\theta'^2 + \mathrm{N_t}\,\theta'\phi') - ME_c f + \mathrm{P_r}\,\delta\exp(-\eta)) \end{pmatrix}, \quad (5.8)$$

$$\phi'' = \gamma S_c \phi^n - S_c(4f'\phi - \phi' f) - \frac{N_t}{N_b}\theta''. \quad (5.9)$$

Boundary conditions in dimensionless form

$$f = S, \quad f'(0) = 1,\ f'(\infty) = 0,$$

$$\theta(0) = 1, \quad \theta(\infty) \to 0,$$

$$\phi(0) = 1, \quad \phi(\infty) \to 0.$$

5.4 Solution Methodology

System of nonlinear PDEs (5.1), (5.2), (5.3) and (5.4) along boundary conditions transmuted into first order ODEs. Then system of first order ODEs with respective boundary conditions is solved by using shooting method by adopting following process

$$f = y_1,$$

$$f' = y_2, \qquad y_1(0) = \mathrm{S},$$

$$f'' = y_3, \qquad y_2(0) = 1,$$

$$y_3 = \left(\frac{1}{1+\frac{1}{\beta}}\right)(My_2 - \lambda y_4(1 + \beta_1 y_4) - N\lambda y_6(1 + \beta_2 y_6) + 2y_2^2 - y_1 y_3),$$

$$\theta = y_4,$$

$$\theta' = y_5,$$

$$y'_5 = \left(\frac{1}{1+\frac{4}{3}}\right)(P_r(4y_2 - y_5 y_1) + \left(\frac{1}{1+\frac{1}{\beta}}\right)P_r E_c f_3^2 - P_r(N_b y_5^2 + N_t y_5 y_7) -$$

$$ME_c y_1 + P_r \delta \exp(-\eta)),$$

$$\phi = y_6,$$

$$\phi' = y_7,$$

$$y_7' = \lambda S_c\, y_6^n - S_c(4y_2 y_6 - y_7 y_1) - \frac{N_t}{N_b} y_5', \qquad y_7(0) = 1.$$

5.5 Results and Discussion

This portion provides a detailed analysis of the numerical data for the variables of velocity, Nusselt number, temperature, concentration, skin friction coefficient and Sherwood number with the aid of graphs and tables, taking into consideration a variety of physical factors.

Table 5.1: Change in $\left(1 + \frac{1}{\beta}\right) f''(0)$, $-\left(1 + \frac{4}{3R}\right) \theta'(0)$ and $-\phi'(0)$ for Newtonian Fluid.

M	λ	N	R	E_c	S_c	γ	P_r	δ	N_t	$-f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.5										1.417389	1.37483	0.676773
1.0										1.581594	1.31150	0.656152
1.5										1.730593	1.25455	0.641657
	0.1									1.417389	1.37483	0.766773
	0.5									1.238374	1.44605	0.707466
	1.0									1.033878	1.50790	0.732737
		0.1								1.417389	1.37483	0.766773
		0.5								1.396412	1.38676	0.686820
		1.0								1.396412	1.38676	0.682820
			0.1							1.417389	1.37483	0.766773
			0.5							1.411840	1.59951	0.859034
			1.0							1.407127	1.81368	0.985390
				0.2						1.417389	1.37483	0.676773
				0.5						1.416412	1.27602	0.747539
				1.0						1.414791	1.11207	0.865022
					0.6					1.417389	1.37483	0.676773
					1.0					1.418375	1.36592	1.258713
					1.5					1.419080	1.35899	1.824731
						0.5				1.417389	1.37483	0.676773
						1.0				1.417651	1.37259	0.800965
						1.5				1.417848	1.37083	0.908624
							0.7			1.417389	1.37483	0.676773
							1.0			1.422353	1.69243	0.466838
							1.5			1.427464	2.114989	0.1719489
								0.5		1.417294	1.374839	1.251085
								1.0		1.416049	1.156522	0.323635
									0.2	1.415959	1.360916	0.013233
									0.3	1.4145881	1.346798	-0.617434

Table 5.2: Change in $\left(1 + \frac{1}{\beta}\right) f''(0)$, $-\left(1 + \frac{4}{3R}\right) \theta'(0)$ and $-\phi'(0)$ for Casson fluid.

M	λ	N	R	E_c	S_c	γ	P_r	δ	N_b	$f''(0)$	$\theta'(0)$	$-\phi'(0)$
0.5										1.744167	1.432971	0.724923
1.0										1.944318	1.377018	0.706179
1.5										2.126035	1.325876	0.692168
	0.1									1.744167	1.432971	0.724931
	0.5									1.549534	1.483512	0.744262
	1.0									1.323357	1.531954	0.761760
		0.1								1.744167	1.432971	0.724923
		0.5								1.377583	1.681649	0.9042583
		1.0								1.731667	1.918981	1.0291780
			0.1							1.74167	1.432971	0.724923
			0.5							1.742689	1.319225	0.803417
			1.0							1.740236	1.130631	0.933675
				0.5						1.745348	1.424050	1.315251
				1.0						1.746151	1.417138	1.884945
					0.6					1.744167	1.432971	0.724923
					1.0					1.744427	1.431071	0.839396
					1.5					1.744632	1.429516	0.940916
						1.0				1.749722	1.749448	0.518567
						1.5				1.755201	2.172262	0.228567
							0.7			1.744167	1.432971	0.724923
							1.0			1.744092	1.322935	1.305886
							1.5			1.742645	1.198653	1.378761
								0.1		1.744167	1.432971	0.724923
								0.5		1.742445	1.417219	0.054242
								1.0		1.740793	1.401407	-0.582723
									0.1	1.417389	1.374839	0.766773
									0.2	1.415959	1.360916	0.013233
									0.3	1.4145881	1.346798	-0.617434

Table 5.3: Change in $\left(1+\frac{1}{\beta}\right)f''(0)$, $-\left(1+\frac{4}{3R}\right)\theta'(0)$ and $-\phi'(0)$ for Newtonian fluid.

s	$\left(1+\frac{1}{\beta}\right)f''(0)$	$-\left(1+\frac{4}{3R}\right)\theta'(0)$	$-\phi'(0)$
0.5	1.6716620	1.48363510	0.65529388
1.0	1.97176105	1.61409297	0.63089928
1.5	2.31383279	1.77002871	0.60792502

Table 5.4: Change in $\left(1+\frac{1}{\beta}\right)f''(0)$, $-\left(1+\frac{4}{3R}\right)\theta'(0)$ and $-\phi'(0)$ for Casson fluid.

s	$\left(1+\frac{1}{\beta}\right)f''(0)$	$-\left(1+\frac{4}{3R}\right)\theta'(0)$	$-\phi'(0)$
0.5	1.997018836	1.559708164	0.712757457
1.0	2.287470610	1.705075438	0.696978709
1.5	2.61329072	1.870607286	0.680491717

Increment in the Casson parameter results in the velocity decrease, temperature and concentration of fluid increased which can be observed in figures (5.2 - 5.4). It affects the viscosity of the fluid and causes resistance for the fluid flow. It is particularly important in determining the behavior of industrial fluids such as lubricants and polymers. The Casson parameter is affected by the concentration of particle suspended in a fluid and can have a significant impact on the viscosity of a fluid at high concentration, it is commonly used in suspension and emulsions.

We observed in figure (5.5), that in boundary layer region the magnetic field parameter increases fluid's velocity. The thickness of the thermal boundary layer grows as the magnetic parameter value increases shown in figure (5.6). It occurred because of an increase in Lorentz' force. Figure (5.7) shows a strong correlation between magnetic field and concentration, while increasing the value of magnetic parameter concentration profile rises gradually. In figure (5.8) we can see that velocity increases due to rise in buoyancy ratio parameter. Velocity profile goes on increasing due to non-linear mixed convection parameter. It shows the fluctuation in velocity profile f for different values of the mixed convection parameter. We investigated whether the velocity f has increased. We can observe that concentration decreases with increasing in mixed convection parameter by providing large λ values. It is also discovered that the mixed convective parameter influences f since the f curve steepens significantly.

Figure (5.9) portrays the impact of the non-linear mixed convention temperature variable on velocity profile. As the value of buoyancy ratio parameter N increases, so does the velocity profile f. The velocity field exhibits increasing features for N which can be seen in figure (5.10). Figure (5.11) shows the impact of non-linear mixed convection variable on velocity profile, due to increment in non-linear mixed convection variable velocity profile goes higher. Surge in the fluid's velocity is observed as the values of this parameter are increased.

Figure (5.12 - 5.13) interprets the impression of the radiation parameter R on the temperature and concentration profiles. It is possible to see an improvement in the temperature profile as the radiation parameter R is increased. This may be due to an increase in the thermal boundary layer's radiation parameter. In addition, a rise in the radiation parameter indicates that the temperature and concentration profiles have a stronger influence. comparing Casson fluid to Newtonian fluid. Figure (5.14 - 5.15) showed that temperature and concentration gradually decreased while increasing in heat generation parameter.

Close examination of figure (5.16) reveals that increasing the Eckert number influences the temperature profile. An increase in the Eckert number appears to raise the temperature of the fluid element above the sheet. Anyone can notice that rise in Eckert number caused in increasing temperature profile. Concentration decreased due to increment in Brownian motion which can be seen in figure (5.17) but this parameter increases the temperature shown in figure (5.18). Figures (5.19 - 5.20) show the impact of thermophores on temperature and concentration profile respectively. We can notice that due to higher values of thermophores temperature profile rises and concentration increases.

The impact of Prandtl number on temperature profile is demonstrated in figure (5.21). As Prandtl number P_r increases, the temperature of the fluid drops. The thermal boundary layer becomes thinner as the Prandtl number increases. As a result, increasing the Prandtl number should enhance the rate of heat diffusion.

By raising the value of the chemical reaction parameter and decreasing the velocity profile the influence of exponential space dependent heat generation parameter on temperature distribution is investigated in Figure (5.22). The viscous force caused by increasing S_c, which corresponds to a weaker concentration profile shown in figure (5.23). depicts the effect of a different number of S_c values on the concentration profile. The increased number of S_c caused the concentration profile curves to deteriorate. Due to suction parameter fluid close to the surface and graphical representation of suction is shown in figures $(5.24 - 5.26)$. We noticed that increasing in suction parameter velocity, temperature and concentration profile decreases.

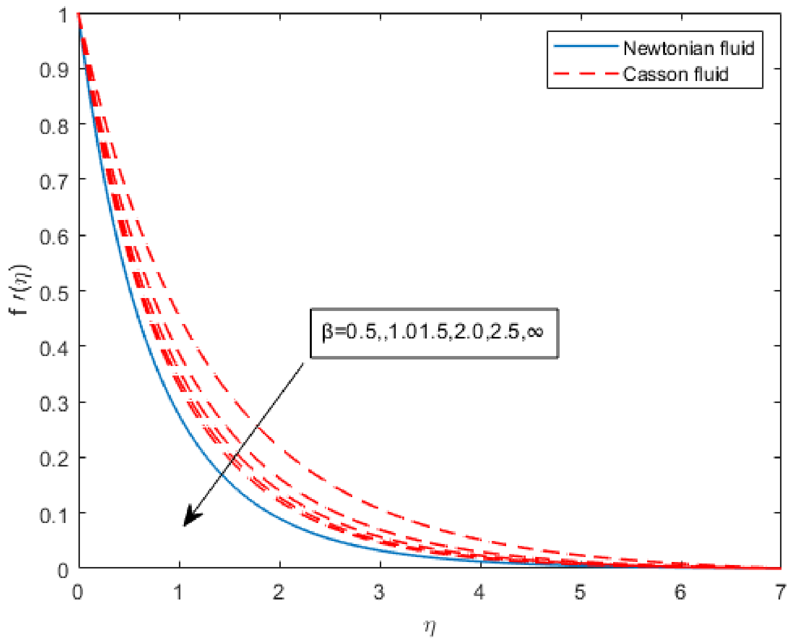

Fig 5.2: Significance of β on the velocity.

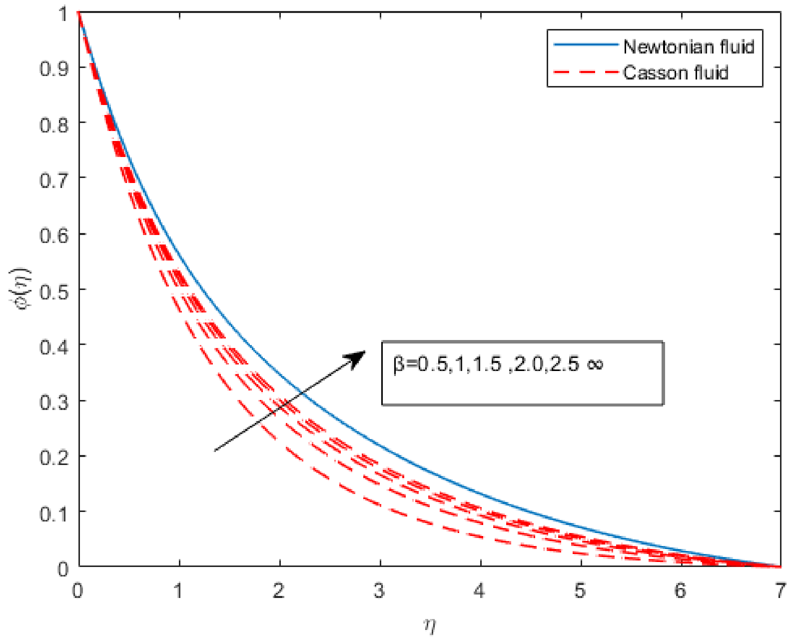

Fig 5.3: Significance of β on concentration.

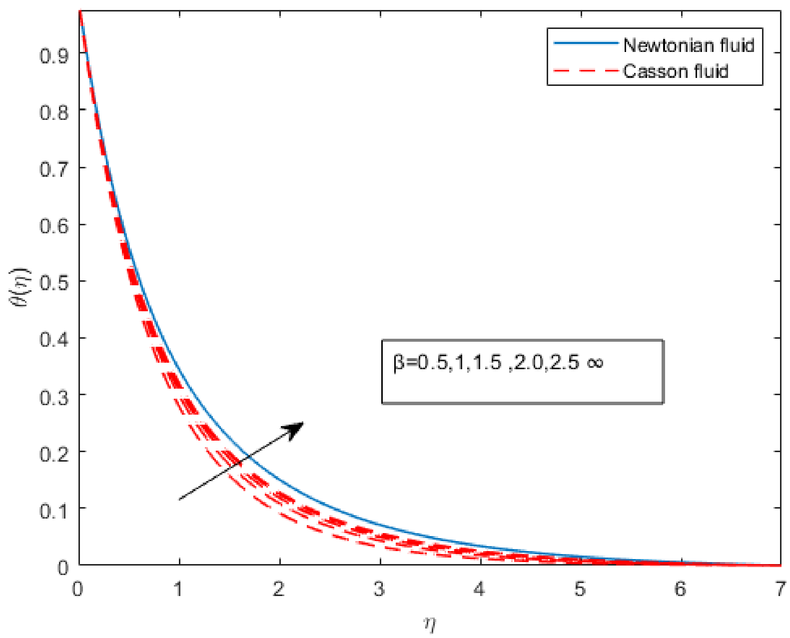

Fig 5.4: Significance of β on temperature.

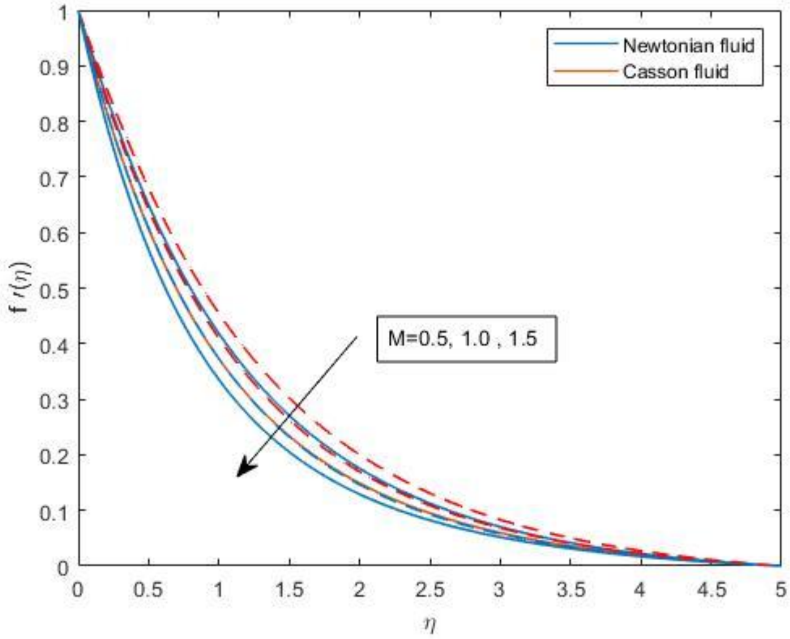

Fig 5.5: Significance of *M* on velocity.

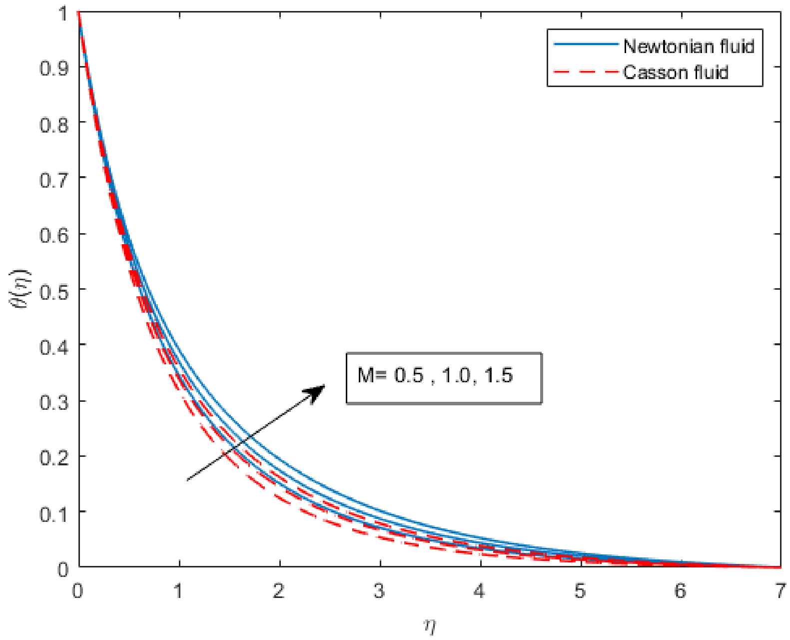

Fig 5.6: Significance of *M* on temperature.

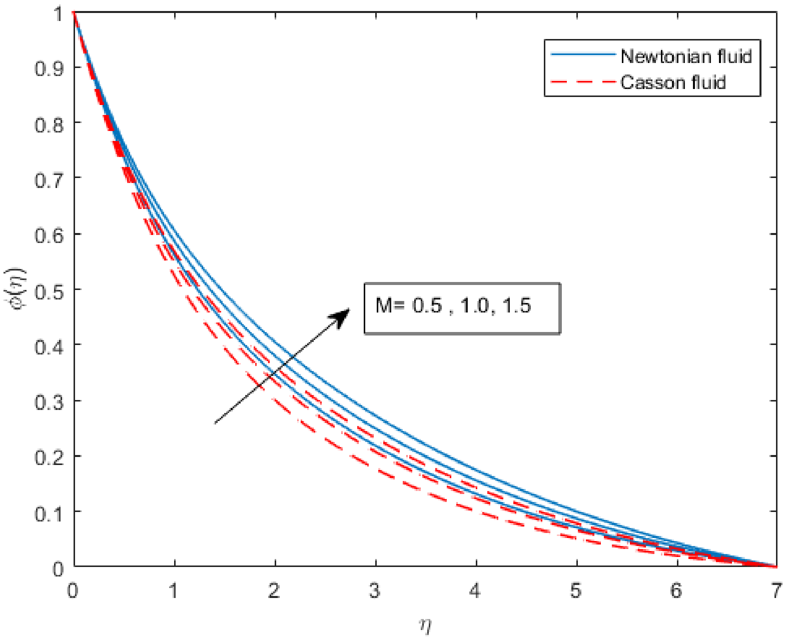

Fig 5.7: Significance of M on concentration.

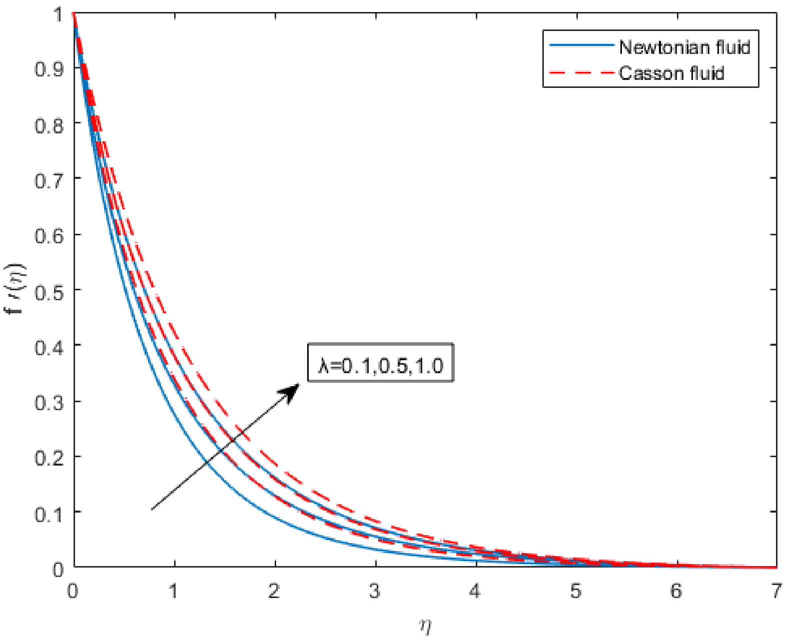

Fig 5.8: Significance of λ on velocity.

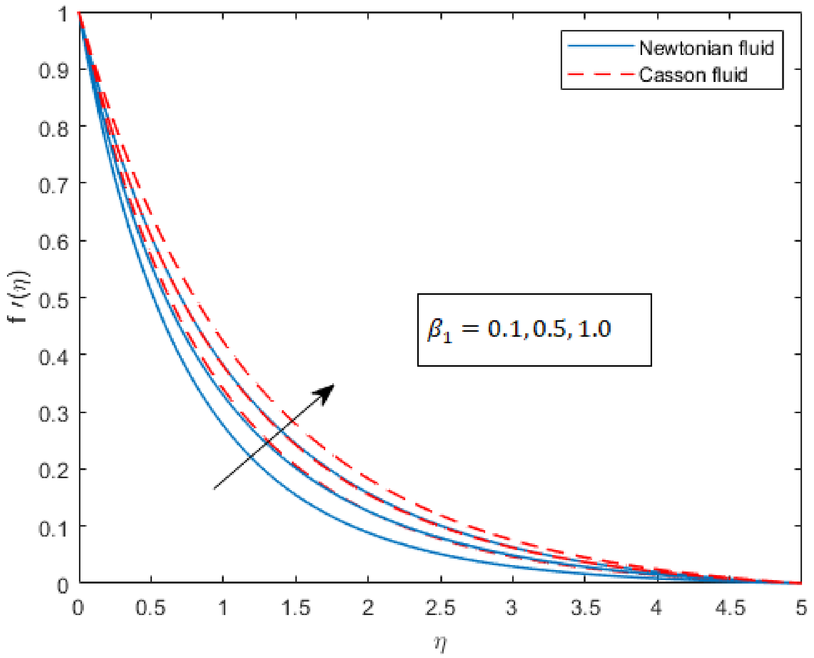

Fig 5.9: Significance of mixed temperature variable on velocity.

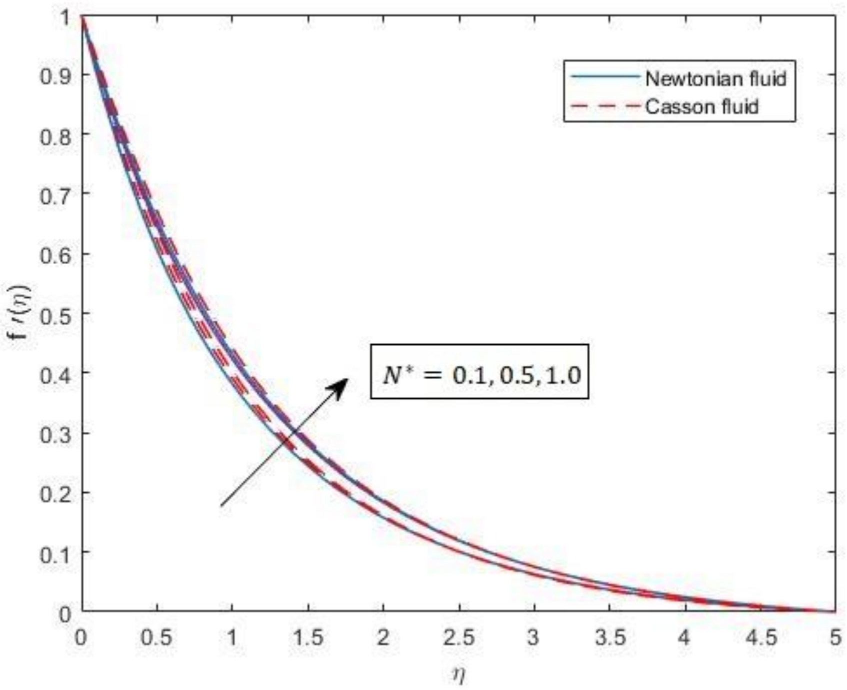

Fig 5.10: Significance of N on velocity.

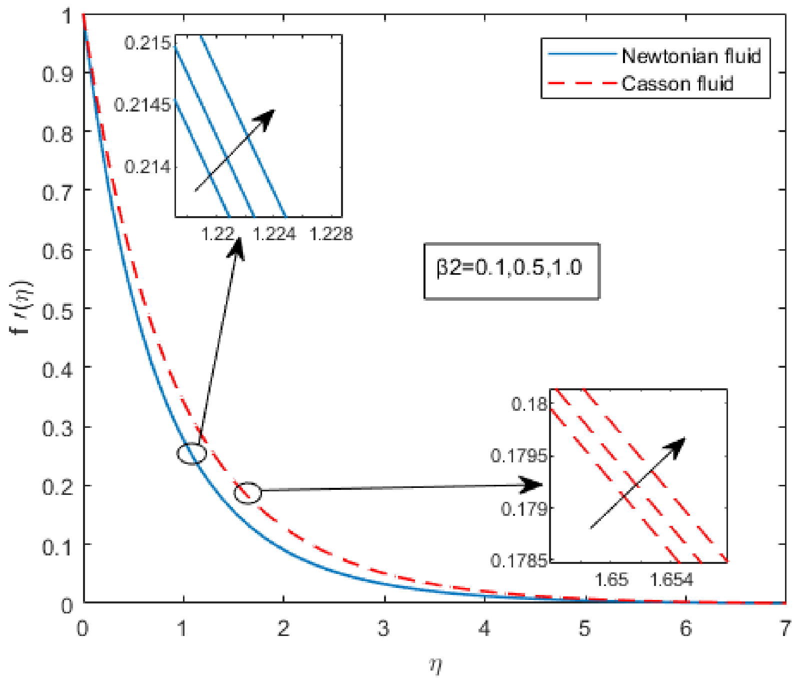

Fig 5.11: Significance of nonlinear mixed convection variable on velocity.

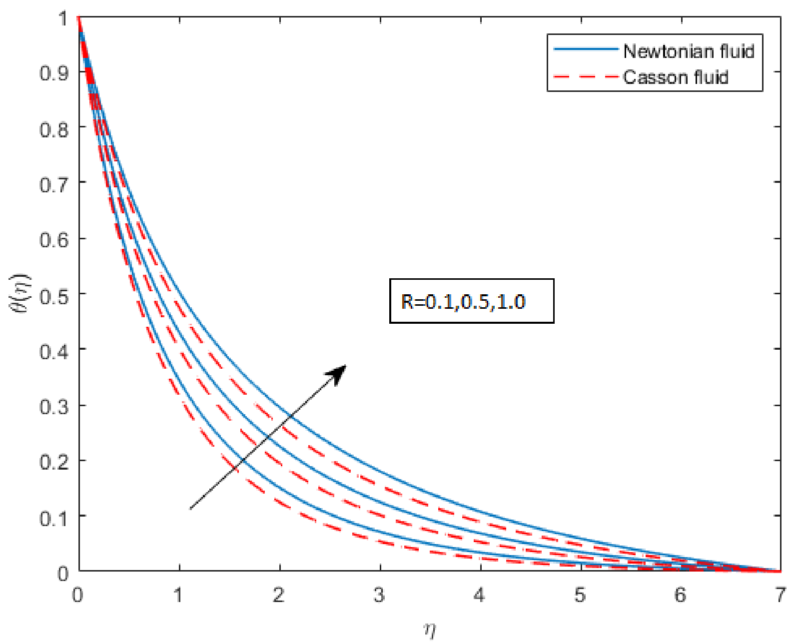

Fig 5.12: Significance of radiation parameter R on temperature.

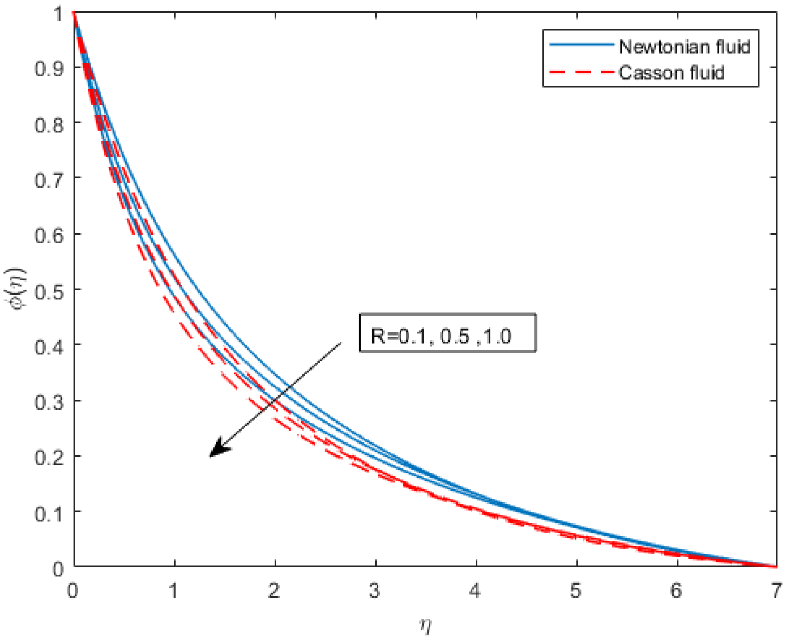

Fig 5.13: Significance of R on concentration.

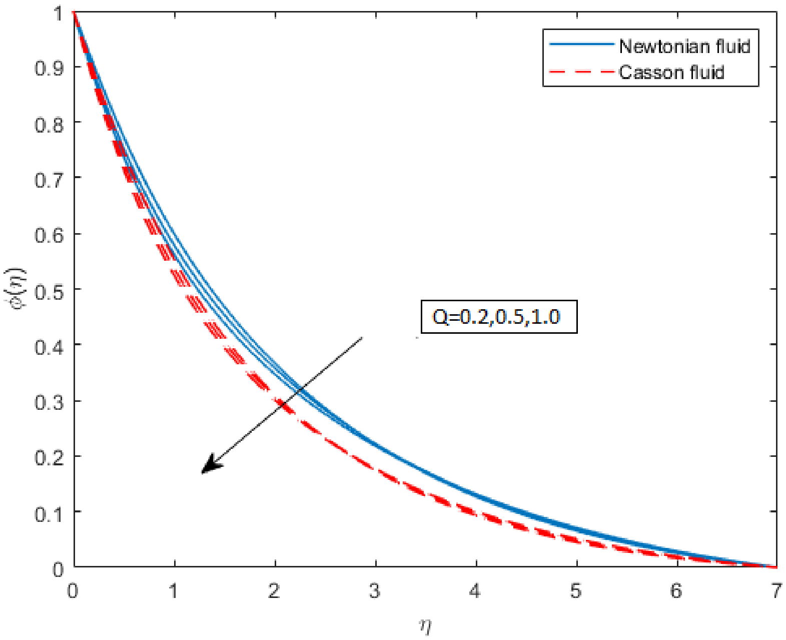

Fig 5.14: Significance of thermal heat parameter Q on concentration.

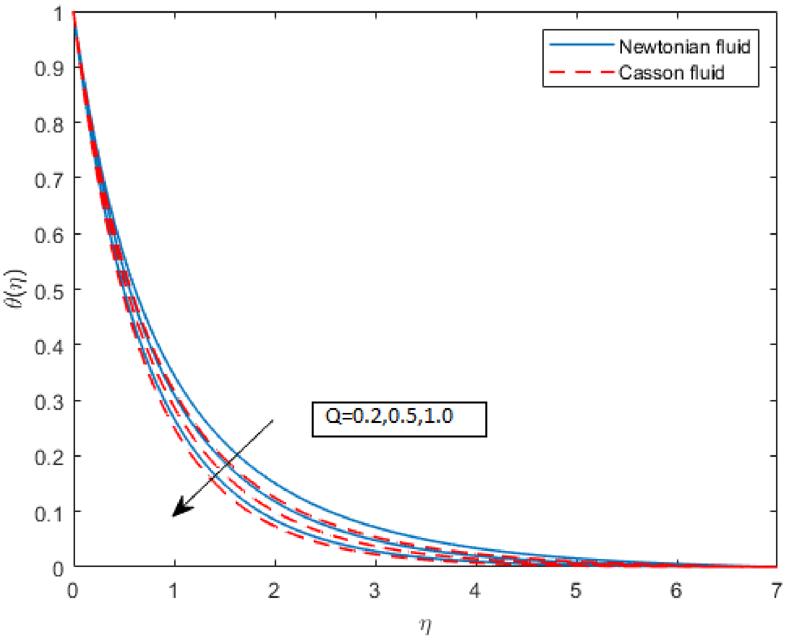

Fig 5.15: Significance of Q on temperature.

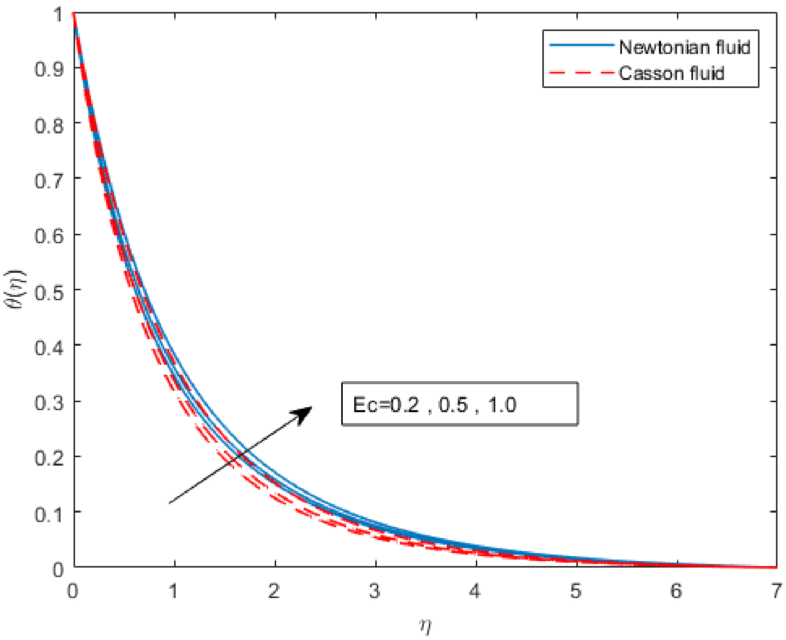

Fig 5.16: Significance of E_c on temperature.

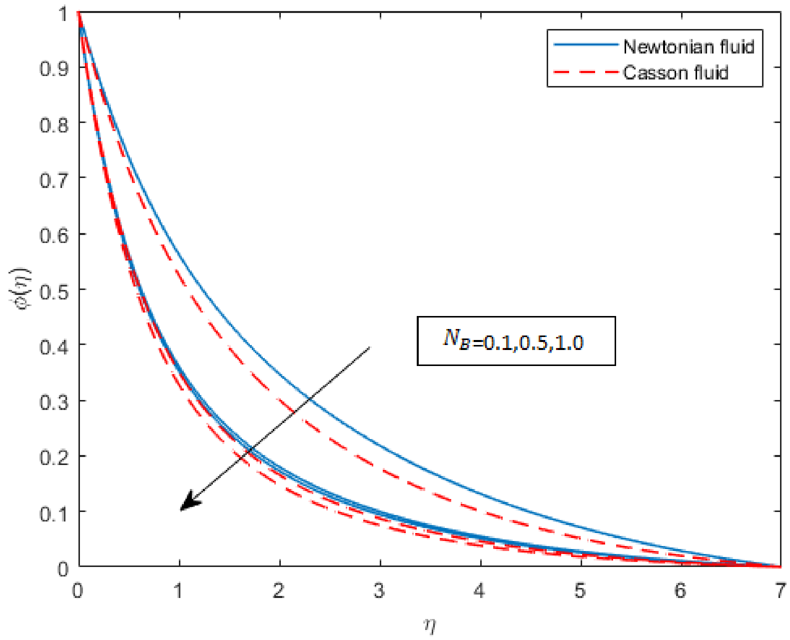

Fig 5.17: Significance of N_B on concentration.

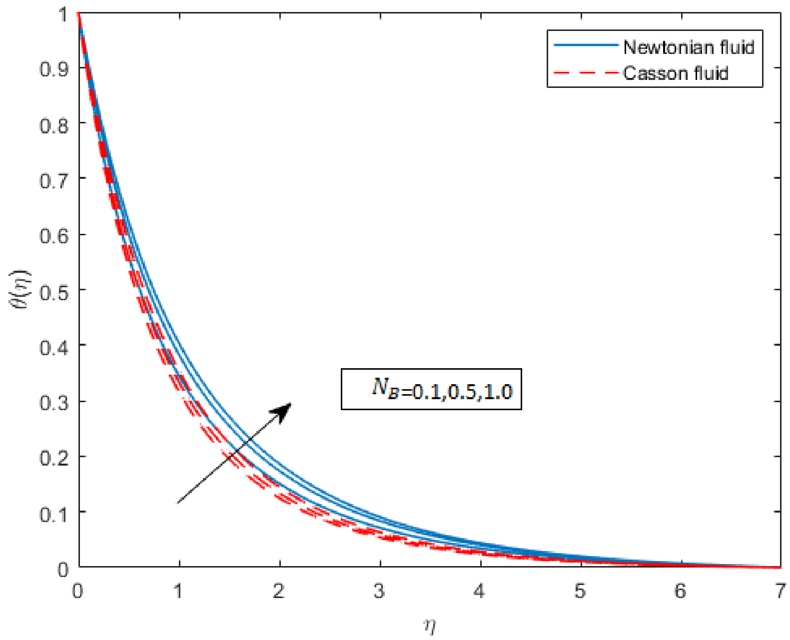

Fig 5.18: Significance of N_B on temperature.

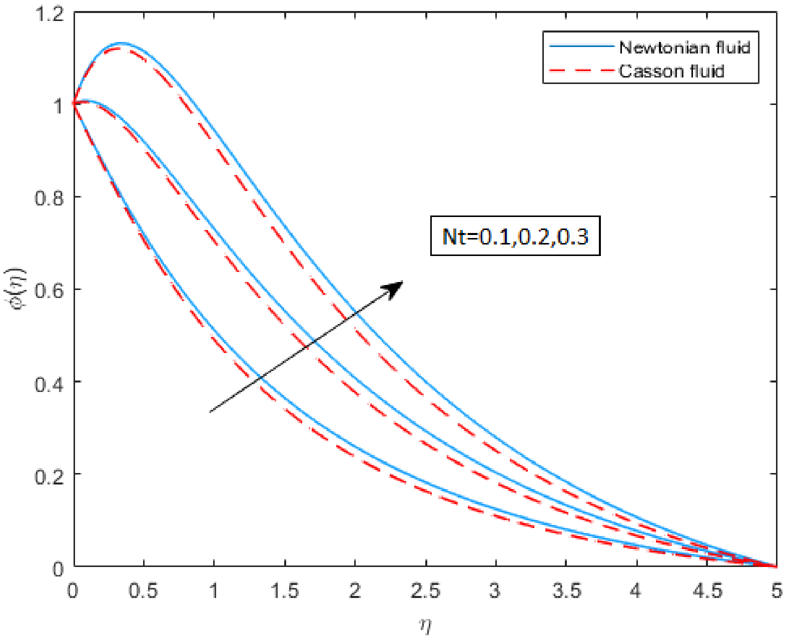

Fig 5.19: Significance of N_T on concentration.

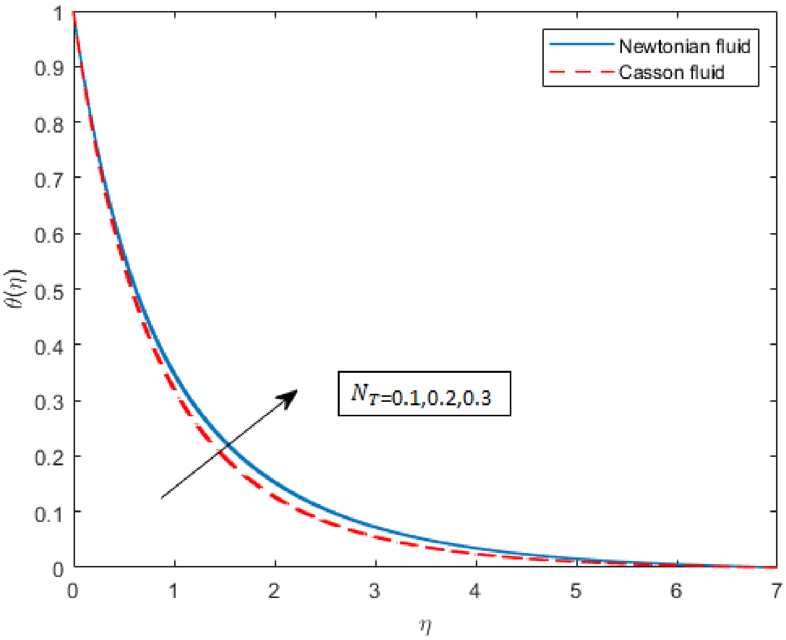

Fig 5.20: Significance of N_T on temperature.

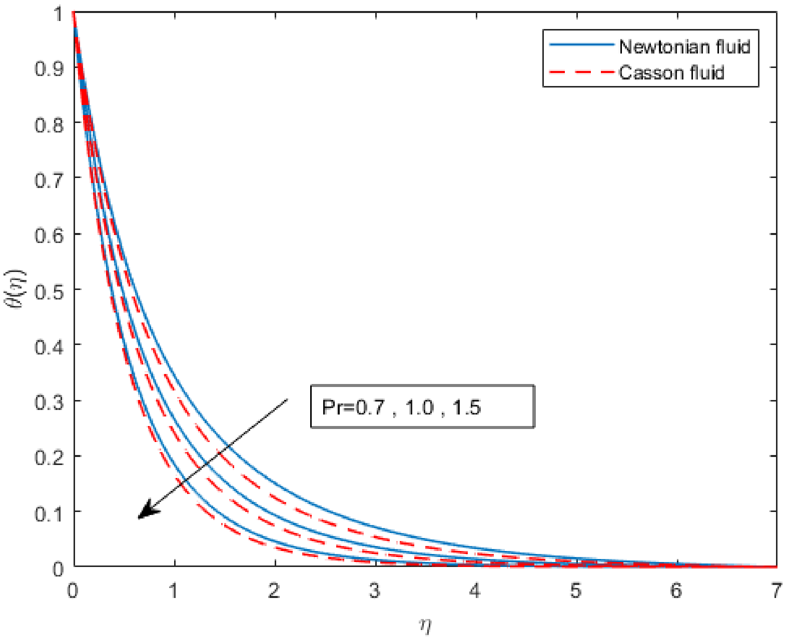

Fig 5.21: Significance of P_r on temperature.

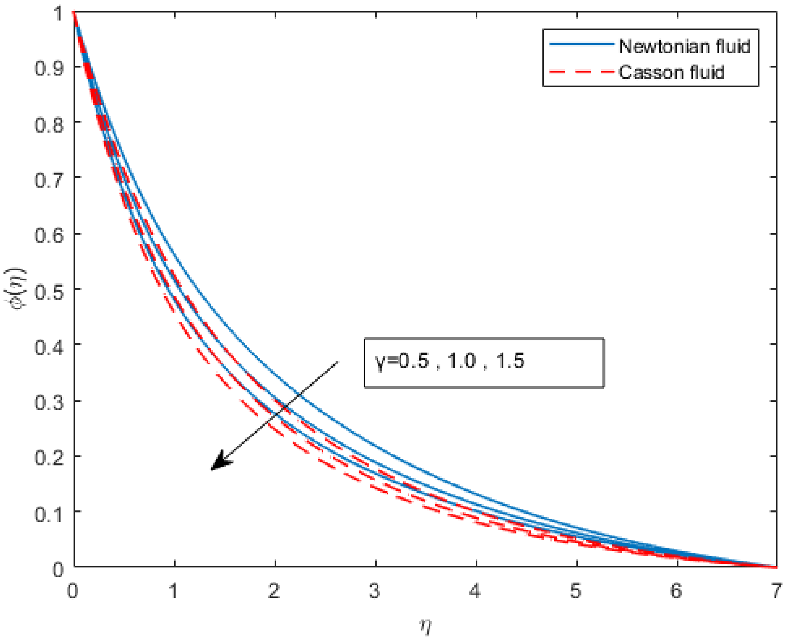

Fig 5.22: Significance of chemical reaction parameter γ on concentration.

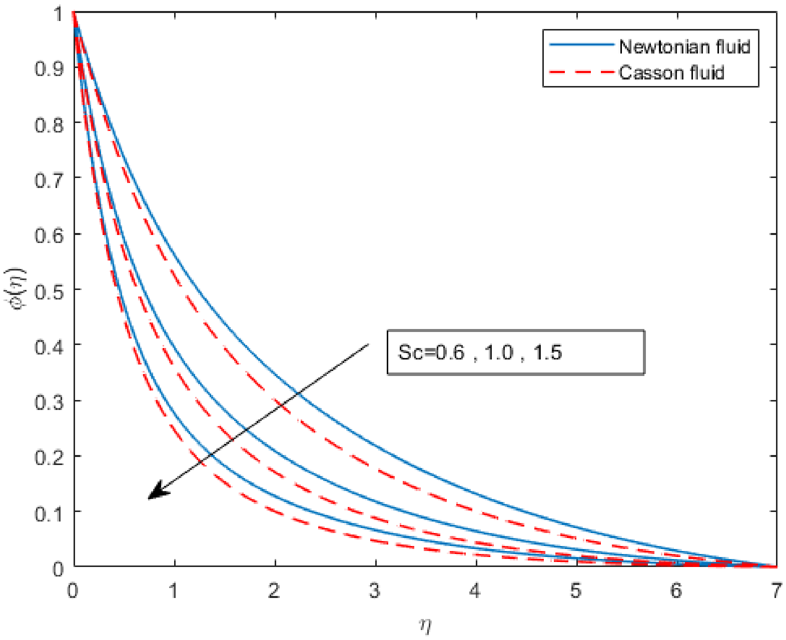

Fig 5.23: Significance of S_c on concentration.

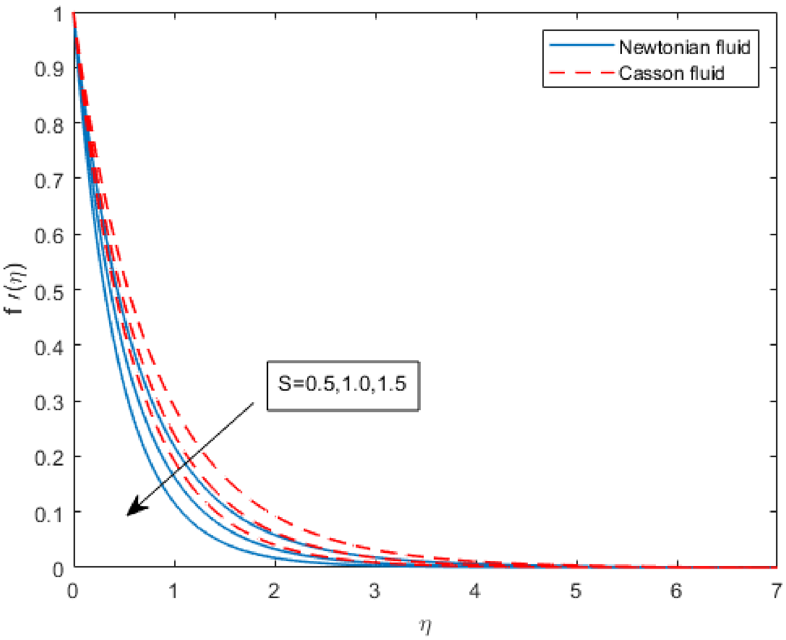

Fig 5.24: Significance of S on velocity.

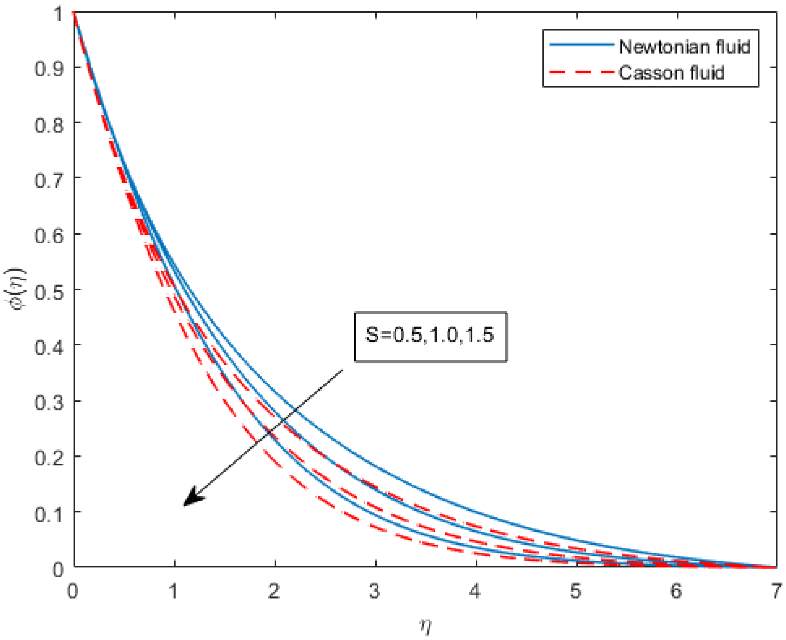

Fig 5.25: Significance of S on concentration.

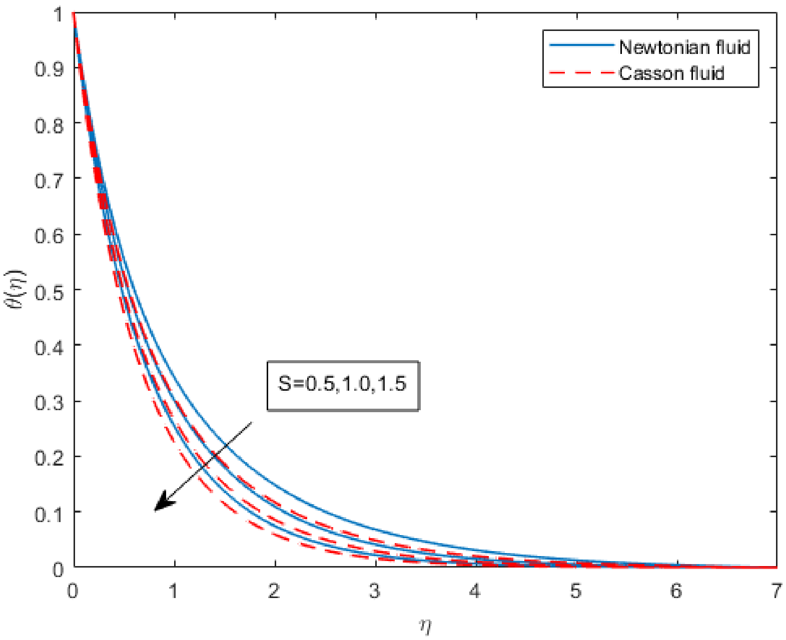

Fig 5.26: Significance of S on temperature.

Chapter 6

6.1 Conclusion

In this thesis, we have investigated the numerical analysis of the flow of Casson Nano fluid by taking into account the effects of non-linear mixed convection and chemical reactions in the presence of a magnetic field. Various physical factors, such as Eckert number, Prandtl number, magnetic field, chemical reaction, radiation, viscous dissipation, and non-linear mixed convection parameter, were considered for the Casson Nano-fluid. We transformed the nonlinear partial differential equations (PDEs) of concentration, momentum, and temperature into a set of ordinary differential equations (ODEs) using the similarity transformation. The numerical results were obtained using the Shooting method in combination with the boundary value problem solver bvp4c. The effect of various physical parameters on the velocity, energy, and concentration distributions was studied in depth using graphs and tables.

6.2 Significant Results

The following summarizes the general conclusion gained from the current work.

Due to the increase in values of Magnetic field parameter M and Casson fluid parameterβ, the momentum boundary layer becomes thinner. We observed that temperature is a decreasing due to increment in values of Prandtl number P_r number and it rises for Casson fluid parameter , Radiation parameter, Eckert number , internal heat source, Brownian motion, and thermophoresis parameter respectively β , $R, E_c,$ δ, N_b and N_t.

In the present study we observed that when we increae the values of Casson fluid parameter β, Magnetic field parameter M and thermophoresis parameter N_t the volume friction of nanoparticles is increased. The liquid velocity increases with increasing the values of chemical reaction parameter *N* and non-linear mixed convection parameter λ.

The Prandtl number can be utilized to accelerate cooling in conducting flows and surface shear stress increases as the Casson parameter is increased. When Variable heat source increase it increase fluid temperature. The fluid temperature near the wall is projected to exceed the wall temperature for non-zero Eckert numbers if heat transfer is received from the fluid to the sheet. The magnitude of the skin friction factor and the heat transport rate are greater in the case of changeable viscosity than in the case of constant viscosity.

Nano particle volume pressure improves heat transfer rate. Decreasing behavior occurs when the Casson fluid parameter is increased, and the same evidence is found in the velocity distribution when the numerical value of the suction parameter is increased. The velocity field decreases when the magnetic parameter is increased, yet the temperature and concentration distribution graph show the reverse tendency. Because of the increased value of the Prandtl number, the temperature distribution slows, and the concentration distribution speeds up.

When the radiation parameter is increased, the temperature profile rises, and similar behavior is evident in the temperature field owing to the action of the thermophoresis parameter. When the value of the inclined magnetic field increases, the velocity profile increases while the temperature and concentration distribution drops. Decreasing behavior observed in velocity, concentration and temperature profile when values of suction parameter increased.

6.2 Future Work

The problem can be possibly expanded by assuming the different fluid models such as Williamson, Burger, Jeffery and Maxwell hyperbolic nanofluid. Other effects such as actiavation of heat source, magnetic field, porous medium and many others can be used to investigate the situation. We may solve the above metioned fluid model by using different techniques and similarity transformations.

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